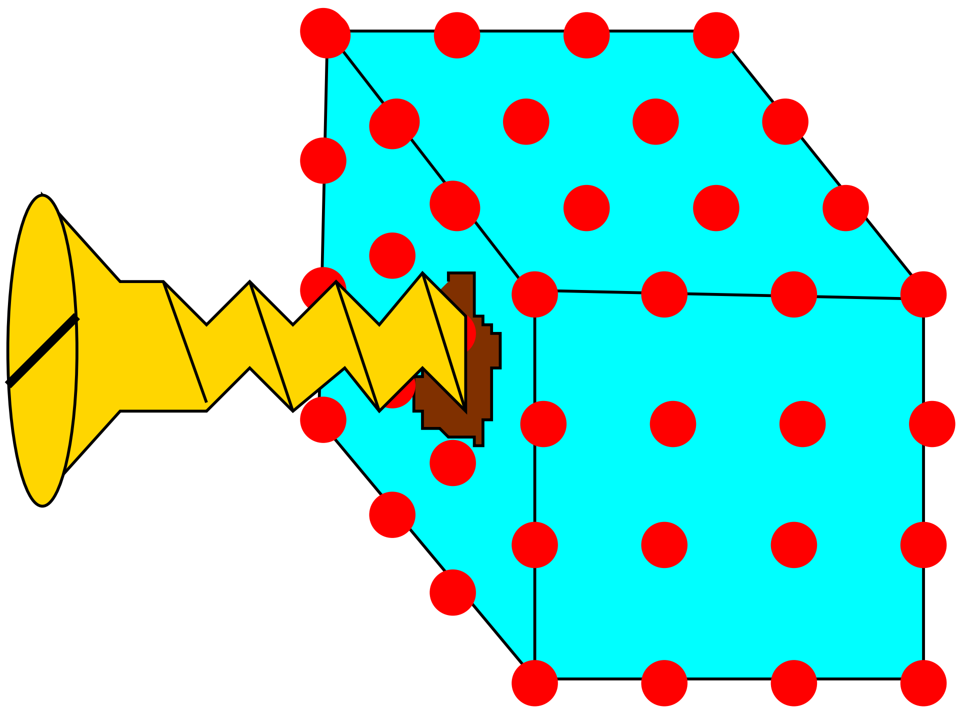
Lecture IV: Chiral controversies

Michael Creutz

BNL



Four closely related questions:

- Could $m_u = 0$ have any fundamental meaning?
- Is topological susceptibility a physical observable?
- Is $\overline{MS}$ valid outside of perturbation theory?
- Do rooted staggered fermions make sense?

The answer to all four is NO!

- tied to gauge field topology and the chiral anomaly

Some background

Consider two flavor QCD with light but non-degenerate masses

- pseudoscalar operators

$$\overline{u}\gamma_5 u$$

$$\overline{d}\gamma_5 d$$

$$\overline{u}\gamma_5 d \sim \pi_+$$

$$\overline{d}\gamma_5 u \sim \pi_-$$

Helicity conservation naively suggests mixing of

$$\overline{u}\gamma_5 u = \overline{u}_L\gamma_5 u_R + \overline{u}_R\gamma_5 u_L$$

- with

$$\overline{d}\gamma_5 d = \overline{d}_L\gamma_5 d_R + \overline{d}_R\gamma_5 d_L$$

- suppressed by $m_u m_d$

Wrong: the anomaly couples u and d through $F\tilde{F}$

- strongly mixes $\overline{u}\gamma_5 u$ and $\overline{d}\gamma_5 d$

  topology induces the effective "t'Hooft vertex"

- physical $\eta' \sim \overline{u}\gamma_5 u + \overline{d}\gamma_5 d$ not a pseudo-Goldstone boson

  $M_{\eta'} \sim \Lambda_{qcd} + O(m_u + m_d)$

  η' also contains glue: $F_{\mu\nu}\tilde{F}_{\mu\nu}$

Leaves the orthogonal combination $\pi_0 \sim \overline{u}\gamma_5 u - \overline{d}\gamma_5 d$

