QCD analysis of the nucleon spin structure function data in next to leading order in _s

The polarized gluon distribution in the nucleon

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Outline

- World data of nucleon spin structure functions
- Q^2 evolution with DGLAP equations
- Results
- Contribution of the polarized gluon distribution
- Direct measurements of the polarized gluon distribution
- Prospective future measurements of the spin structure functions
- Conclusions

History

SLAC-YALE E80/E130

G. Baum et al. 1983

0.2 < x < 0.7

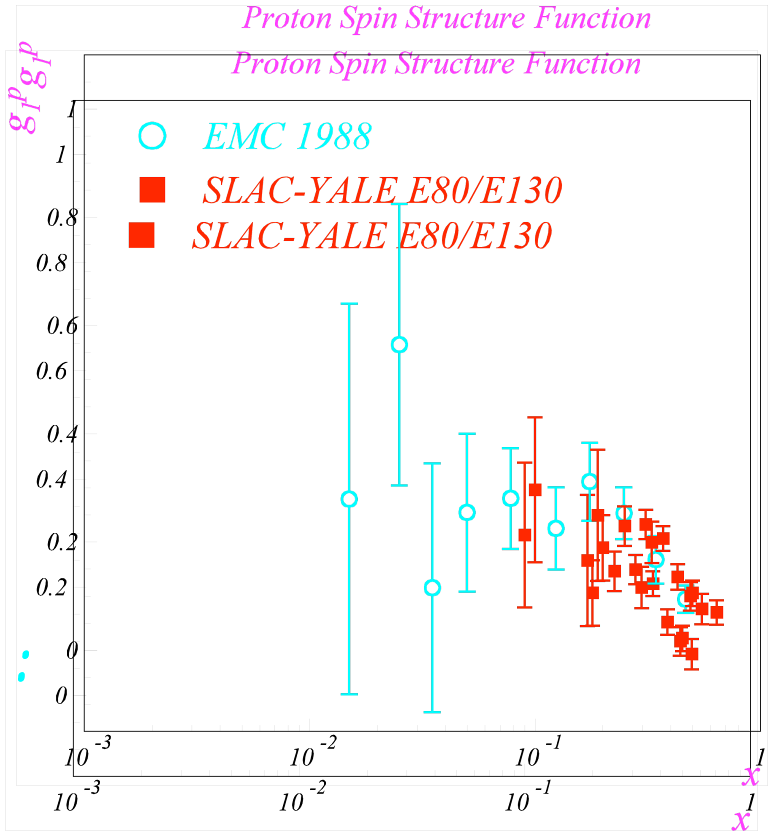
EMC 1988

J. Ashman et al; 198

0.01 < x < 0.7

Determine the first moment :

$$\Gamma_1^p = \int_0^1 g_1(x)\mathrm{d}x$$



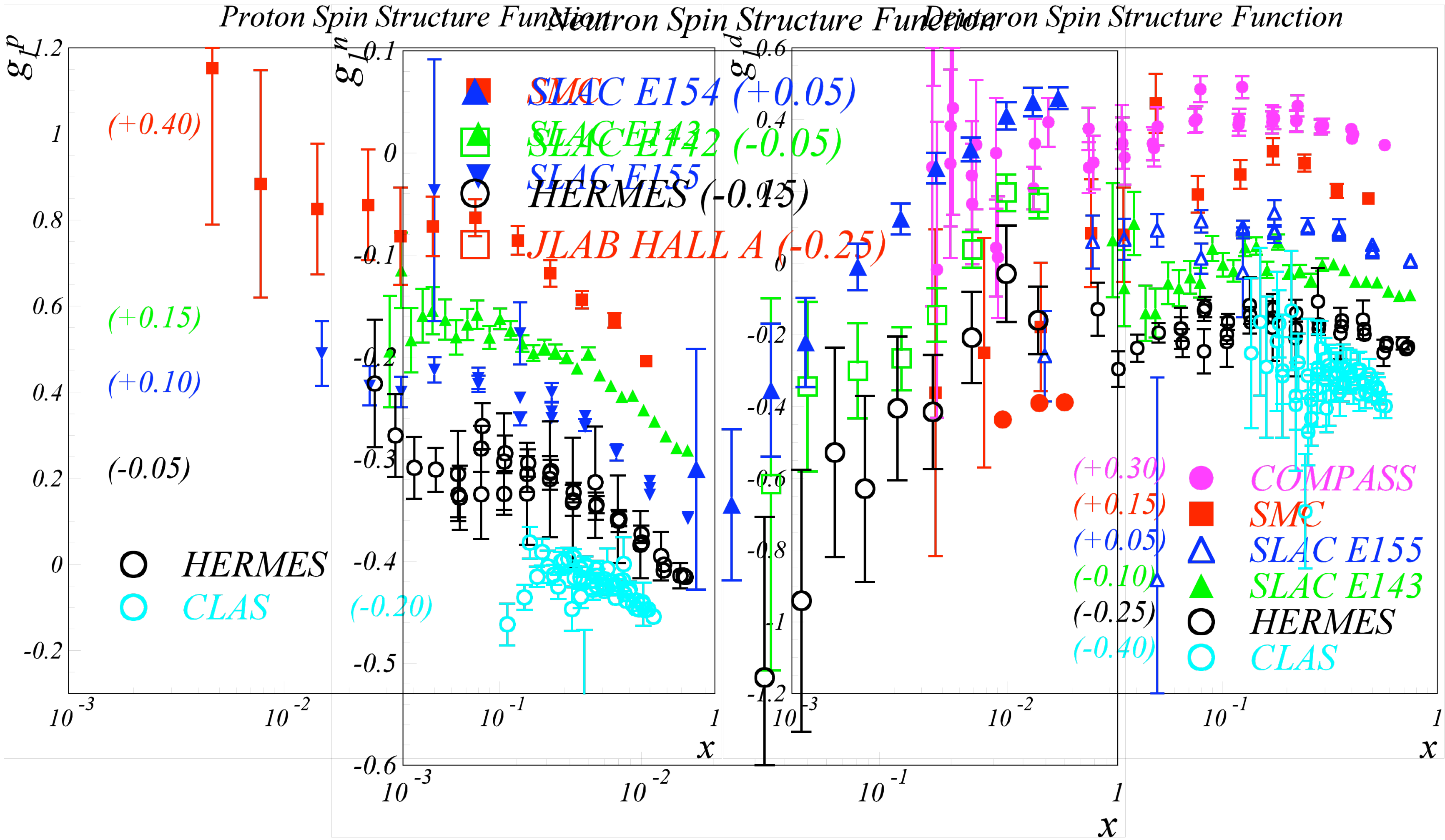
World data of the Spin Structure Functions

Proton:EMC, SMC, SLAC-E143, E154,
HERMES, JLAB-CLAS

Deuteron: SMC, SLAC-E143, E155, HERMES,
JLAB-CLAS

Neutron: SLAC-E142, HERMES,JLAB-MIT

Spin structure function data



QCD Evolution - DGLAP equations

- In the parton model, the spin structure function g_1 is related to the polarized quark and gluon distributions in the nucleon by an integral equation involving quark and gluon coefficient functions C^i_q and C_g
  (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi)

QCD Evolution - DGLAP equations (Cont.)

$$g_1(x,t) = \frac{1}{2}\sum_{k=1}^{n_f}\frac{e_k^2}{n_f}\int_x^1\frac{dy}{y}[C_q^S(\frac{x}{y},\alpha_s(t))\Delta\Sigma(y,t)$$

$$+2n_f C_g(\frac{x}{y},\alpha_s(t))\Delta G(y,t) + C_q^{NS}(\frac{x}{y},\alpha_s(t))\Delta q^{NS}(y,t)]$$

Where: $t = \ln(Q^2/\Lambda^2)$, and _ is a scale parameter

_G(x) - the polarized gluon distribution

$$\Delta\Sigma(x,t) = \sum_{i=1}^{n_f}[\Delta q_i(x,t) + \Delta\overline{q}_i(x,t)]$$

(singlet)

$$\Delta q^{NS}(x,t) = \left(\sum_{k=1}^{n_f}\frac{e_k^2}{n_f}\right)^{-1}\sum_{i=1}^{n_f}\left(e_i^2 - \sum_{k=1}^{n_f}\frac{e_k^2}{n_f}\right)\left[\Delta q_i(x,t) + \Delta\overline{q}_i(x,t)\right]$$

(non-singlet)

QCD analysis- DGLAP equations (Cont.)

- Parameterization at initial scale (1 $(GeV/c)^2$)

$$\Delta f(x)= \_ \ N(\_{f},\_{f},a_f,b_f)\ x_{-}\ (1-x)_{-}(1+ax+bx^{1/2})$$

for __(x), _G(x), $\Delta q_p^{NS}(x)$, $\Delta q_n^{NS}(x)$

- Normalization

$\int N x_{-}(1-x)_{-}(1+ax+bx^{1/2})dx = 1$

_ the first moment of the parton distribution

- Fit procedure:

Set parameters at initial scale

Evolve parton distributions –using the DGLAP equations

and calculate $g_1(x,Q^2)$ for each data

QCD analysis- DGLAP equations (Cont.)

