# Global Analysis of Polarized PDFs by Asymmetry Analysis Collaboration

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**Global Analysis of Polarized RHIC and DIS Data**
**BNL, Upton, U.S.A.**
**http://www.bnl.gov/riken/**

**October 8, 2007**

# Contents

- **Introduction to polarized PDFs (~ 5 min)**

  **Global analysis of longitudinally polarized PDFs**

- **Polarized PDFs of AAC (~ 20 min)**

  **AAC (Asymmetry Analysis Collaboration)**

# Papers on polarized PDFs


T. Gehrmann and W. J. Stirling, Z. Phys. C65 (1995) 461; Phys. Rev. D53 (1996) 6100.

D. de Florian, L. N. Epele, H. Fanchiotti, C. A. Garcia Canal, and R. Sassot (+G. A. Navarro),
Phys. Rev. D51 (1995) 37; D57 (1998) 5803; D62 (2000) 094025; D71 (2005) 094018.

M. Glück, E. Reya, M. Stratmann, and W. Vogelsang,
Phys. Lett. B359 (1995) 201; Phys. Rev. D53 (1996) 4775; D63 (2001) 094005.

G. Altarelli, R. D. Ball, S. Forte, and G. Ridolfi,
Nucl. Phys. B496 (1997) 337; Acta Phys. Pol. B29 (1998) 1145.

C. Bourrely, F. Buccella, O. Pisanti, P. Santorelli, and J. Soffer,
Prog. Theor. Phys. 99 (1998) 1017; Eur. Phys. J. C23 (2002) 487; C41 (2005) 327;
Phys. Lett. B648 (2007) 39.

L. E. Gordon, M. Goshtasbpour, and G. P. Ramsey, Phys. Rev. D58 (1998) 094017.

SMC (B. Adeva et al.), Phys. Rev. D58 (1998) 112002.

E. Leader, A. V. Sidrov, and D. B. Stamenov,
Phys. Rev. D58 (1998) 114028; Eur. Phys. J. C23 (2002) 479; PRD67 (2003) 074017;
JHEP 0506 (2005) 033; PRD73 (2006) 034023; D75 (2007) 074027.

AAC (Y. Goto et al., M. Hirai et al.), PRD62 (2000) 034017; D69 (2004) 054021; D74 (2006) 014015.

J. Bartelski and S. Tatur, Phys. Rev. D65 (2002) 034002.

J. Blümlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.


It is likely that I miss some papers!
(Analyses of experimental groups)

# Situation of data for polarized PDFs

| Process/ Experiment | Leading order subprocess | Parton behaviour probed |
|---|---|---|
| **DIS ($\mu N \to \mu X$)** $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)* | $\gamma^* q \to q$ | Four structure functions → $u+\bar{u}$ $d+\bar{d}$ $\bar{u}+\bar{d}$ $s$ (assumed $=\bar{s}$), but only $\int xg(x,Q_0^2)dx \simeq 0.35$ and $\int(\bar{d}-\bar{u})dx \simeq 0.1$ |
| **DIS ($\nu N \to \mu X$)** $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)* | $W^* q \to q'$ | |
| **DIS (small $x$)** $F_2^{ep}$ (H1, ZEUS)* | $\gamma^*(Z^*)q \to q$ | $\lambda$ $(x\bar{q} \sim x^{-\lambda_S},\ xg \sim x^{-\lambda_g})$ |
| **DIS ($F_L$)** NMC, HERA | $\gamma^* g \to q\bar{q}$ | $g$ |
| **$\ell N \to c\bar{c}X$** $F_2^c$ (EMC; H1, ZEUS)* | $\gamma^* c \to c$ | $c$ $(x \gtrsim 0.01;\ x \lesssim 0.01)$ |
| **$\nu N \to \mu^+\mu^- X$** (CCFR)* | $W^* s \to c$ $\hookrightarrow \mu^+$ | $s \approx \frac{1}{4}(\bar{u}+\bar{d})$ |
| **$pN \to \gamma X$** (WA70*, UA6, E706, ...) | $qg \to \gamma q$ | $g$ at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2-0.6$ |
| **$pN \to \mu^+\mu^- X$** (E605, E772)* | $q\bar{q} \to \gamma^*$ | $\bar{q} = ...(1-x)^{\eta_S}$ |
| **$pp, pn \to \mu^+\mu^- X$** (E866, NA51)* | $u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$ | $\bar{u}-\bar{d}$ $(0.04 \lesssim x \lesssim 0.3)$ |
| **$ep, en \to e\pi X$** (HERMES) | $\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$ | $\bar{u}-\bar{d}$ $(0.04 \lesssim x \lesssim 0.2)$ |
| **$p\bar{p} \to WX(ZX)$** (UA1, UA2; CDF, D0) | $ud \to W$ | $u, d$ at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13;\ 0.05$ |
| → **$\ell^\pm$ asym** (CDF)* | | slope of $u/d$ at $x \approx 0.05-0.1$ |
| **$p\bar{p} \to t\bar{t}X$** (CDF, D0) | $q\bar{q}, gg \to t\bar{t}$ | $q, g$ at $x \gtrsim 2m_t/\sqrt{s} \simeq 0.2$ |
| **$p\bar{p} \to$jet $+X$** (CDF, D0) | $gg, qg, qq \to 2j$ | $q, g$ at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05-0.5$ |