$M_{\pi_0}^2 \sim \frac{m_u+m_d}{2}$

- isospin breaking suppressed to higher order

  $M_{\pi_0}^2 = M_{\pi_\pm}^2 - O((m_u - m_d)^2)$

Fix m_d , vary m_u

$$M_\pi^2 \propto \frac{m_u+m_d}{2} + O(m_q^2)$$

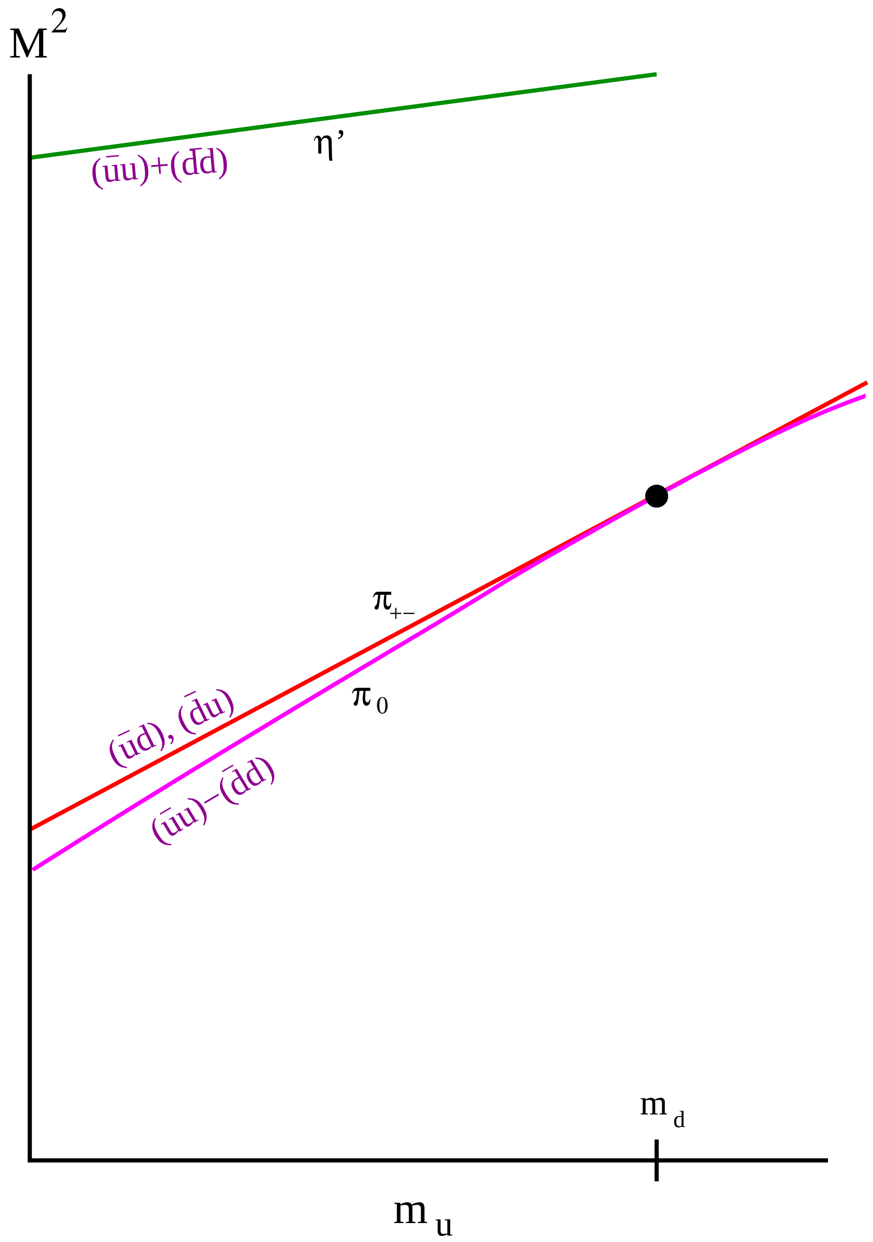
$$M_{\eta'} \sim \Lambda_{qcd}$$

With isospin broken

$$M_{\pi_\pm}^2 - M_{\pi_0}^2 \propto (m_d - m_u)^2$$

η', π_0, glueballs all mix

Mass gap survives at $m_u = 0$



The Dashen phenomenon

No singularity at $m_u = 0$

- extrapolate to negative m_u
- $M^2_{\pi_0}$ can go negative
- pion condensate forms