QCD analysis in the Adler-Bardeen scheme:

First moment of C_g -

$$C_g^1 = \frac{\alpha_s}{4\pi}$$

(G. Altarelli, R. Ball, DS. Forte and G. Ridolfi Nucl. Phys. B 496 (1997) 337.)

Compare also to the $\overline{MS}$ scheme – $C^1{}_g = 0.$

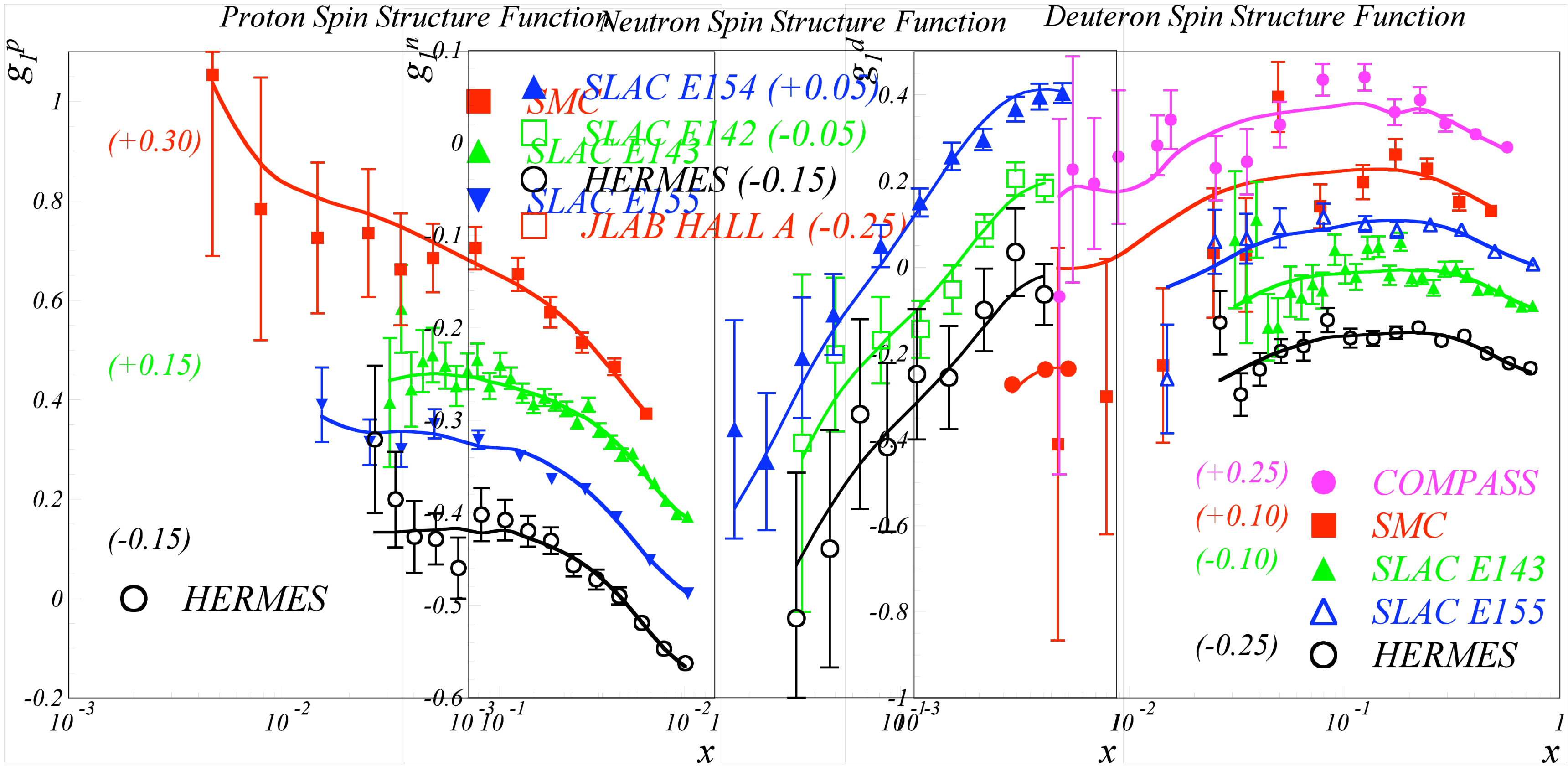
The first moments of the singlet distributions differ by:

$$a_0 = \Delta\Sigma_{\overline{MS}}(Q^2) = \Delta\Sigma_{AB} - n_f \frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2)$$

(a_0– the singlet axial current matrix element)

QCD analysis in NLO- results

- Fit to data (shown at measured Q^2)

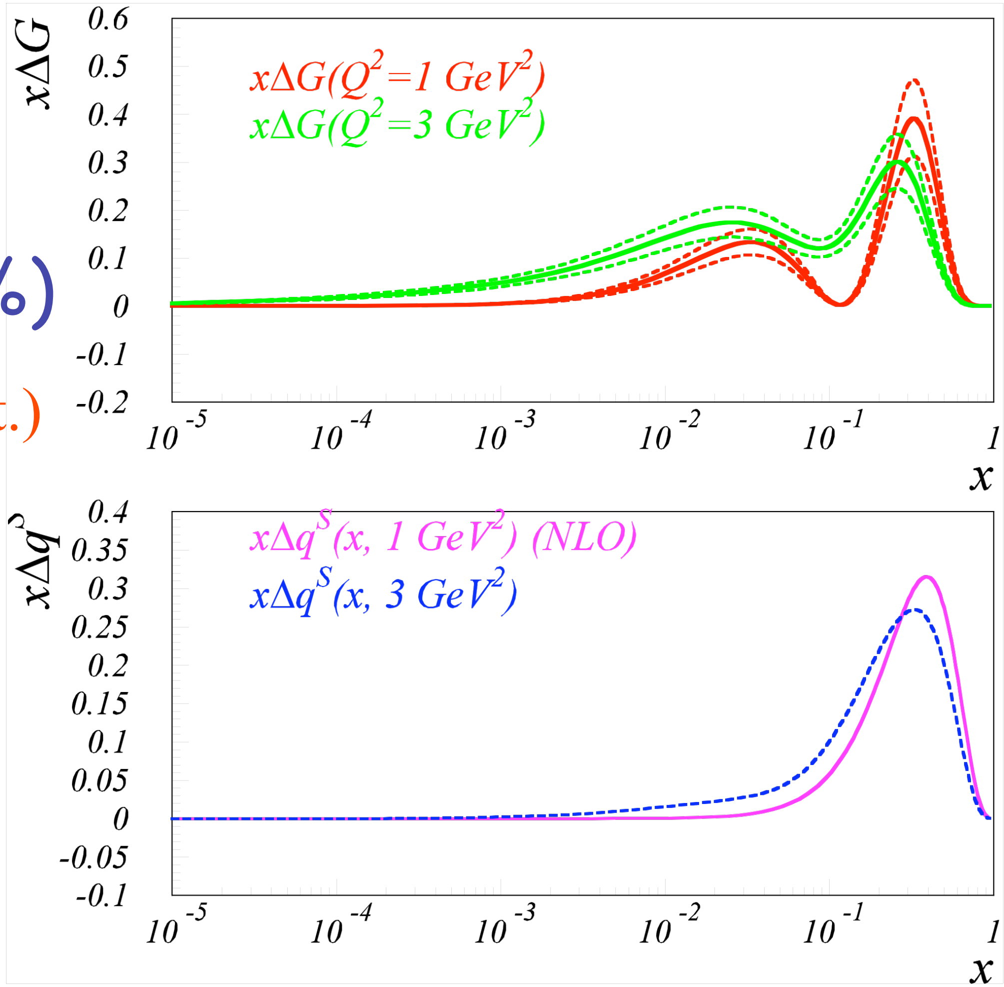


QCD analysis in NLO - results

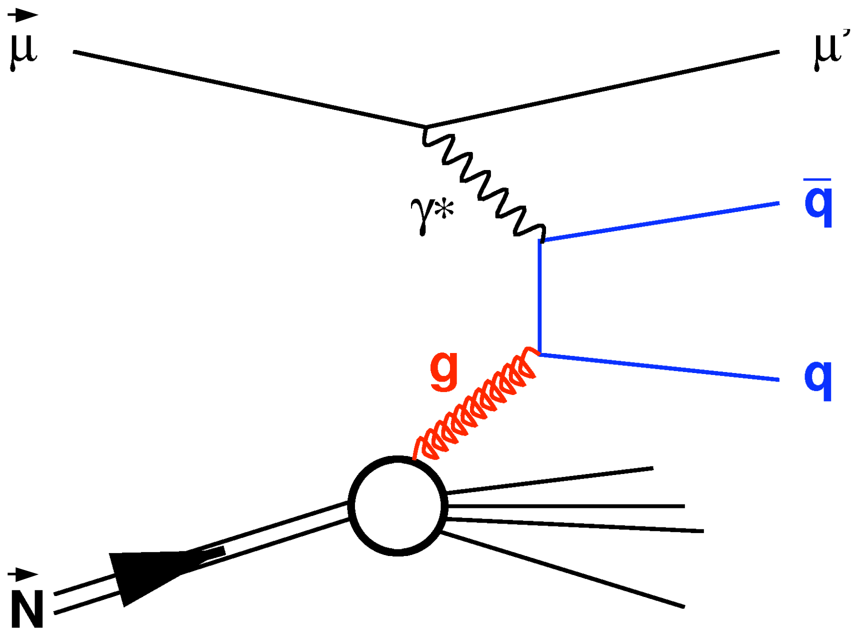
- 12 parameters
- 286 data
- $\Sigma\chi^2$=251 (C.L.=84%)