**(MRST, hep/ph-9803445)**

→ **Neutrino factory: 10~15 years later**

→ **Small-x: eRHIC, eLIC ?**

→ **Charm: eRHIC, eLIC ?**

→ **RHIC**

**RHIC-Spin program should play an important role in determining polarized PDFs.**

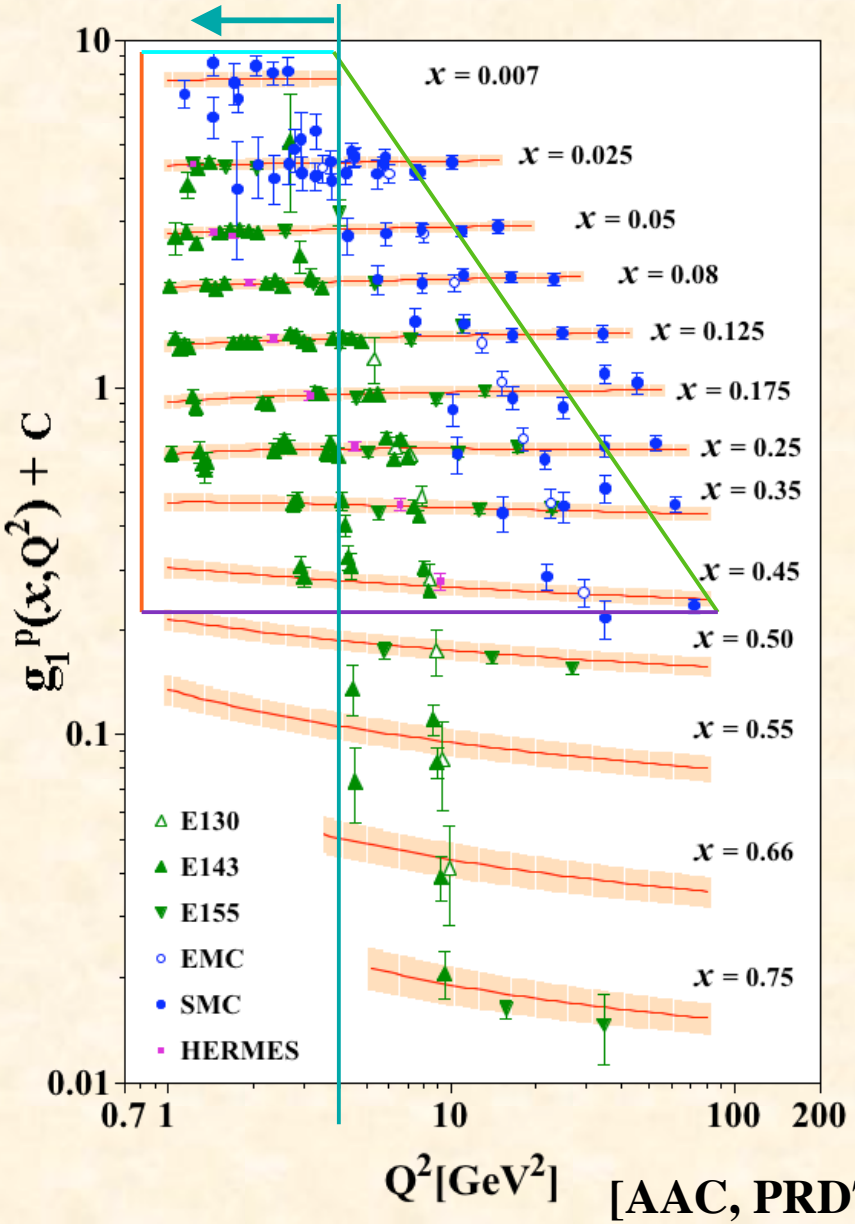
→ **pp: RHIC**

**J-PARC**

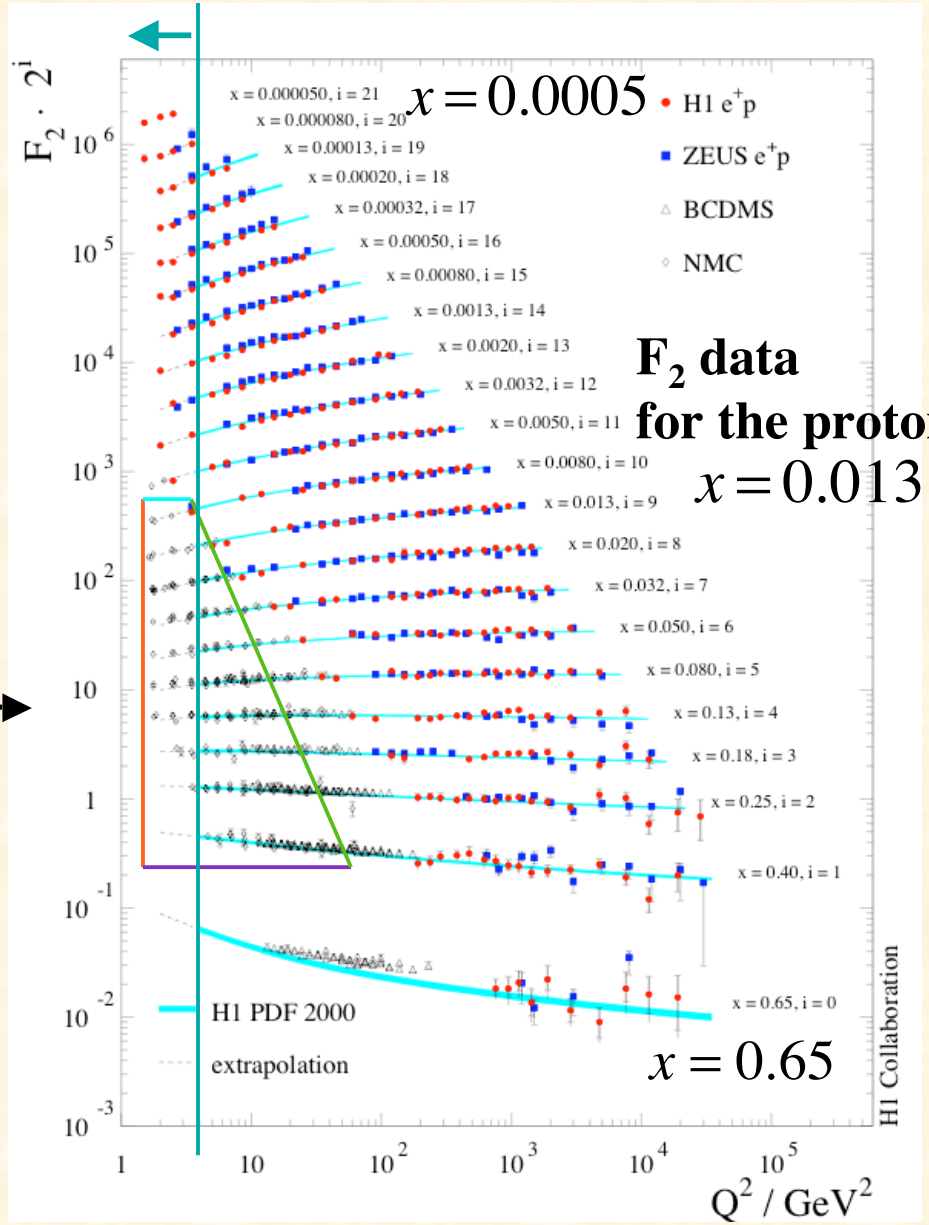
**GSI-FAIR**

→ **pp: RHIC**

**It may be possible to remove small $Q^2$ data in an analysis of unpolarized PDFs, whereas it is difficult to remove them in $g_1$ (or $A_1$).**




**(from H1 and ZEUS, hep-ex/0502008)**




**[AAC, PRD74 (2006) 014015]**

## Comments on the previous figure

- lack of small-$x$ data $\rightarrow$ makes the determination of $\lim_{x\to 0}\Delta q(x)$ difficult