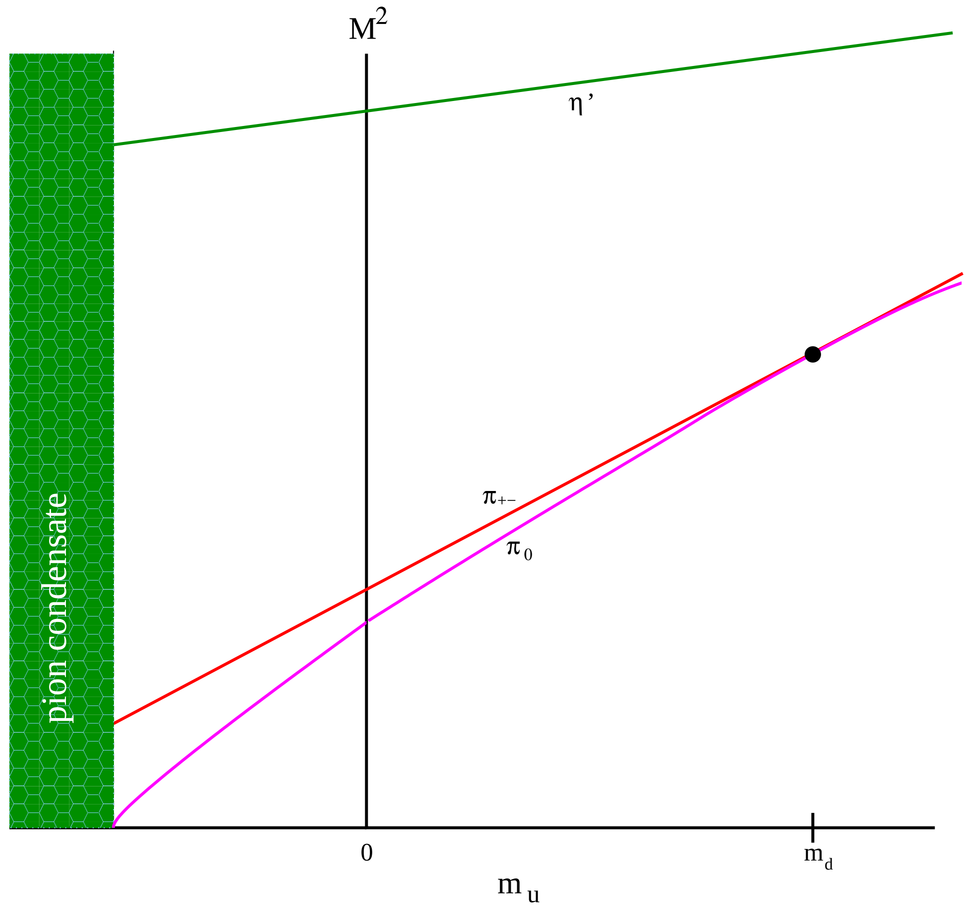
  $\langle \pi_0 \rangle \neq 0$

  CP broken

- formally at $\Theta = \pi$

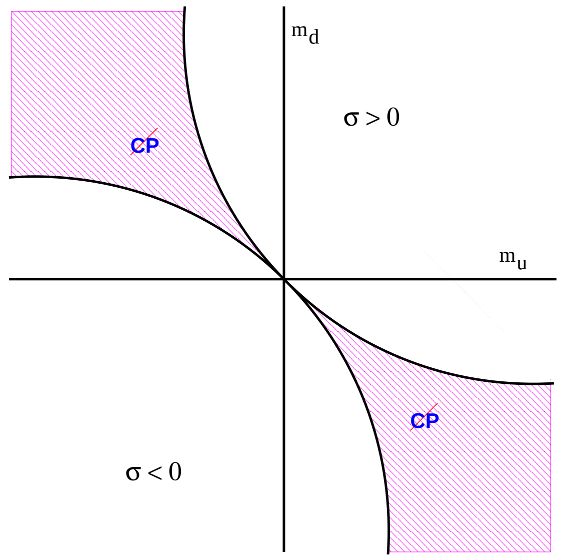
  $\prod_q m_q < 0$

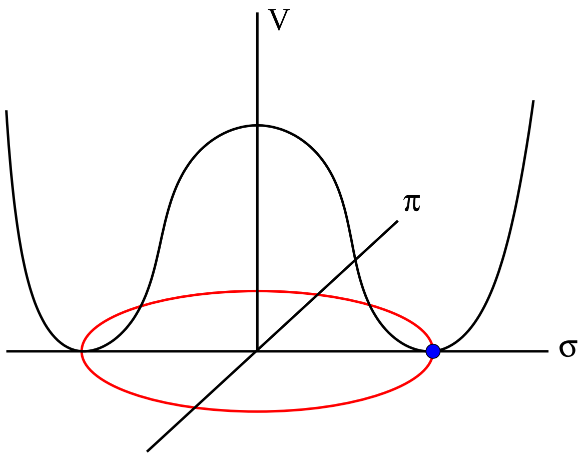
Dashen 1971

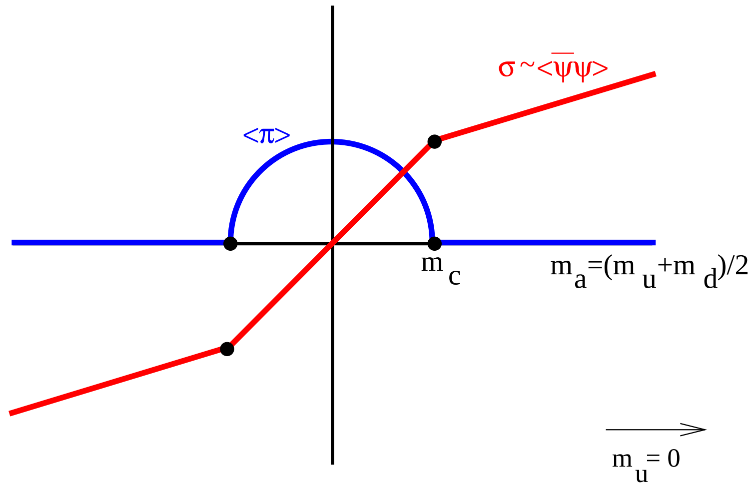