$$\Delta G = \int_0^1 \Delta G(x)\mathrm{d}x = 0.62^{+0.13}_{-0.12}\,(\mathrm{stat.})$$

at Q^2=1 (GeV/c)^2



_G/G from photon gluon fusion



1) Open charm production:

q = c

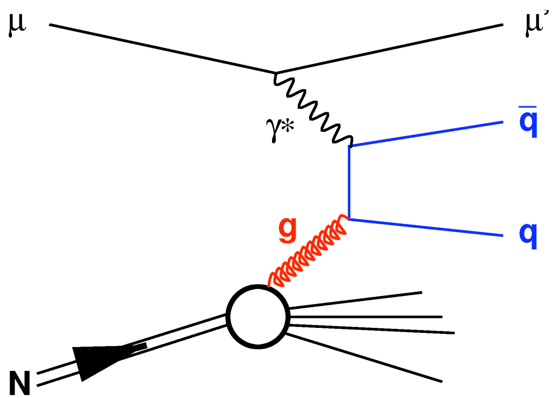
charm fragmentation:

- D^0, D^* (60%)
- D^+ (20%)
- D_S^+, _c$^+$ (10% each)

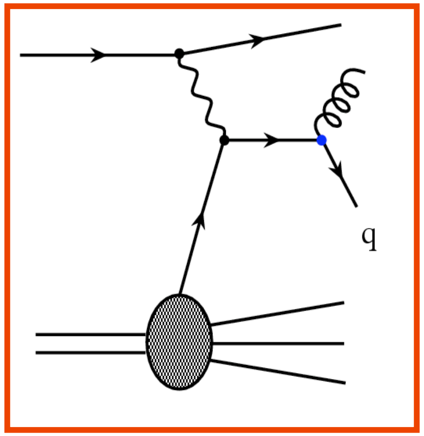
2) High-P_t hadron pair production:

Measurements of _G/G

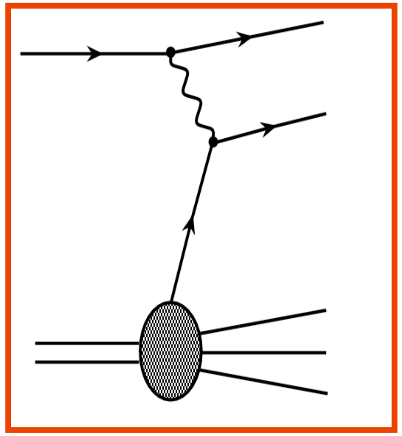
- Determination of _G/G from high P_T hadron pairs



PGF



QCD Compton



Leading order DIS

Measurements of the gluon polarization $\Delta G/G(x)$

Photoproduction of High P_T hadron pairs

SMC: Phys. Rev C **D70** (2004) 012002

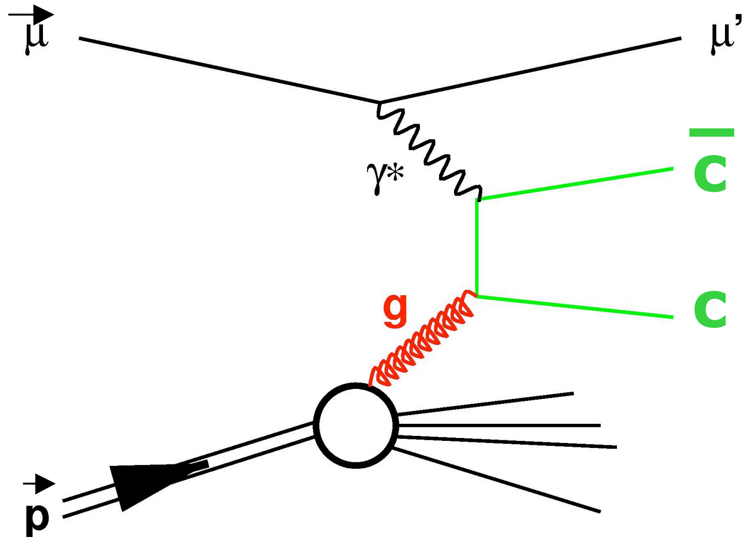
$\Delta G/G(x_g=0.07;\ \_^2=3\ GeV^2) = -0.20 \pm 0.28 \pm 0.10$

HERMES: Phys. Rev. Lett. **84** (2000) 2584

$\Delta G/G(x_g=0.17;\ \_^2=2\ GeV^2) = 0.41 \pm 0.18 \pm 0.03$

Determination of _G/G from open charm production

COMPASS at CERN



$D^0 \Rightarrow K^-\pi^+$ BR = 4%

$D^{*+} \Rightarrow D^0\pi_s^+ \sim 30\%^{(*)}$

$\Rightarrow K^-\pi^+\pi_s^+$

Measurements of the gluon polarization $\Delta G/G(x)$ (Cont.)

- COMPASS:

High PT (low Q2)

$\Delta G/G(x_g=0.085;\ \_^2=3\ GeV^2) = 0.016 \pm 0.058 \pm 0.055$

High PT (high Q2)

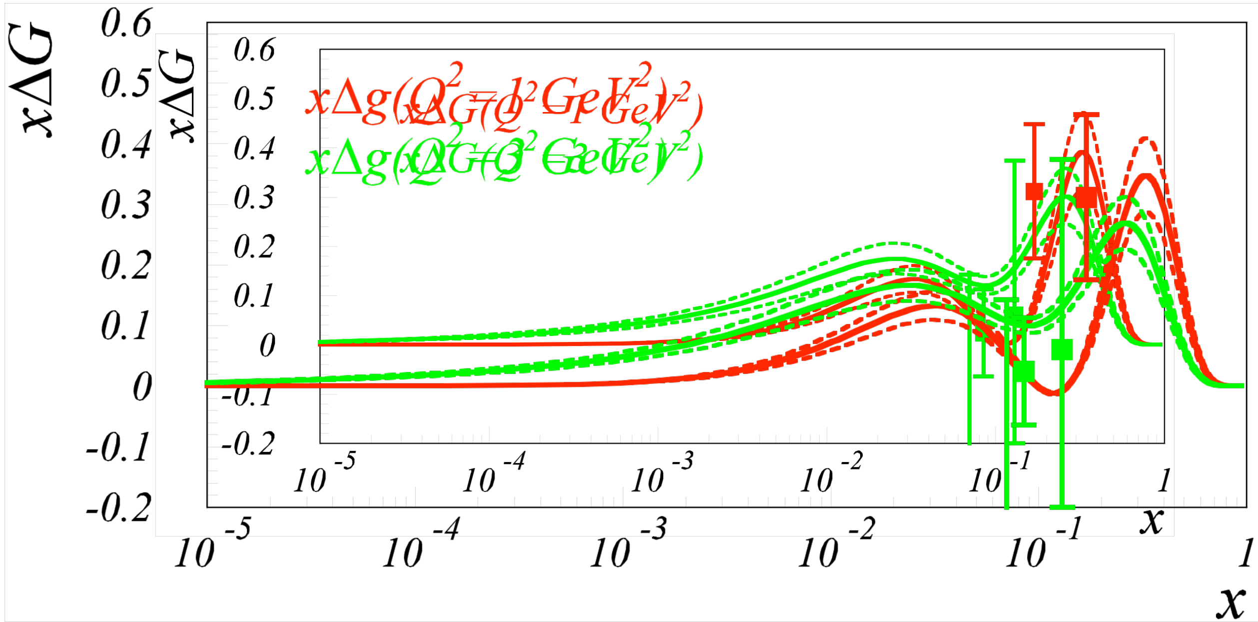
$\Delta G/G(x_g=0.13;\ \_^2=3\ GeV^2) = 0.06 \pm 0.31 \pm 0.06$