  $\rightarrow$ quark spin content cannot be well determined

- lack of data in a wide range of $Q^2$ at small $x$

  $\rightarrow$ makes the determination of $\Delta g(x)$ difficult

  $\rightarrow$ gluon contribution to the proton spin cannot be determined

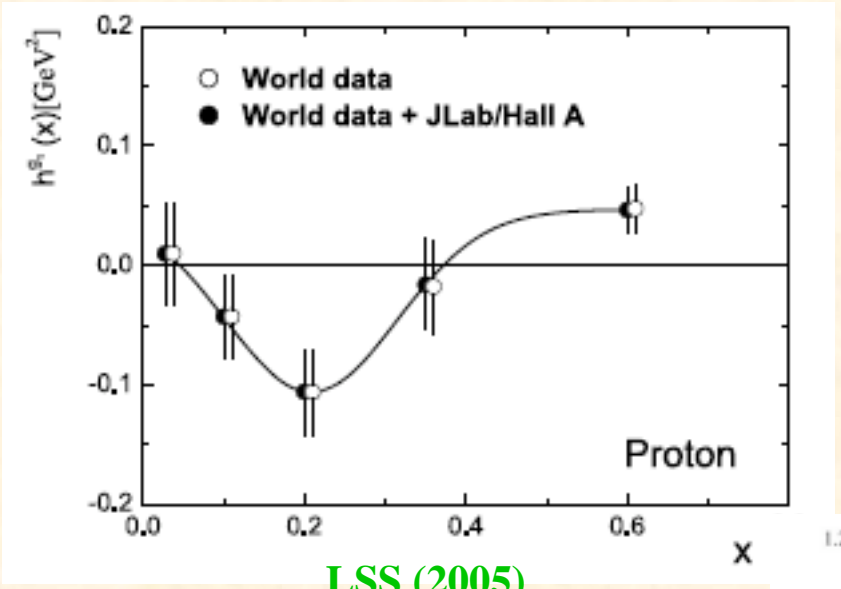
- lack of data in a wide range of $Q^2$

  $\rightarrow$ difficult to remove the small $Q^2 (< 4\ \mathrm{GeV}^2)$ data,

  which may contain higher-twist effects, from the anlysis data set

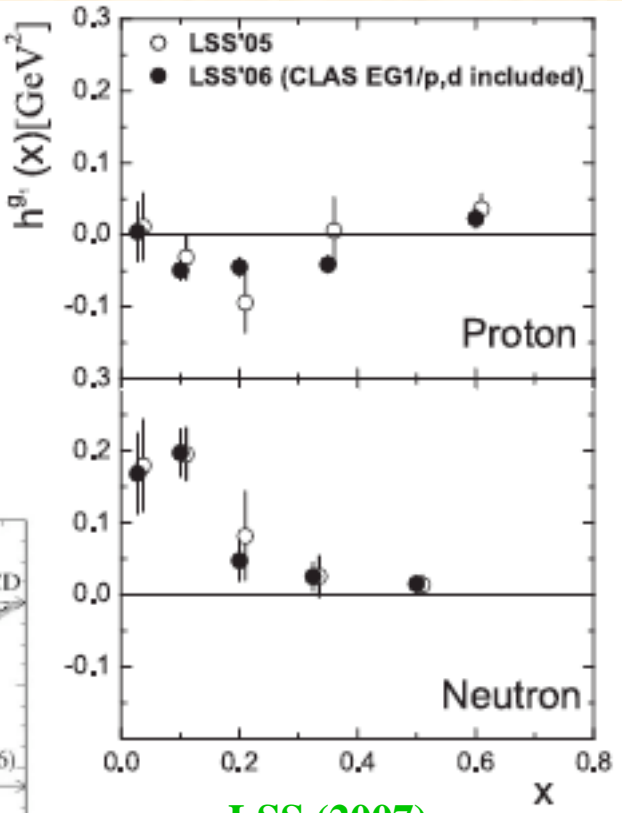
# Higher-twist effects

$$A_1(x,Q^2) = \frac{g_1(x,Q^2)_{LT} + h(x)/Q^2}{F_2(x,Q^2)_{\exp}} 2x\left[1 + R(x,Q^2)_{\exp}\right]$$

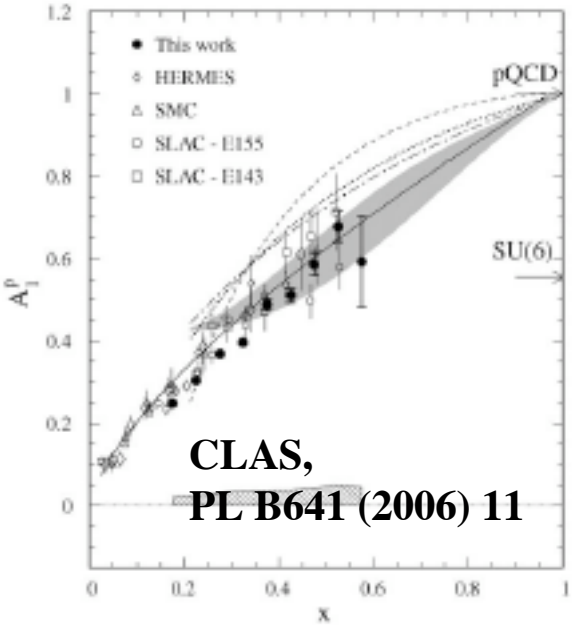



LSS (2005)

with 2006 CLAS data




LSS (2007)



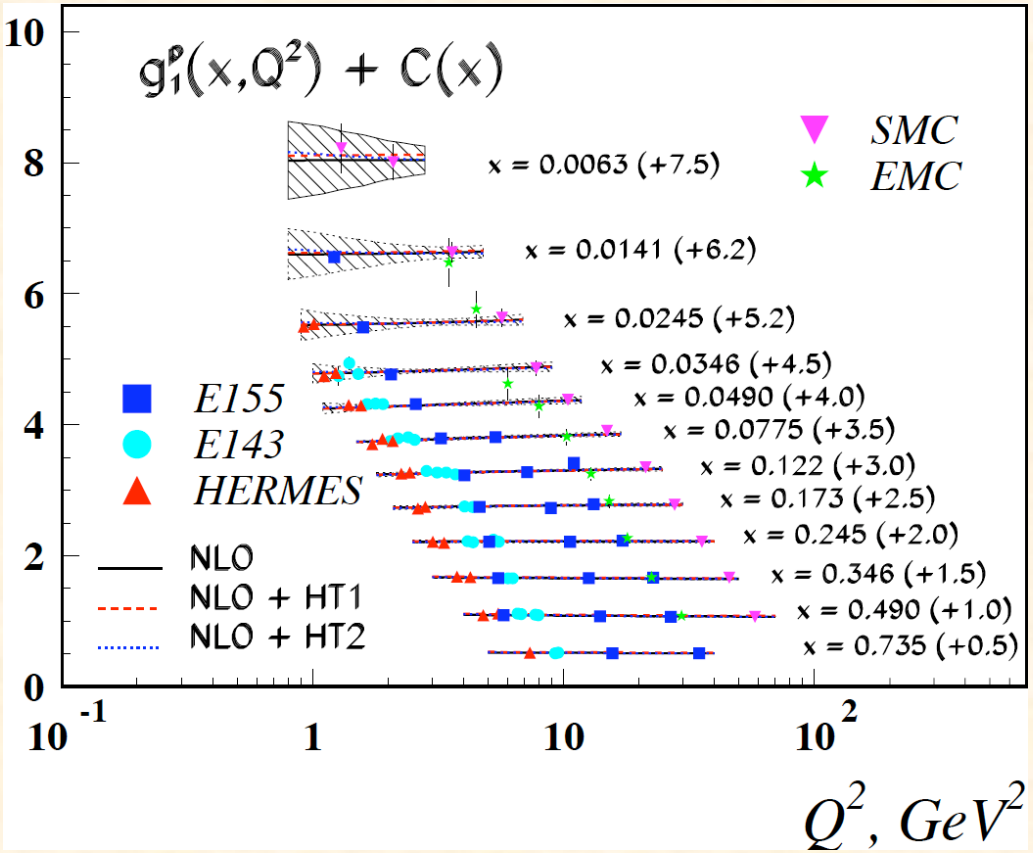

CLAS, PL B641 (2006) 11

According to LSS-2007 analysis, the CLAS-2006 data improve our knowledge of higher-twist effects.

E. Leader, A. V. Sidrov, and D. B. Stamenov, JHEP 0506 (2005) 033; D75 (2007) 074027.

# Higher-twist effects

$$h_I(x,Q^2) = 1 + \frac{A}{Q^2} x^{\alpha}(1-x)^{\beta}, \quad h_{II}(x,Q^2) = 1 + \frac{1}{Q^2}(A + Bx + cx^2)$$




**Higher-twist effects become conspicuous at small $Q^2$; however, they are within the error bands.**

**The present data with $Q^2 > 1$ GeV$^2$ do not contain significant higher -twist contributions.**

**J. Blümlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.**

# Asymmetry Analysis Collaboration (AAC)

http://spin.riken.bnl.gov/aac/

**Original Members:**

Y. Goto, N. Hayashi, M. Hirai, H. Horikawa,
S. Kumano, M. Miyama, T. Morii, N. Saito,
T.-A. Shibata, E. Taniguchi, T. Yamanishi

(Theorist, Experimentalist)

**Three publications:**

(AAC00) Y. Goto *et al.*, Phys. Rev. D62 (2000) 034017.