Ising-like transition at $m_u < 0$

- order parameter $\langle \pi_0 \rangle \neq 0$
- breaks CP spontaneously



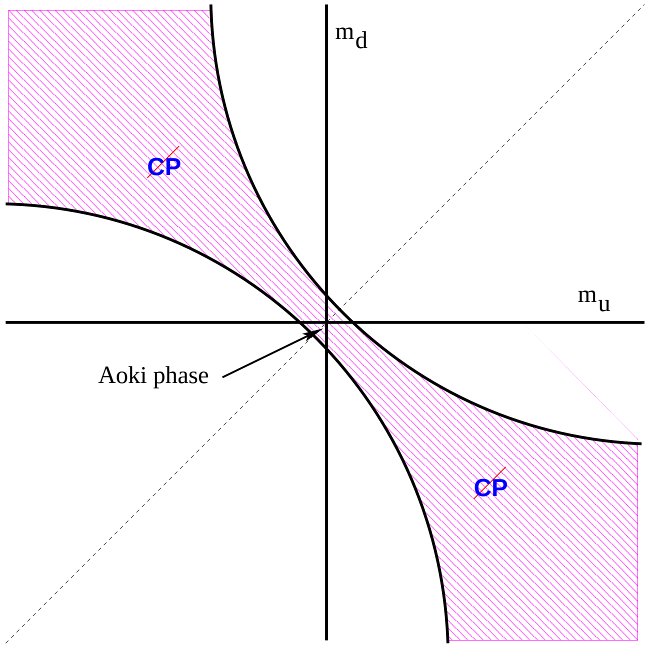




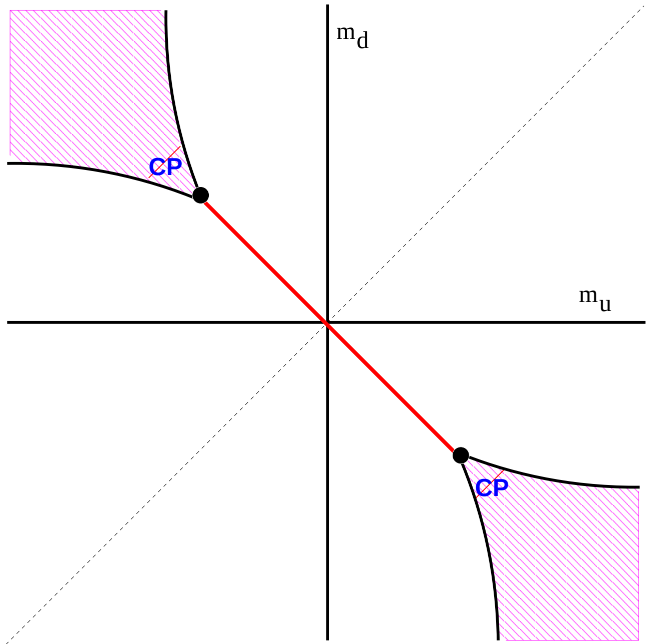
Structure manifest in both “linear” and “nonlinear” sigma models