Open Charm

$\Delta G/G(x_g=0.15;\ \_^2=13\ GeV^2) = -0.57 \pm 0.41 \pm 0.17$

Use CTEQ parameterization of $G(x)$ to obtain $\Delta G(x)$

Comparison of best-fit polarized gluon distribution with ΔG measurements



Best fit compatible with presently measured ΔG values

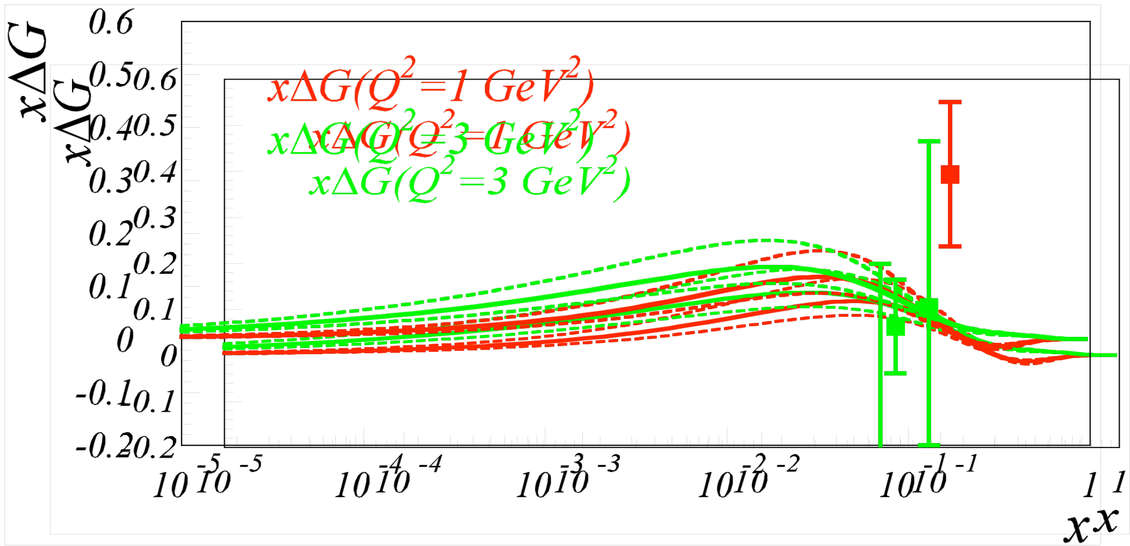
Including ΔG data in fit has a very small effect.

$\Delta G = 0.57 \pm 0.11$ (stat.)

x dependence of the polarized gluon distribution

- Fit data with 11 parameters:

  no mid-range node



- 286 data
- $\Sigma\chi^2$=261

  (C.L.=72%)
- ΔG = 0.48±0.14

- Not quite compatible with measured ΔG
- (C.L. -> 67%)
- Additional measurements of ΔG are essential

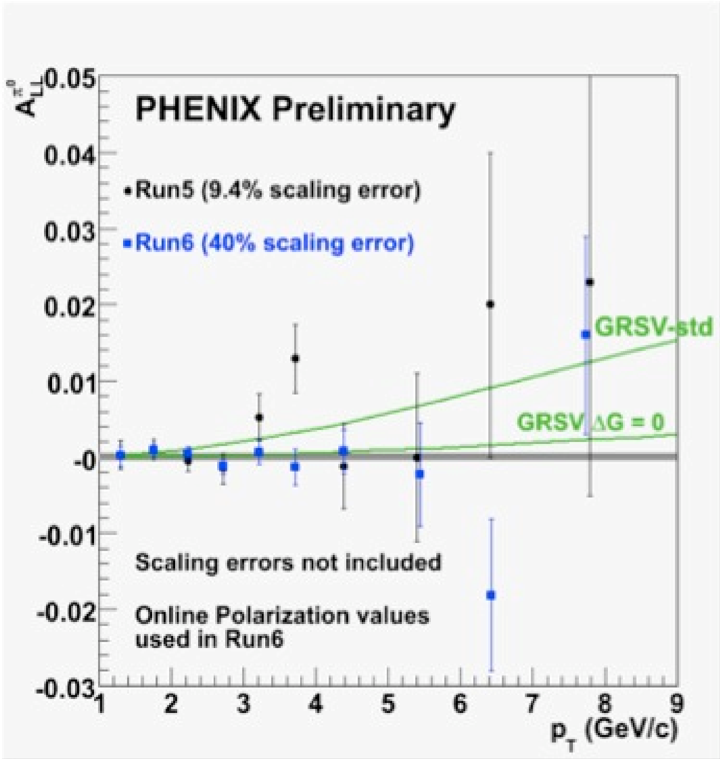
PHENIX at RHIC

Asymmetry in π^0 production from p+p collisions

$$\vec{p} + \vec{p} \; at \; \sqrt{s} = 200 \; GeV$$

Calculations: NLO with pQCD
Glück, Reya, Stratmann and Vogelsang

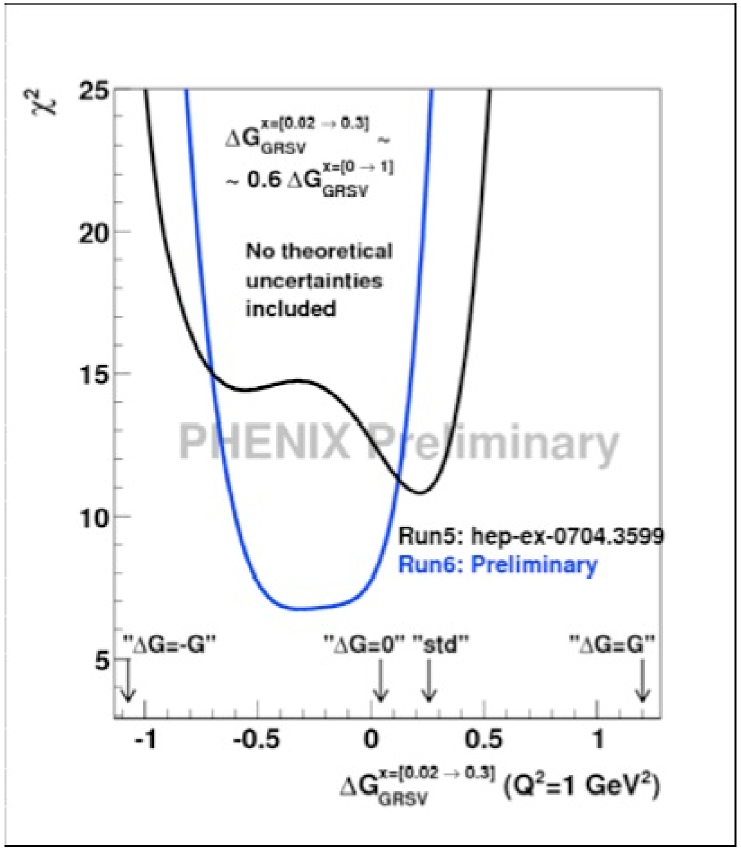
$0.02 < X_g < 0.3$ for each P_T bin.



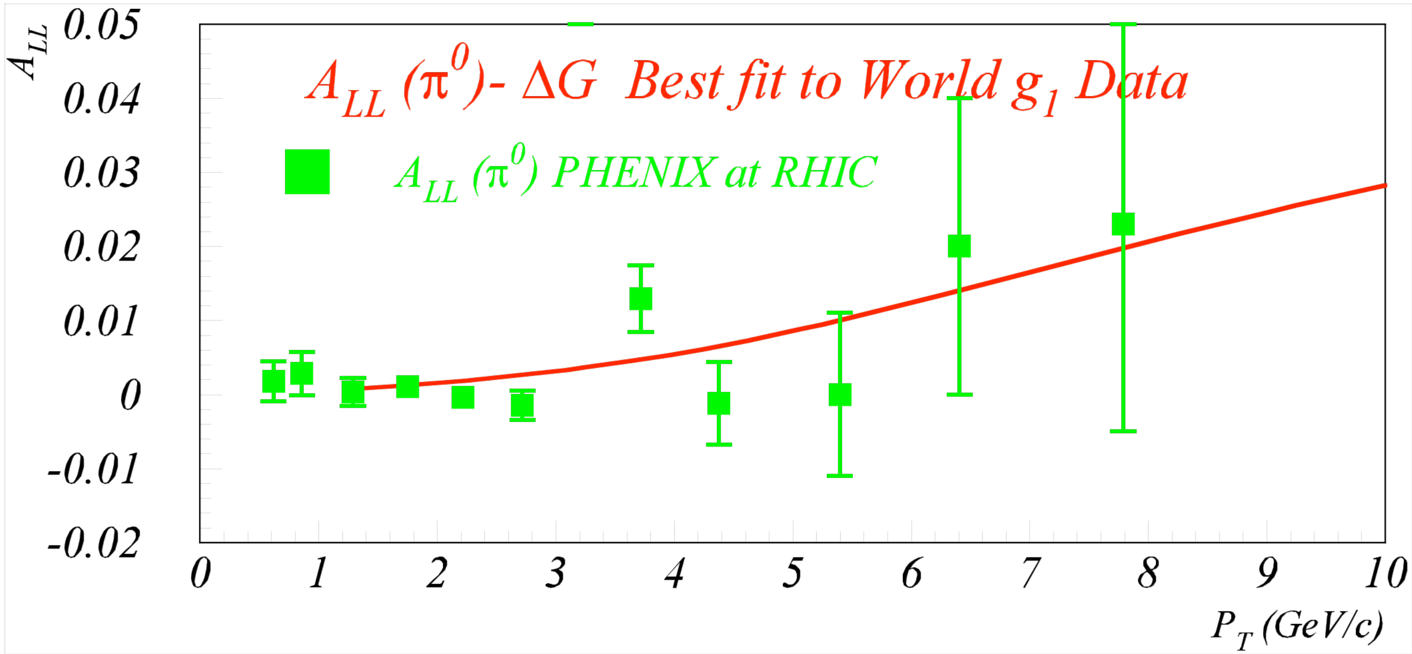
Sensitivity to $\Delta G^{x=(0.02-0.3)}$