(AAC03) M. Hirai, S. Kumano, N. Saito, Phys. Rev. D69 (2004) 054021;

(AAC06) D74 (2006) 014015.

Active members now

- **2000 version (AAC00)**
  - **$Q^2$ dependence of $A_1$, positivity**
  - **$\Delta\bar{q}$ at small and large $x \rightarrow \Delta\Sigma$ issue**
- **2004 version (AAC03)**
  - **uncertainty estimation**
    **(very large $\Delta g$ uncertainty, impact of accurate $g_1^p$ (E155))**
  - **error correlation between $\Delta g$ and $\Delta\bar{q}$**
- **2006 version (AAC06)**
  - **include RHIC-Spin $\pi^0$ ($\Delta g$ uncertainty is significantly reduced)**
  - **$\Delta g$ at large $x$ ?**
    **($Q^2$ difference between HERMES and COMPASS)**
  - **$\Delta g < 0$ solution**

# General strategies for determining polarized PDFs

$$MW_1 = F_1$$

$$A_1 \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{M\nu G_1 - Q^2 G_2}{M^3 W_1} \simeq \frac{g_1}{F_1} = g_1 \frac{2x(1+R)}{F_2}$$

$$\frac{\nu}{M^2} G_1 = g_1 \qquad \frac{\nu^2}{M^2} G_2 = g_2$$

$\sigma_{\lambda+\lambda_N} \propto \varepsilon_\lambda^{\mu *} \varepsilon_\lambda^{\nu} W_{\mu\nu}(\lambda_N)$ : photoabsorption cross section

$$R \equiv \frac{F_L}{2xF_1} = \frac{F_2 - 2xF_1}{2xF_1}$$

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \int_x^1 \frac{dy}{y}\left[\Delta q(x/y,Q^2) + \Delta\bar{q}(x/y,Q^2)\right]\left[\delta(1-y) + \frac{\alpha_s(Q^2)}{2\pi}\Delta C_q(y) + \cdots\right]$$

$$+\frac{1}{2}\left\langle e_q^2\right\rangle \int_x^1 \frac{dy}{y}\Delta g(x/y,Q^2)\left[n_f \frac{\alpha_s(Q^2)}{2\pi}\Delta C_g(y) + \cdots\right]$$

$$\left\langle e_q^2\right\rangle = \frac{1}{n_f}\sum_q e_q^2$$

**Leading Order (LO)** **Next to Leading Order (NLO)**

$\Delta C_q$ $(\Delta C_g)$ = quark (gluon) coefficient function

$$F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y}\left[q(x/y,Q^2) + \bar{q}(x/y,Q^2)\right]\left[\delta(1-y) + \frac{\alpha_s(Q^2)}{2\pi}C_q^{(2)}(y) + \cdots\right]$$

$$+ x\left\langle e_q^2\right\rangle \int_x^1 \frac{dy}{y} g(x/y,Q^2)\left[n_f \frac{\alpha_s(Q^2)}{2\pi}C_g^{(2)}(y) + \cdots\right]$$

**Unpolarized PDFs**

$R(x,Q^2)$ : taken from experimental measurements

# Initial distributions

$$\Delta f_i(x,Q_0^2) = A_i\, x^{\alpha_i} (1 + \gamma_i\, x^{\lambda_i})\, f_i(x,Q_0^2)$$

$i = u_v, d_v, \bar{q}, g$ $\qquad A_i\ ,\ \alpha_i\ ,\ \gamma_i\ ,\ \lambda_i$ : parameters

**$\chi^2$ fit to the data [p, n ($^3$He), d]**

$$\chi^2 = \sum_i \frac{(A_{1\,i}^{data} - A_{1\,i}^{calc})^2}{(\sigma_{A_{1\,i}}^{data})^2}$$

$$A_1 \simeq \frac{g_1}{F_1} = g_1 \frac{2x(1+R)}{F_2} \qquad R = \frac{F_L}{2xF_1} = \frac{F_2 - 2xF_1}{2xF_1}$$

**We analyzed the data with the following conditions.**

- **unpolarized PDF** **GRV98**
- **initial $Q^2$** **$Q_0^2 = 1$ GeV $^2$**
- **number of flavor** **$N_f = 3$**
- **positivity** **$|\Delta f(x)| \le f(x)$ (to be precise, $|\Delta\sigma| \le \sigma$)**
- **antiquark flavor:** **$\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s}$**

(1) $\Delta q(x,Q_0^2)$, $\Delta\bar{q}(x,Q_0^2)$, $\Delta g(x,Q_0^2)$ : expressed in terms of parameters

$\rightarrow$ evolve them to experimental $Q^2$ points

$\rightarrow$ calculate $g_1$

(2) $q(x,Q_0^2)$, $\bar{q}(x,Q_0^2)$, $g(x,Q_0^2)$ : taken from a typical unpolarized PDFs (CTEQ, GRV, MRST)

$\rightarrow$ evolve them to experimental $Q^2$ points

$\rightarrow$ calculate $F_2$

or

(2′) $q(x,Q^2)$, $\bar{q}(x,Q^2)$, $g(x,Q^2)$ : calculated by a library code of a typical unpolarized PDFs

$\rightarrow$ calculate $F_2$

(3) $R(x,Q^2)$ : typical parametrization for explaining experimental data

(e.g. SLAC parametrization)

(4) calculate $A_1^{theo} = g_1 \dfrac{2x(1+R)}{F_2}$, then $\chi^2 = \sum_i \dfrac{(A_{1i}^{\exp} - A_{1i}^{theo})^2}{\sigma_{A_{1i}}^2}$

(5) parameters are determined so as to minimize $\chi^2$

# Positivity of polarized PDFs

$|\Delta\sigma| \le \sigma \;\rightarrow\; |\Delta f_i(x,Q^2)| \le f_i(x,Q^2)$ ?

Ref. G. Altarelli, S. Forte, G. Ridolfi, Nucl. Phys. B534 (1998) 277.

$$|A_1| = \left|\frac{\sigma_{1/2}-\sigma_{3/2}}{\sigma_{1/2}+\sigma_{3/2}}\right| = \left|\frac{g_1}{F_1}\right| \le 1 \quad \rightarrow \quad |g_1| \le F_1$$

$$g_1(x,Q^2) = \frac{1}{2}\sum_i e_i^2 \Delta C_q \otimes (\Delta q_i + \Delta\bar{q}) + 2n_f \langle e^2 \rangle \Delta C_g \otimes \Delta g$$

$$F_1(x,Q^2) = \frac{1}{2}\sum_i e_i^2 C_q \otimes (q_i + \bar{q}) + 2n_f \langle e^2 \rangle C_g \otimes g$$

$$\langle e^2 \rangle \equiv \frac{1}{n_f}\sum_i e_i^2$$

$$f \otimes g \equiv \int_x^1 \frac{dy}{y} f(y) g(x/y)$$

in LO $\quad C_q = \Delta C_q = \delta(1-x), \quad C_g = \Delta C_g = 0$

$$\rightarrow \left|\sum_i e_i^2 \left[\Delta q_i(x,Q^2) + \Delta\bar{q}_i(x,Q^2)\right]\right| \le \sum_i e_i^2 \left[q_i(x,Q^2) + \bar{q}_i(x,Q^2)\right]$$