CP breaking phase related to the Aoki phase and Wilson lattice artifacts

- CP breaking in isospin limit

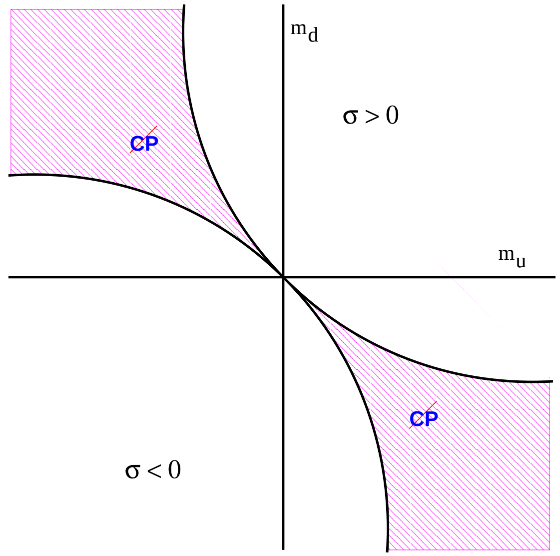


- First order alternative



Which alternative remains controversial

can depend on lattice action



Second order transition at non-vanishing m_u and m_d of opposite sign

- long distance physics without small Dirac eigenvalues

No structure at $m_u = 0$ when $m_d \neq 0$

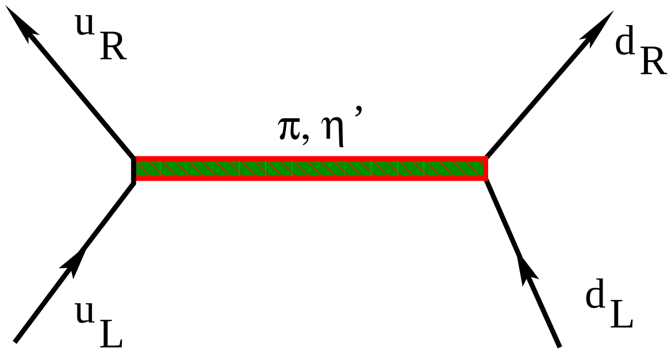
- no long distance physics despite possible small Dirac eigenvalues

Question: Can any experiment tell if $m_u = 0$?

The $m_u = 0$ issue

Eta prime and neutral pion: distinct mixtures of $\overline{u}u$, $\overline{d}d$, and glue

- consider quark-quark spin flip scattering



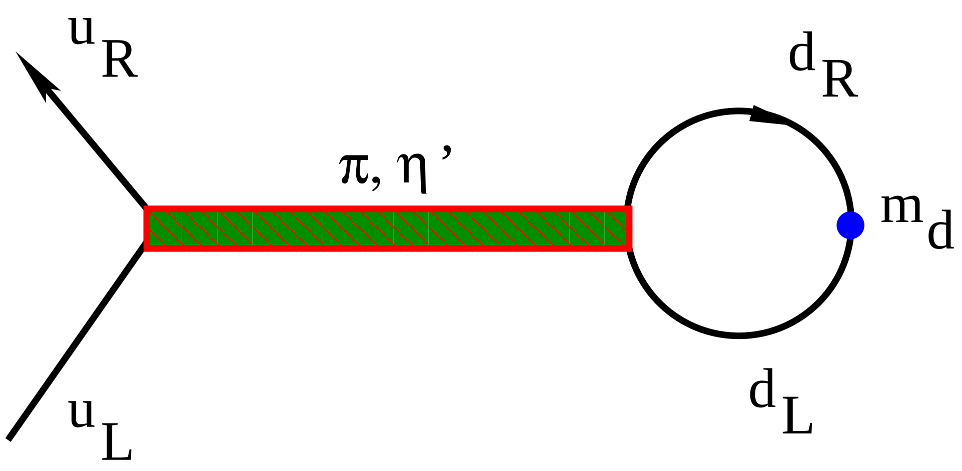
- anomaly: π_0 and η' not degenerate
- four point function $\langle \overline{u}_L u_R \, \overline{d}_L d_R \rangle$ does not vanish

Helicity-flip quark-quark scattering does not vanish in the chiral limit

axial anomaly $\Leftrightarrow$ “ ’t Hooft vertex”

Now turn on a small d quark mass

- closing d loop induces $u_L\ u_R$ mixing



Non-zero d quark mass induces an effective mass for the u quark

Non-perturbative effects renormalize $\frac{m_u}{m_d}$

- quark mass ratios not renormalization group invariant

$$\frac{m_u}{m_d} \to \frac{m_u + \epsilon m_d}{m_d + \epsilon m_u}$$

Effect automatically included in lattice simulations

Old point

- Georgi, McArthur, 1981 (unpublished)
- Choi, Kim, Sze, 1988 (PRL)
- Banks, Nir, Seiberg, 1994 (conference proceedings)
- MC, 2003 (unpublished)
- MC, 2004 (PRL)

Intense consternation from the perturbative community

- effect not seen perturbatively, i.e. in the $\overline{MS}$ scheme
- consequences

  mass renormalization is not flavor blind

  mass independent regularization is tricky

  inherent ambiguities defining $m_u = 0$

$\overline{MS}$ is only a perturbative regulator

- when $m_u \neq m_d$

  while matching perturbative lattice masses to $\overline{MS}$ is OK

  matching to non-perturbative lattice results is not valid!