- Compare data with $A_{LL}(p_T)$ calculated with different $\Delta G(x)$ and

$$\Delta G_{GRSV}^{x=(0.02-0.3)} = \int_{0.02}^{0.3} \Delta G(x)\mathrm{d}x$$



$A_{LL}(\pi^0)$ with ΔG from best-fit to world g_1 data



Caculation by W. Vogelsang

Systematic errors

Preliminary (rough) estimates

- Systematic errors in g_1 data ~ ±0.10
- Factorization and renormalization scales ~ ±0.05
- Functional form ~ ±0.15

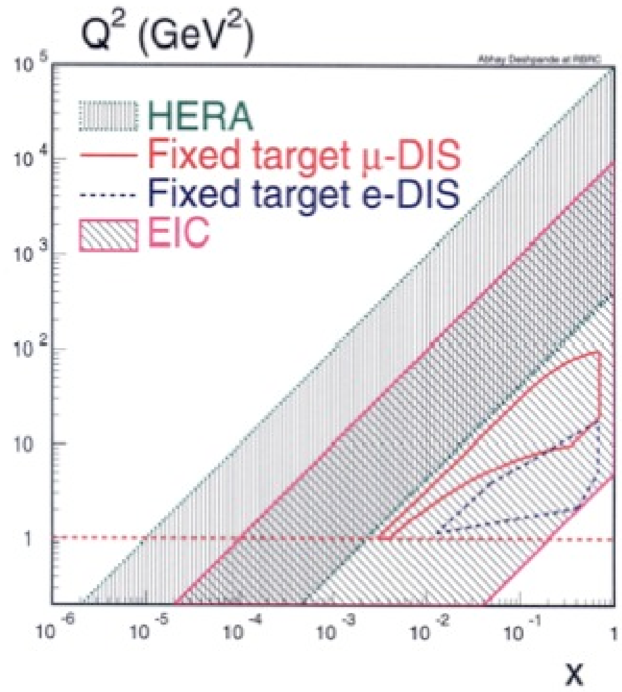
- Total estimate ~ ±0.20

Measurements with a future Electron Ion Collider

- Extend kinematic range:
- High luminosity
- $E_e = 10\ GeV$
- $E_p = 250\ GeV$

$\sqrt{s} = 100\ GeV$

- $\int L = 2fb^{-1}$



Projected EIC data in QCD analysis

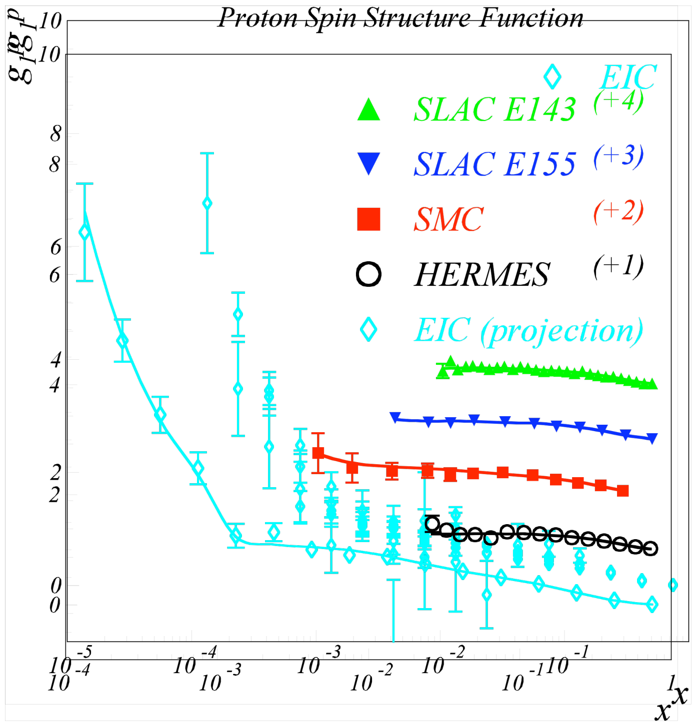
- Use best-fit distributions

  to calculate asymmetries over EIC kinematic region

- Randomize data
- Repeat QCD analysis

  with projected data

$\Delta G = 0.64 \pm 0.05$ (stat.)



CONCLUSIONS (I)

- New measurements of the polarized gluon distribution are compatible with g_1 data.
- Need more g_1 data at low x (proton, deuteron).
- Need more precise ΔG data to determine the x dependence of the gluon distribution

  (Expected in near future)

CONCLUSIONS (II)

- Determine the axial vector coupling constant – The Bjorken sum-rule.

$$g_1^p(x,Q^2) - g_1^n(x,Q^2) = \frac{1}{2}\langle e^2\rangle \int_x^1 \frac{dy}{y}\left[C_1^{NS}\left(\frac{x}{y},\alpha_s(t)\right)\Delta q_{NS}(y,t)\right]$$

with

$$\Delta q_{NS}(x,Q^2) = \frac{3}{2}\left|\frac{g_A}{g_V}\right| N x^{\alpha}(1-x)^{\beta}(1+ax+bx^{1/2})$$

Note: The non-singlet distribution evolves independently! (Measurements at same (x,Q^2))

CONCLUSIONS (III)

Remaining tasks:

- Evaluate systematic error.

Future tasks

- Introduce model-independent SF's (Fourier-Bessel, Sum of Gaussians) to get a realistic estimate on the uncertainty (ΔG is not determined at low x)
- Incorporate RHIC measurement ($\Delta G^{(x:0.02-0.3)}$) in the analysis (iterative procedure)
- Fit strong coupling constant α_s

The last slide

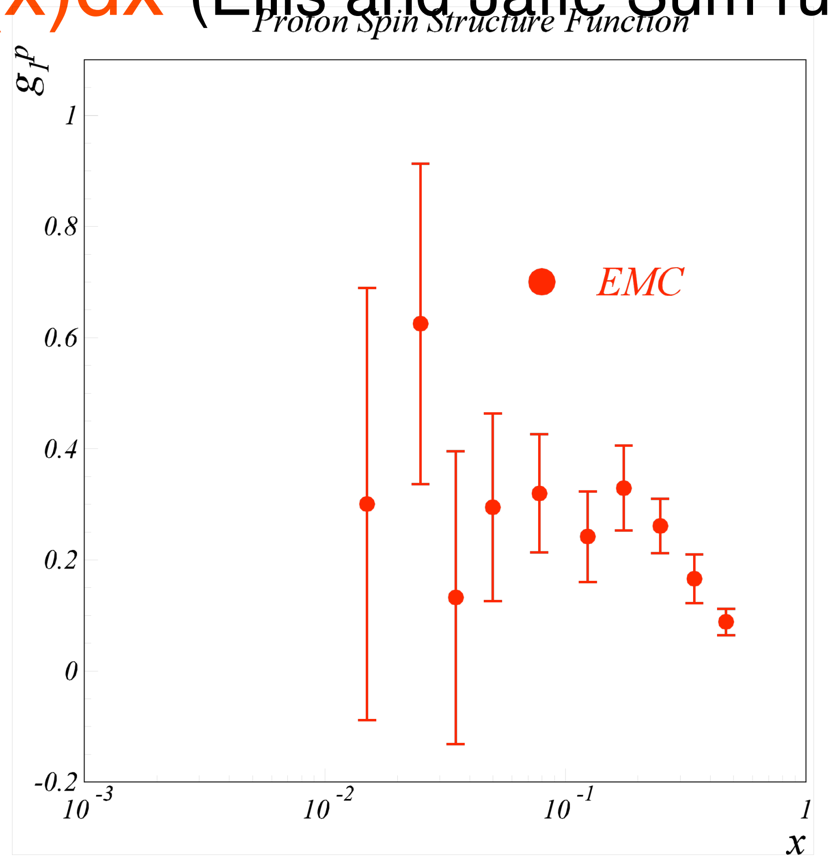
- Thanks to many people who were involved and contributed to this work:

- A. Deshpande
- W. Vogelsang
- R. Windmolders

History - EMC 1988

- Measurement of the proton spin structure function and determination of its first moment: $\_ = \int g_1^P(x)dx$ (Ellis and Jaffe Sum rule)

$0.01 \leq X \leq 0.7$



QCD Evolution - DGLAP equations (Cont.)