This bound should be satisfied for any target in electron/muon and neutrino scattering processes, i.e. for any combination of quark and antiquark distributions. Therefore, we have

$$|\Delta q_i(x,Q^2)| \le q_i(x,Q^2), \quad |\Delta\bar{q}_i(x,Q^2)| \le \bar{q}_i(x,Q^2) \quad \text{in LO}$$

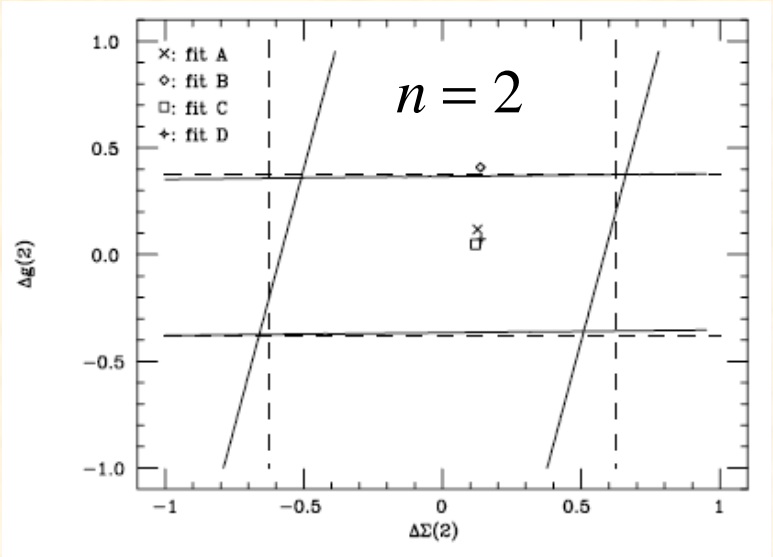
# NLO numerical results for positivity

(from Nucl. Phys. B534 (1998) 277)

$$\Delta\Sigma(n,Q^2) = \int_0^1 dx\, x^{n-1} \sum_i \left[\Delta q_i(x,Q^2) + \Delta\bar{q}_i(x,Q^2)\right]$$

$$\Delta g(n,Q^2) = \int_0^1 dx\, x^{n-1} \Delta g(x,Q^2)$$

A, B, C, D = Altarelli-Ball-Forte-Ridolfi (1998) parametrizations



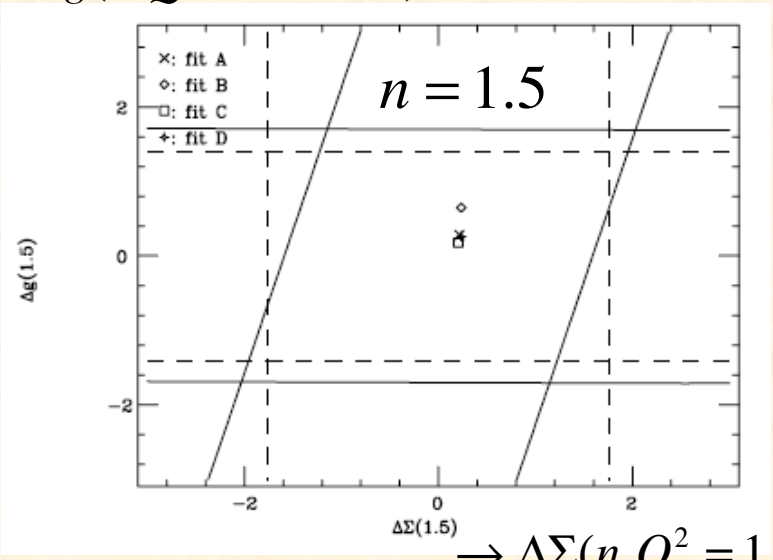

- NLO effects modify the positivity bound
- NLO bound is either more or less restrictive
- NLO corrections are reasonably small
- NLO corrections are negligible for large n

——— **NLO positivity bound**

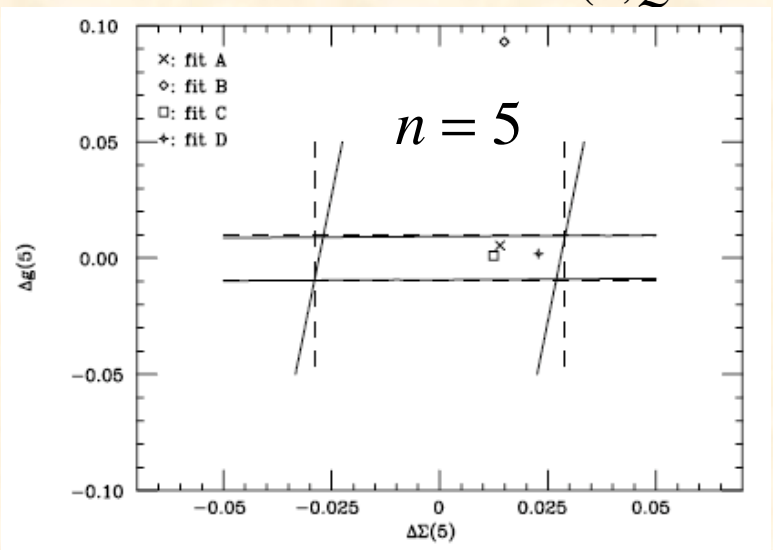
- - - - - - - - - - **LO**

NLO ($\overline{\text{MS}}$)