Specific critiques

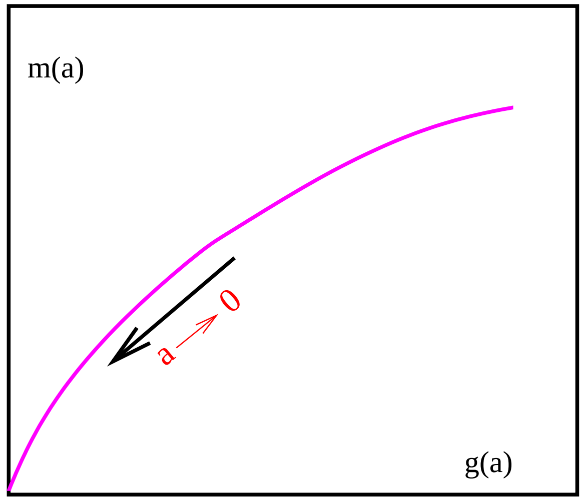
Complaint 0

- $m = 0$ corresponds to the bare mass

Response

- the bare mass always vanishes
- RG: $m_0 \propto g_0^{\gamma_0/\beta_0}(1 + O(g_o^2))$

$$\beta_0 = \frac{11 - 2n_f/3}{(4\pi)^2}$$
$$\gamma_0 = \frac{8}{(4\pi)^2}$$



- asymptotic freedom: $g_0 \to 0 \quad \Rightarrow \quad m_0 \to 0$
- must define mass at some finite scale

Complaint 1:

- Use a mass independent regularization

$$a\frac{dm_i}{da} = \gamma(g)m_i \;\Rightarrow\; \frac{m_i}{m_j} = \text{constant}$$

Response:

- allowed, but obscures above off-diagonal m_d effect on m_u
- no guarantee that $\frac{m_i}{m_j}$ universal between schemes
- lattice is not a mass independent scheme

unclear how to do matching

Complaint 2:

- Do matching at 100 GeV
- instantons exponentially suppressed and irrelevant

Response:

- the lattice simulations are not done at miniscule scales

  instanton effects must be included

- $1/g^2 \sim \log(\mu) \sim \log(1/a)$

  exponential suppression in $1/g^2 \rightarrow$ power in scale μ

Effect controlled by

- $M_{\eta'} - M_{\pi_0}$
- also proportional to $m_d - m_u$
- estimate at scale $\mu = 2$ GeV

$$\Delta m_u(\mu) \sim \frac{(M_{\eta'} - M_{\pi_0})\ (m_d - m_u)}{\mu} = O(1 \text{ MeV})$$

same magnitude as quoted “results”

Note

$$M_{\eta'} \propto \Lambda_{qcd} \propto \mu \, g^{-\beta_1/\beta_0^2} \, e^{-1/(2\beta_0 g^2)}$$

comes from non-trival topology

- exponential behavior controlled by

$$\frac{1}{2\beta_0 g^2} = \frac{8\pi^2}{(11-2n_f/3)g^2} \quad << \quad \frac{8\pi^2}{g^2} = \text{classical instanton action}$$

- topological excitations above quantum, not classical, vacuum

Classical instanton action strongly overestimates suppression

Topological effects are not ‘‘soft’’

Complaint 3:

- at $m_u = 0$ there is no Θ dependence

Response

- $\mathrm{Re}\, m_u$ and $\mathrm{Im}\, m_u$ are independent parameters
- $\mathrm{Re}\, m_u = 0$ is not RG stable

  $\Theta = \arccos(\mathrm{Im}\, m_u / \mathrm{Re}\, m_u)$

  non-perturbative scheme dependence
- the strong CP problem only involves $\mathrm{Im}\, m_u$

  any real m_u is an equivalent “solution” for strong CP problem

Note:

- rotating all phases into m_u leaves three parameters

  $$\mathrm{Re}\, m_u,\ \mathrm{Im}\, m_u,\ m_d$$

- mapping to conventional parameters is singular

  $$m_u,\ m_d,\ \Theta$$

- no singularity at $m_u = 0$

  no natural origin for "polar" coordinates

Polar coordinates should use a natural origin

La Gare de Perpignan, the center of the universe (Dali)

Complaint 4:

- define $m_u = 0$ by vanishing topological susceptibility

Response

- topology has the same scheme dependence
- 1). count small real eigenvalues of the Wilson operator

  How to define “small”?

  At finite cutoff only a minimum, not a zero
- 2). cooling (Wilson flow) to remove short distance fluctuations

  With which action should we cool? How long?