The t dependence of the distributions follows the DGLAP equations:

Gluon and singlet:

Note: The non-singlet evolves independently of the singlet and the gluon distributions:

The coefficient functions C^i_q, C_g, and splitting functions $P_{i,j}$ can be expanded in powers of $\_{s}$.

Use next to leading order (NLO) coefficient functions.

QCD Evolution - DGLAP equations (Cont.)

The t dependence of the distributions follows the DGLAP equations:

Singlet and gluon distributions:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta\Sigma(x,t)=\frac{\alpha_s}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[P_{qq}^S(\frac{x}{y},\alpha_s(t))\Delta\Sigma(y,t)+2n_f P_{qg}(\frac{x}{y},\alpha_s(t))\Delta G(y,t)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta G(x,t)=\frac{\alpha_s}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[P_{gq}^S(\frac{x}{y},\alpha_s(t))\Delta\Sigma(y,t)+P_{gg}(\frac{x}{y},\alpha_s(t))\Delta G(y,t)\right]$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta q^{NS}(x,t)=\frac{\alpha_s}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[P_{qq}^{NS}(\frac{x}{y},\alpha_s(t))\Delta q^{NS}(y,t)\right]$$

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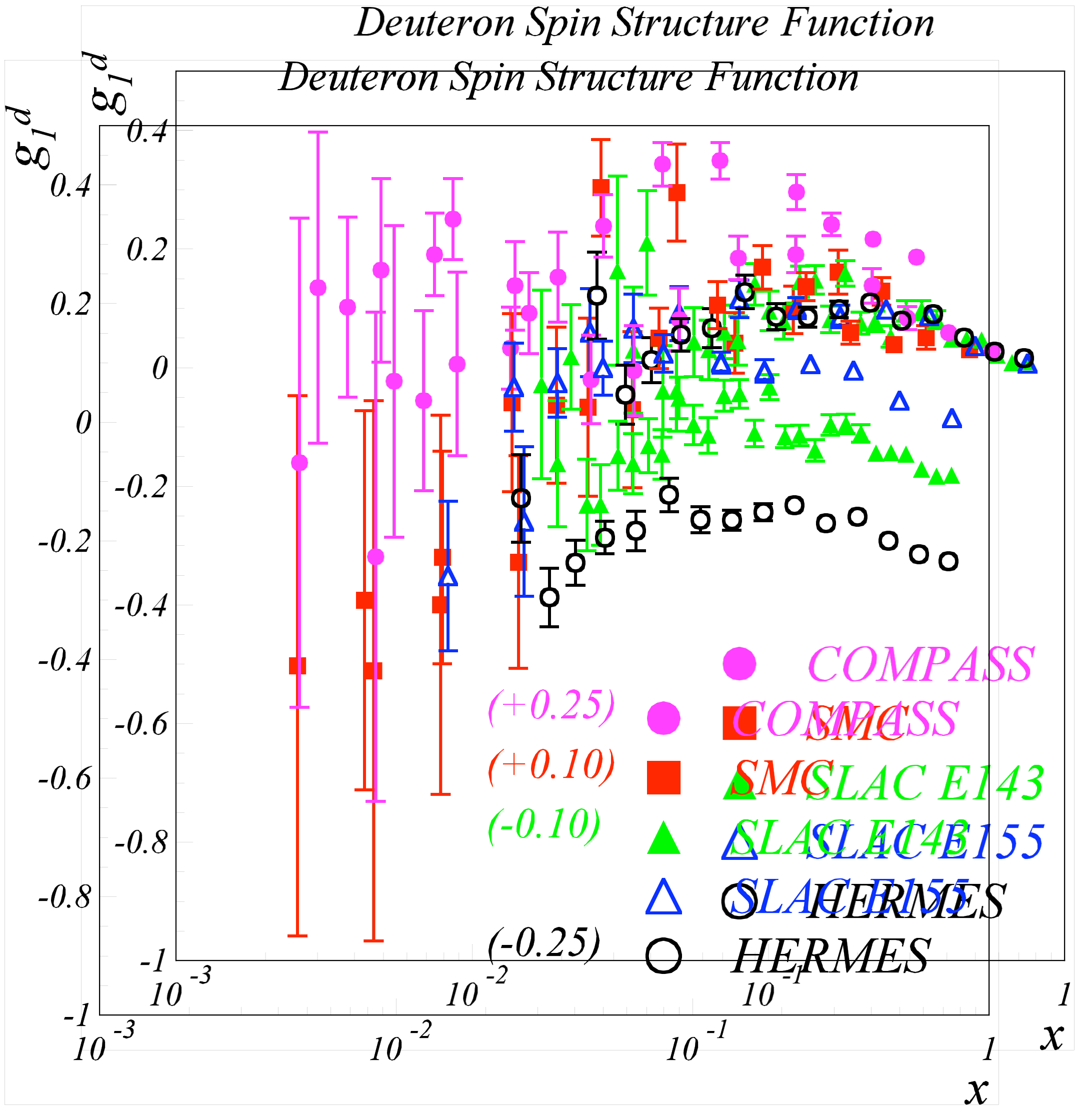
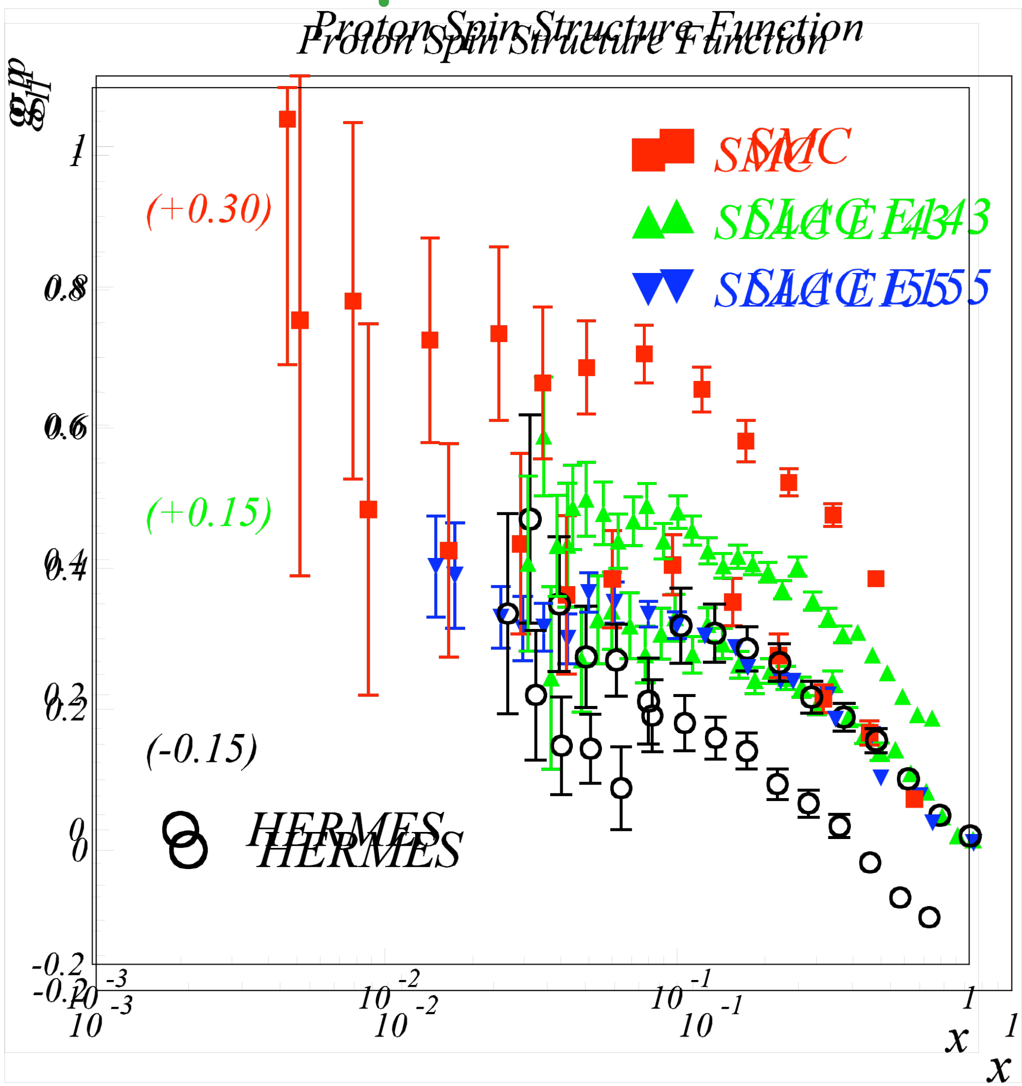
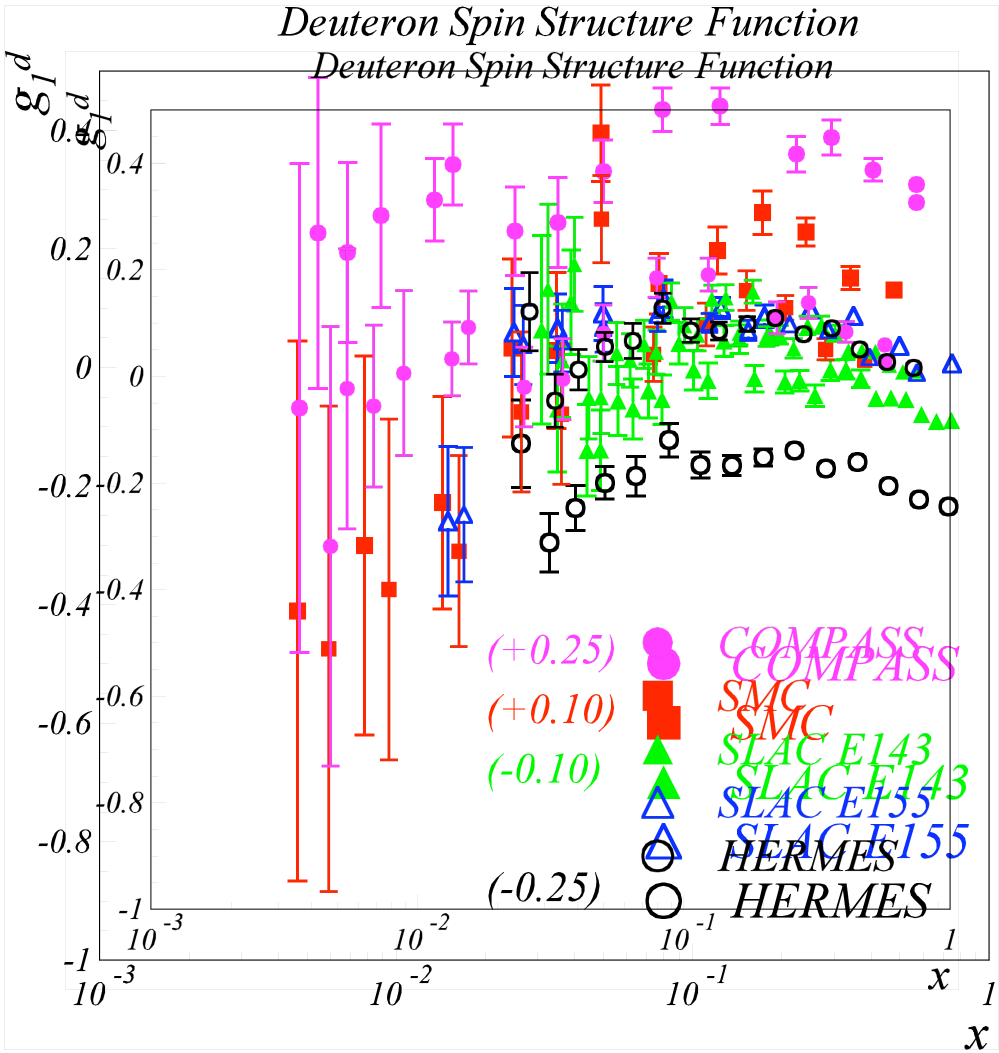
Spin structure function data-deuteron