$\uparrow \Delta g(n,Q^2 = 1\ \text{GeV}^2)$



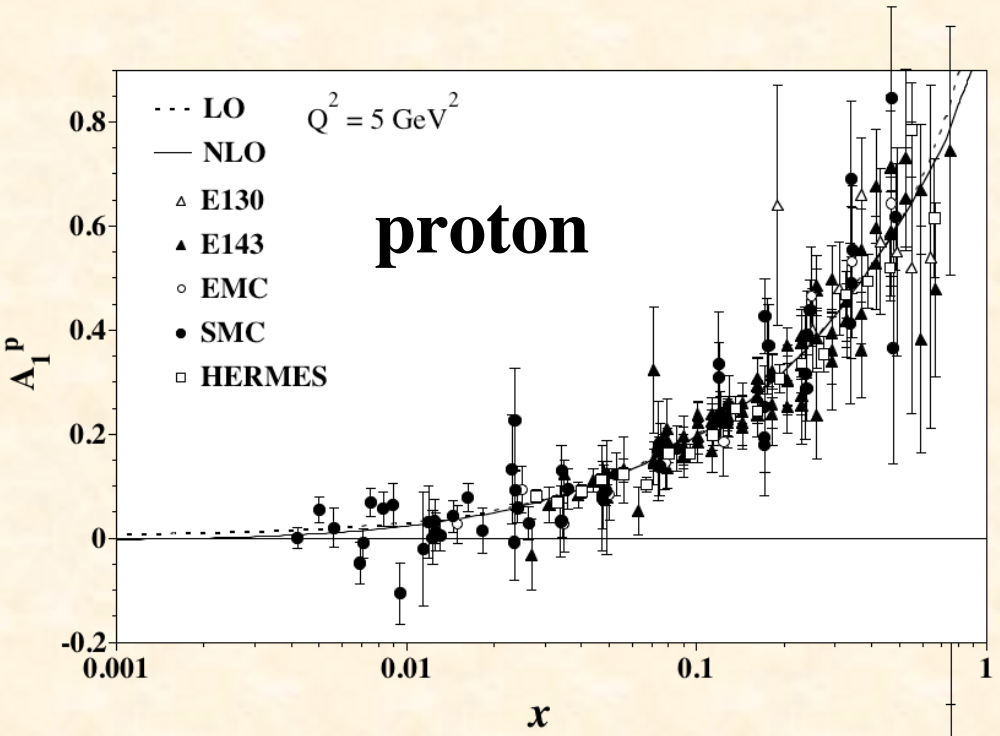

$\rightarrow \Delta\Sigma(n,Q^2 = 1\ \text{GeV}^2)$

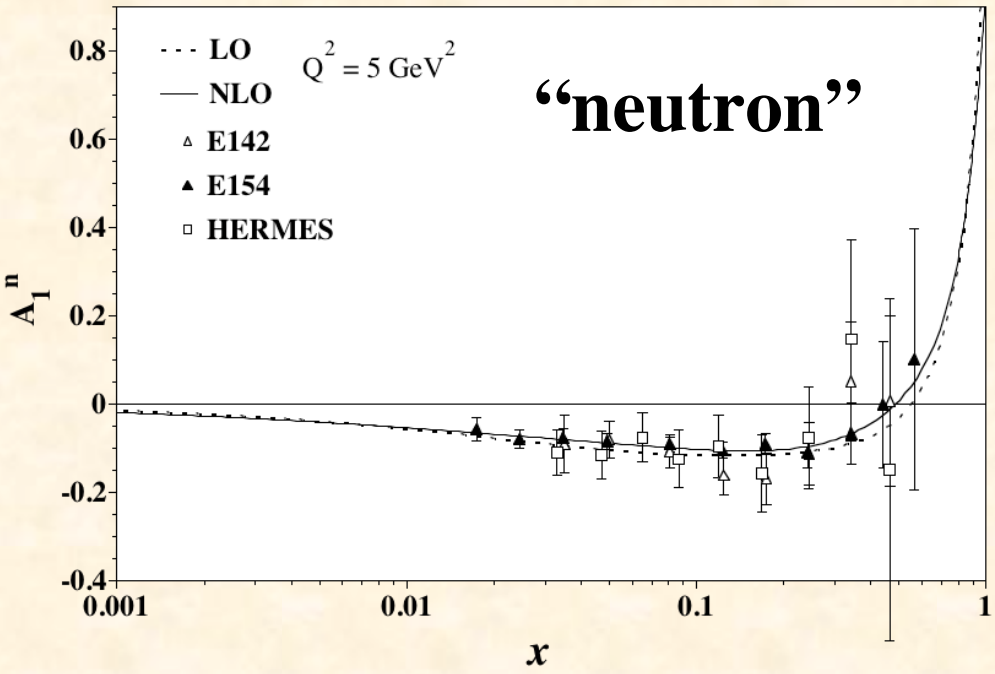


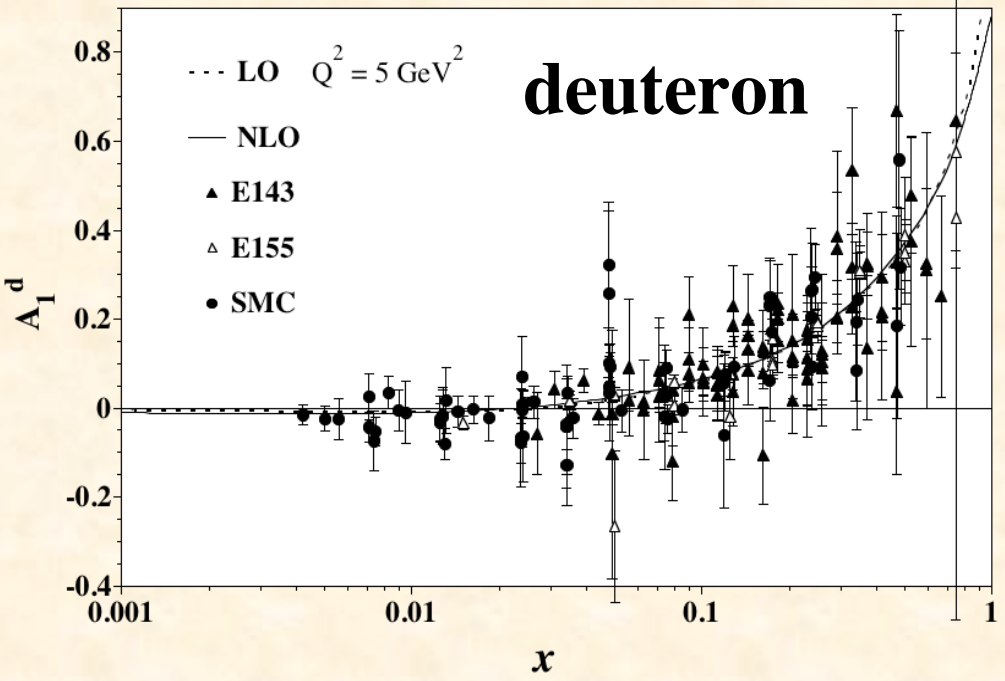

**Conclusion: Positivity is a useful condition for determining polarized PDFs although it is not a strict one if higher-order corrections are taken into account.**