  Can small “instantons” fall through the lattice?
- 3). the overlap operator not unique: “domain wall height”

Note on the “admissibility condition”

- Luscher: if plaquettes restricted $P <\sim .03$

  unique continuum continuation of gauge fields

  instantons cannot collapse, unique winding number

This constraint requires a non-Hermitian Hamiltonian

- $Z = \mathrm{Tr} e^{-\beta H} = \mathrm{Tr}(e^{-aH})^{N_t}$
- Hermitan H implies $\langle\psi|e^{-aH}|\psi\rangle > 0$ for every ψ
- requires plaquette weight to be analytic over the gauge group
- inconsistent with the admissibility constraint

MC, Phys. Rev. D 70, 091501(R) (2004), hep-lat/0409017

Complaint 5:

- staggered fermions do have a chiral symmetry at $m_u = 0$

Response

- rooted staggered fermions are not QCD

  15 taste non-singlet $\overline{u}\gamma_5 u$ pseudoscalars with $M^2 \sim m_u$

  not at physical $M_\pi^2 \sim (m_u + m_d)/2$

  will appear in scattering processes

Four “tastes” per flavor

- symmetry $(SU(4)_u, SU(4)_d)$
- well separated spurious states

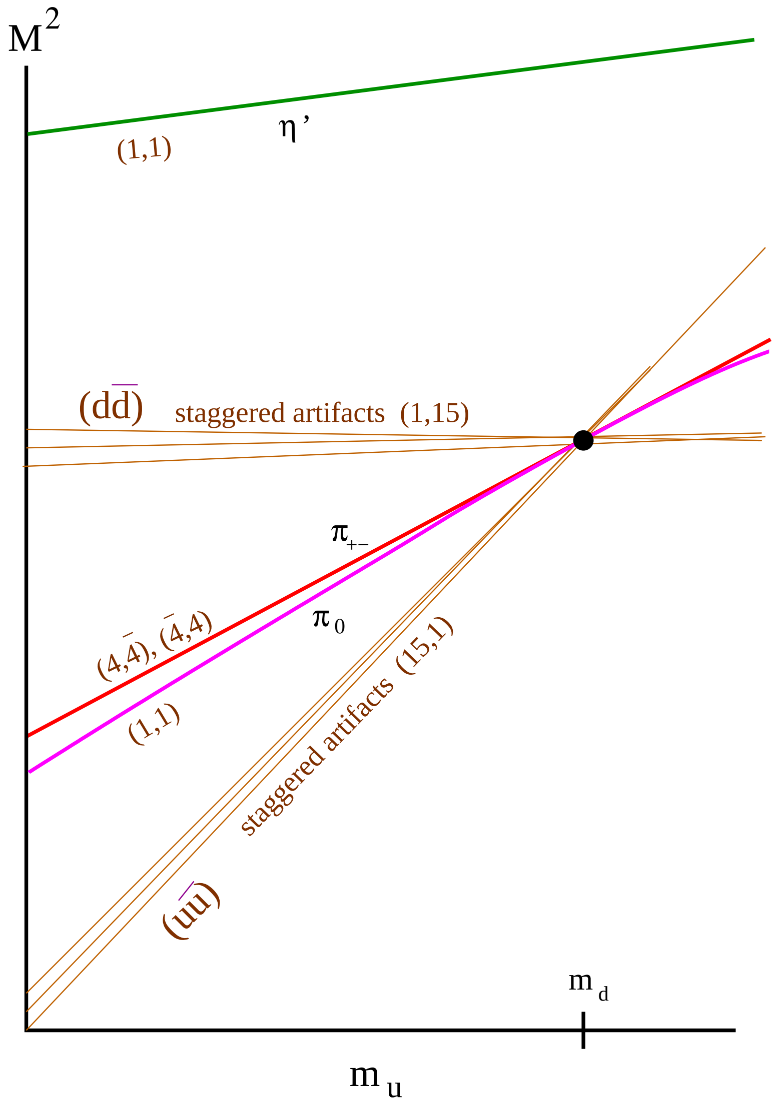
  $\overline{u}_i \gamma_5 u_j$ and $\overline{d}_i \gamma_5 d_j$