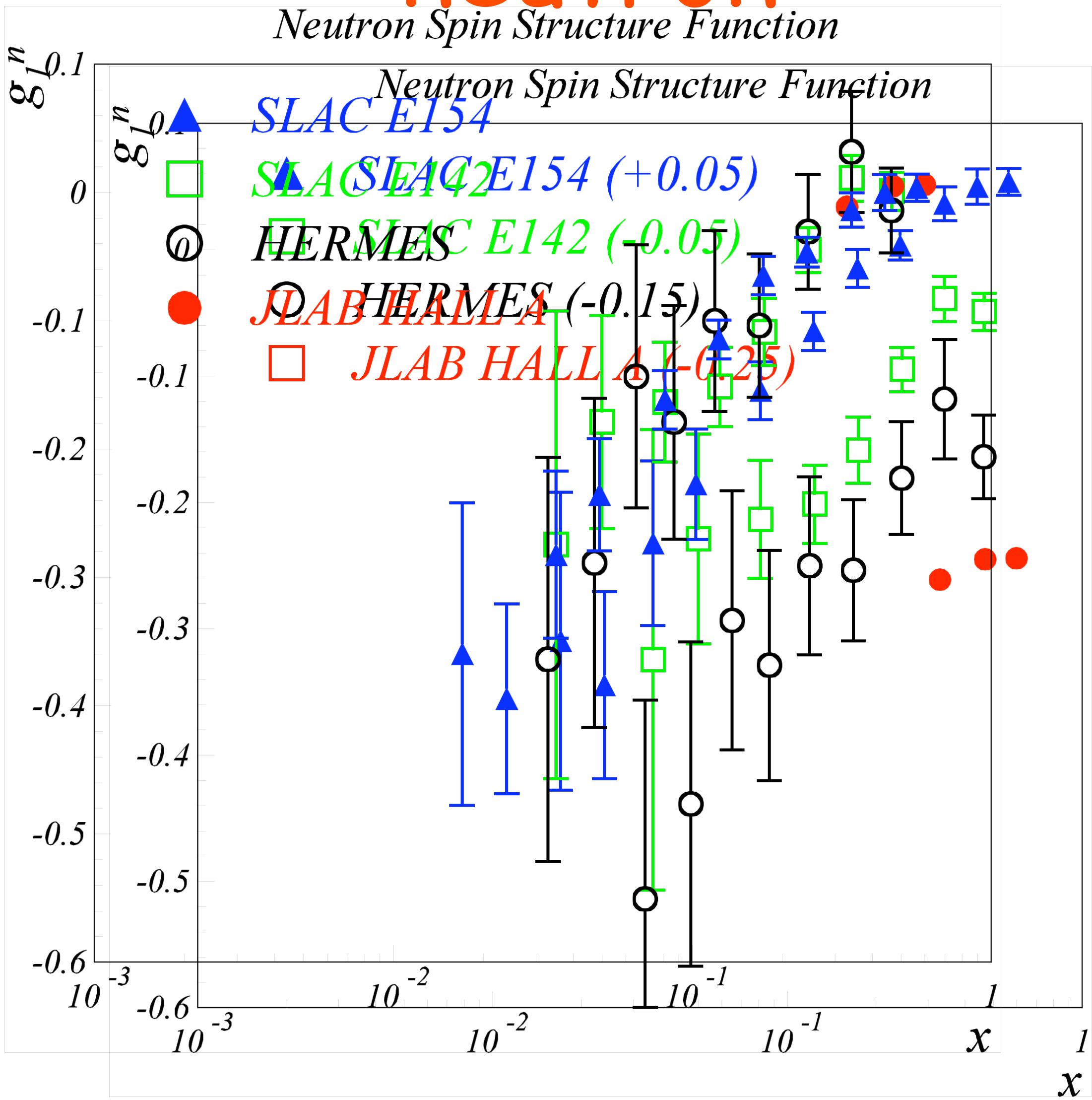
Spin structure function data - proton



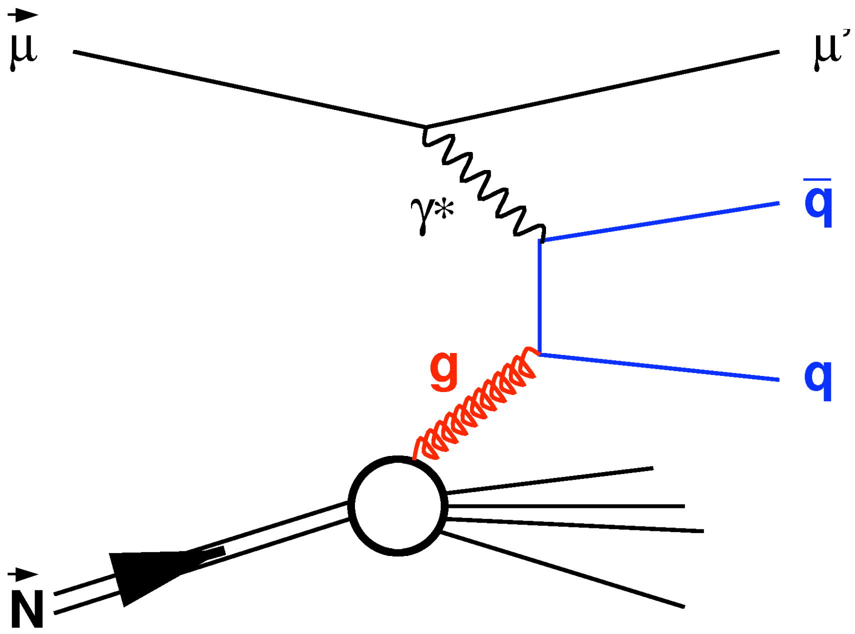
Spin structure function data-deuteron (301)