# AAC00
# (Some Highlights)

# Actual fit to $A_1$ data (AAC00)



# $Q^2$ dependence of $A_1$

**$Q^2$ independence assumption was sometimes used for getting $g_1$ from $A_1$ data.**

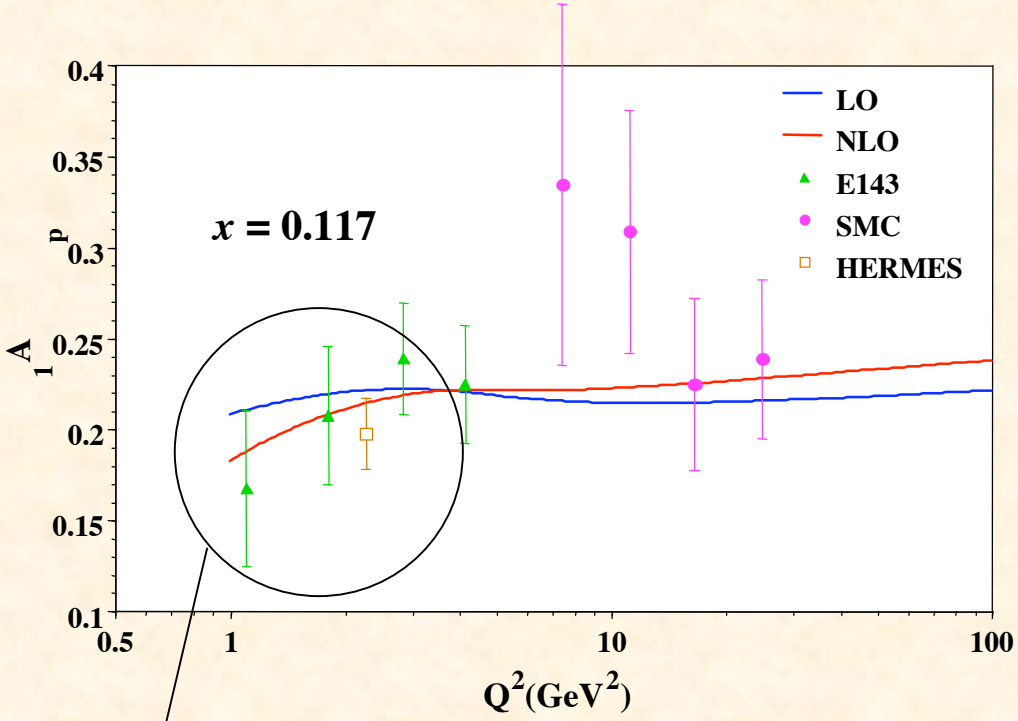



**Figure from AAC00 analysis**

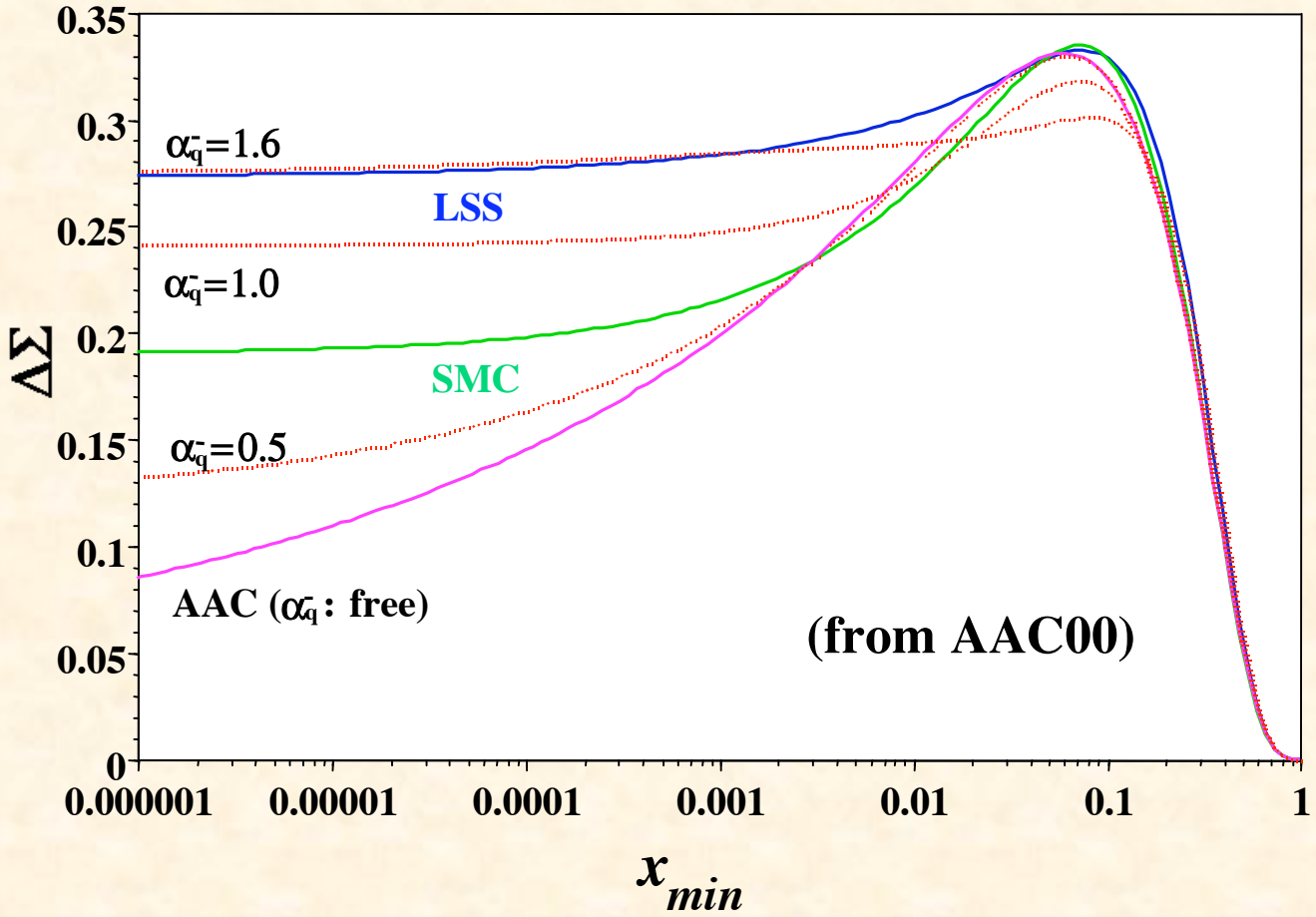
**$A_1$ is $Q^2$ dependent especially at small $Q^2$ ($< 2$ GeV$^2$).**

**The lack of accurate $Q^2$ dependent data at small *x* makes the determination of $\Delta g$ very difficult.**

# “Spin content” $\Delta\Sigma$

$$\Delta\Sigma(x_{\min}) = \int_{x_{\min}}^{1} \Delta\Sigma(x)\, dx$$

$$\frac{\Delta\bar{q}}{\bar{q}} \propto x^{\alpha_{\bar{q}}} \quad (x \to 0)$$



# AAC03
# (Some Highlights)

# Error estimation

## Hessian method

$$\chi^2 = \sum_i \frac{(A_{1i}^{\exp} - A_{1i}^{theo})^2}{\sigma_{A_{1i}}^2}$$

**$\chi^2(\xi)$ is expanded around its minimum $\xi_0$ ($\xi$ =parameter)**

$$\chi^2(\xi_0 + \delta\xi) = \chi^2(\xi_0) + \sum_i \frac{\partial \chi^2(\xi_0)}{\partial \xi_i}\delta\xi_i + \frac{1}{2}\sum_{i,j} \frac{\partial^2 \chi^2(\xi_0)}{\partial \xi_i \partial \xi_j}\delta\xi_i \delta\xi_j + \cdots$$

**where the Hessian matrix is defined by**

$$H_{ij} = \frac{1}{2}\frac{\partial^2 \chi^2(a_0)}{\partial \xi_i \partial \xi_j}$$

**In the $\chi^2$ analysis, 1σ standard error is**

$$\Delta\chi^2 = \chi^2(\xi_0+\delta\xi) - \chi^2(\xi_0) = \sum_{i,j} \delta\xi_i H_{ij} \delta\xi_j$$

$P(s)_N$: $\chi^2(= s)$ distribution with N degrees of freedom

$$\int_0^{\Delta\chi^2} ds\, P(s)_{N=11} = 0.6826 \rightarrow \Delta\chi^2 = 12.64 \quad (N = 1 \text{ case}, \ \Delta\chi^2 = 1)$$

**The error of a distribution $F(x)$ is given by**

$$\left[\delta F(x)\right]^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F(x)}{\partial \xi_i} H_{ij}^{-1} \frac{\partial F(x)}{\partial \xi_j}$$

$\chi^2 (= s)$ distribution with N degrees of freedom

$$P(s) = \frac{1}{2^{N/2}\Gamma(N/2)}\, s^{N/2-1} \exp(-s/2)$$

(1) N = 1 case, $\Delta\chi^2 = 1$

$$\int_0^{\Delta\chi^2 = 1} ds\, P(s)_{n=1} = \mathrm{erf}\,(1/\sqrt{2}) = 0.6826...$$

where $\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} \int_0^z dt\, e^{-t^2}$

(2) our N = 11 case, $\Delta\chi^2 = ?$

$$\int_0^{\Delta\chi^2} ds\, P(s)_{n=11} = \int_0^{\Delta\chi^2} ds\, \frac{s^{9/2}\, e^{-s/2}}{2^{11/2}\,\Gamma(11/2)} = 0.6826...$$

Solving this equation, we have $\Delta\chi^2 = 12.64...$

**Refs. http://wwwasdoc.web.cern.ch/wwwasdoc/minuit/node33.html**
**http://www.library.cornell.edu/nr/bookfpdf/f15-6.pdf**
**http://ccwww.kek.jp/pdg/2005/reviews/statrpp.pdf**