  15 with $M^2 \sim m_u$

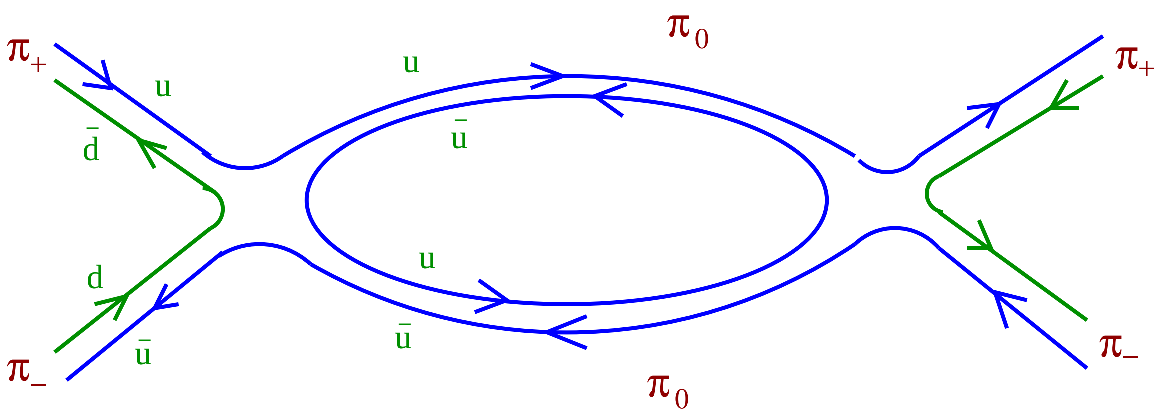
  15 with $M^2 \sim m_d$

- one massless at $m_u = 0$

  required by symmetry



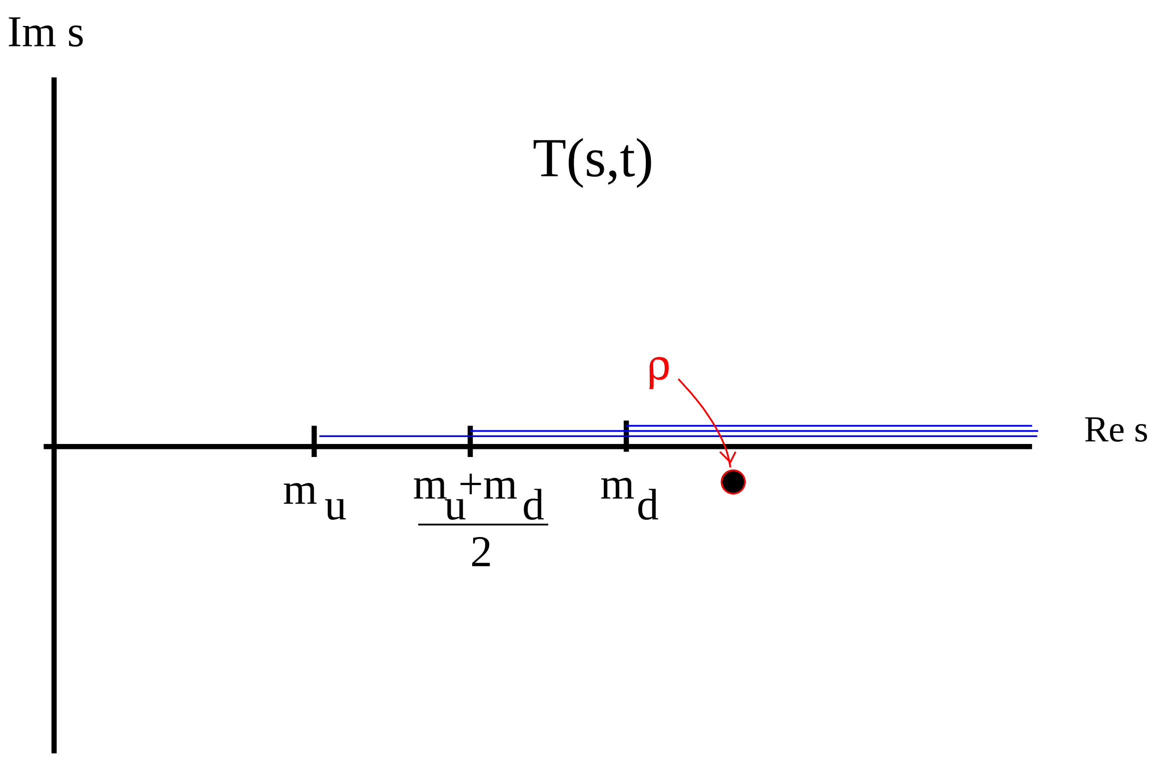
Scattering will create the unphysical mesons



Unphysical thresholds appear in $T(s,t)$

- locations controlled by m_u and m_d separately

  not only the average quark mass



Incorrect analytic structure

- widely separated cuts at unphysical locations

Spurious states cannot mix

- independent taste symmetries for up and down quarks

Note:

- Rooting OK for replicated Wilson fermions
- additive mass shift breaks spurious symmetry

Staggered quarks are not replicated fermions

- chiral symmetry is ‘‘flavored’’ (tasted?)

  four tastes are not equivalent

  rooting mixes inequivalent fields
- taste breaking is not the issue
- the chiral limit is not the issue

Summary

Careful chiral analysis resolves several controversies

$m_u = 0$, rooting, topological susceptibility

Perturbation theory can mislead

- mass mixing effects absent in perturbation theory
- inappropriate to match lattice and perturbative masses

No structure at $m_u = 0$ when $m_d \neq 0$

- $m_u = 0$ not an appropriate solution to the strong CP problem
- ambiguity in defining topology

Interesting phase structure with negative mass quarks

- possible pion condensation