Spin structure function data - neutron



_G/G from photon gluon fusion



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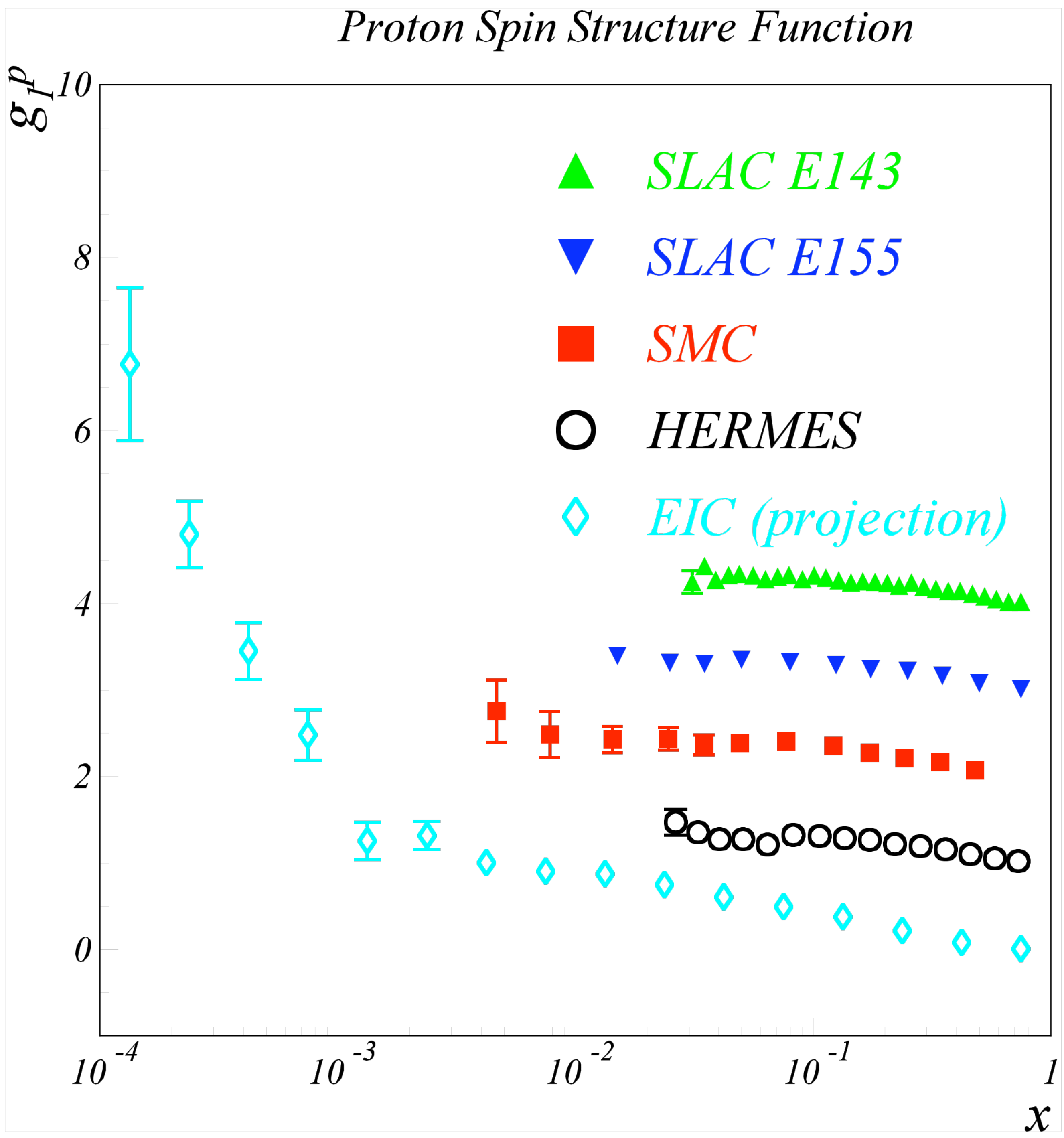
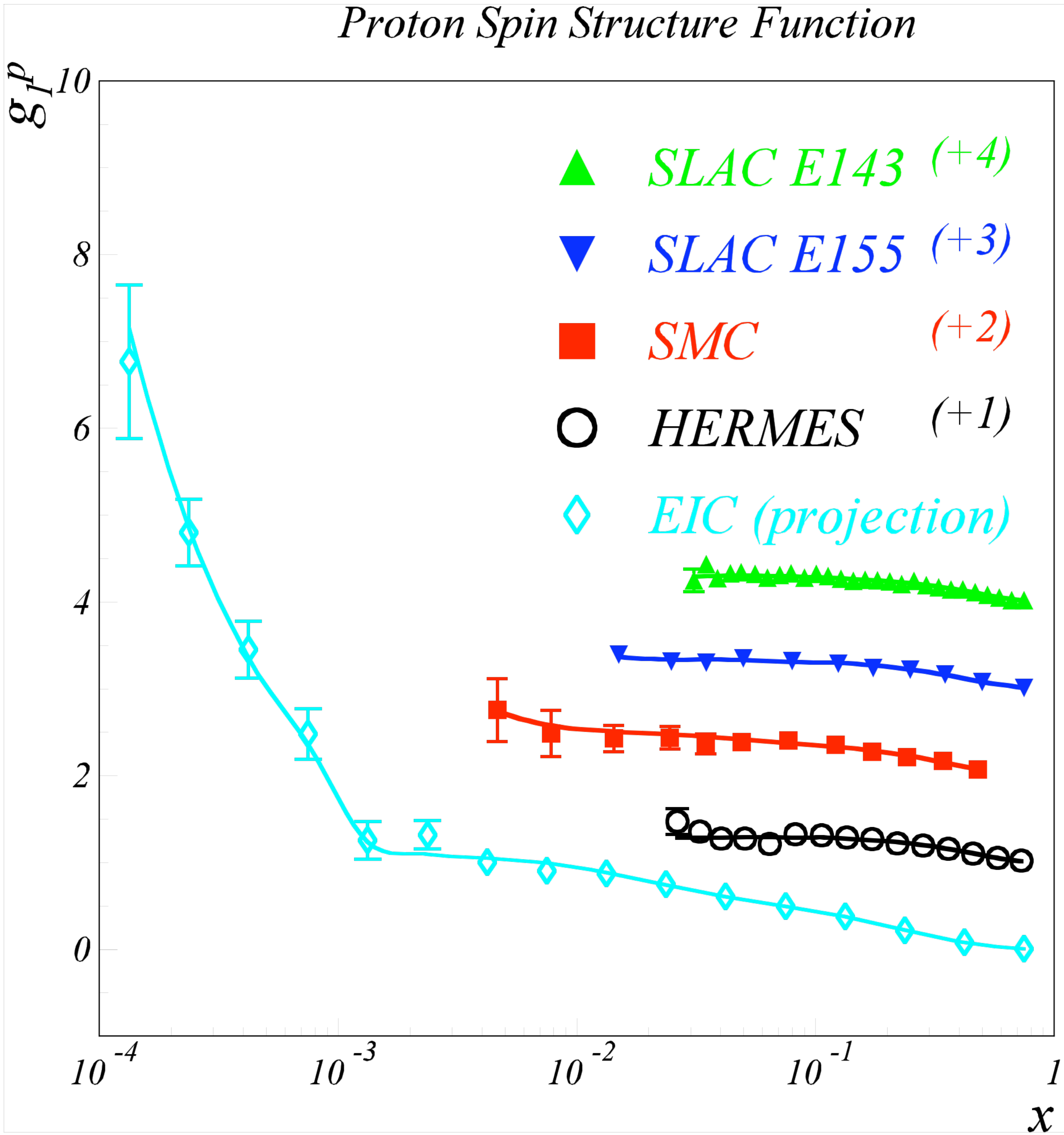


Proton Spin Structure Function
g_1^p
10
8
6
4
2
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
SLAC E143
SLAC E155
SMC
HERMES
EIC (projection)


Proton Spin Structure Function
g_1^p
SLAC E143 (+4)
SLAC E155 (+3)
SMC (+2)
HERMES (+1)
EIC (projection)
x
10
8
6
4
2
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1