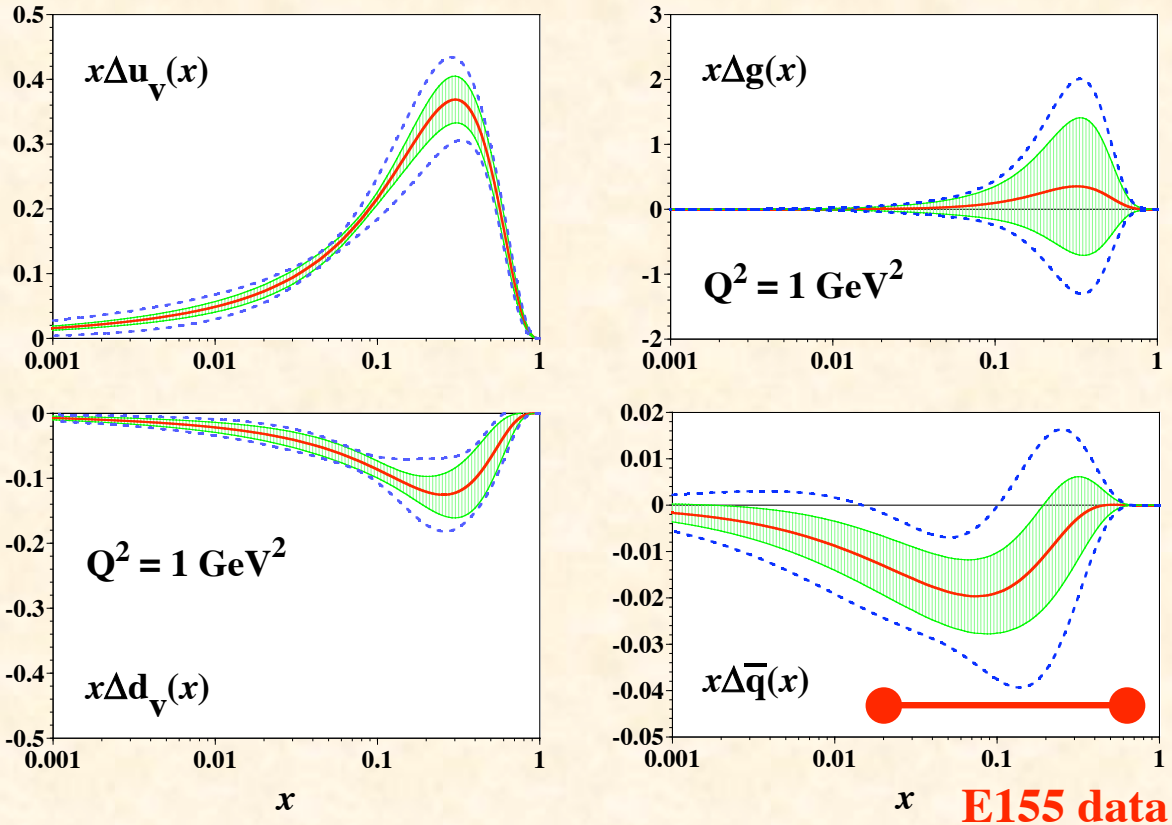
# Polarized PDFs (AAC03)

- **PDF uncertainties are reduced by including precise (E155-p) data**
- **Valence-quark distributions are well determined**
  - **Small uncertainties of $\Delta u_v$, $\Delta d_v$**
- **Antiquark uncertainty is significantly reduced**
  - $g_1^p \propto 4\Delta u_v + \Delta d_v + 12\Delta\bar{q}$
- **$\Delta g(x)$ is not determined**
  - **Large uncertainty**
  - **Indirect contribution to $g_1^p$**
  - **Correlation with antiquark**

**AAC03 uncertainties**

**AAC00 uncertainties**

**AAC00 ↔ AAC03 (with E155-p)**




**E155 data**

# Correlation between $\Delta\bar{q}(x)$ and $\Delta g(x)$

- **analysis with $\Delta g(x)=0$ at $Q^2=1$ GeV$^2$**

  $\chi^2$/d.o.f. = 0.915

- **$\Delta q_v(x)$ uncertainties are not affected**

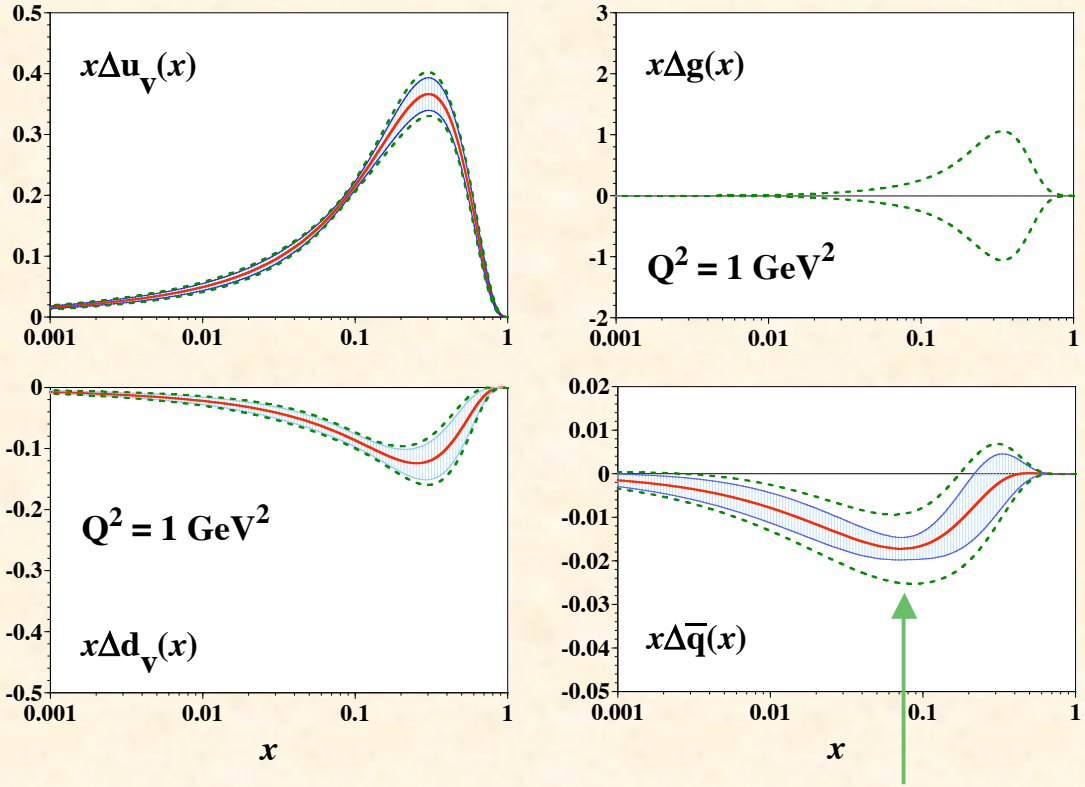
- **antiquark uncertainties are reduced**

  **Strong correlation with $\Delta g(x)$**

  **Note: correlation with $\Delta g(x)$ is almost terminated in the $\Delta g=0$ analysis**

**$\Delta g=0$ uncertainties**

**AAC03 uncertainties**




**The error band shrinks due to the correlation with $\Delta g(x)$.**

# AAC06

# Gluon polarization at large $x$

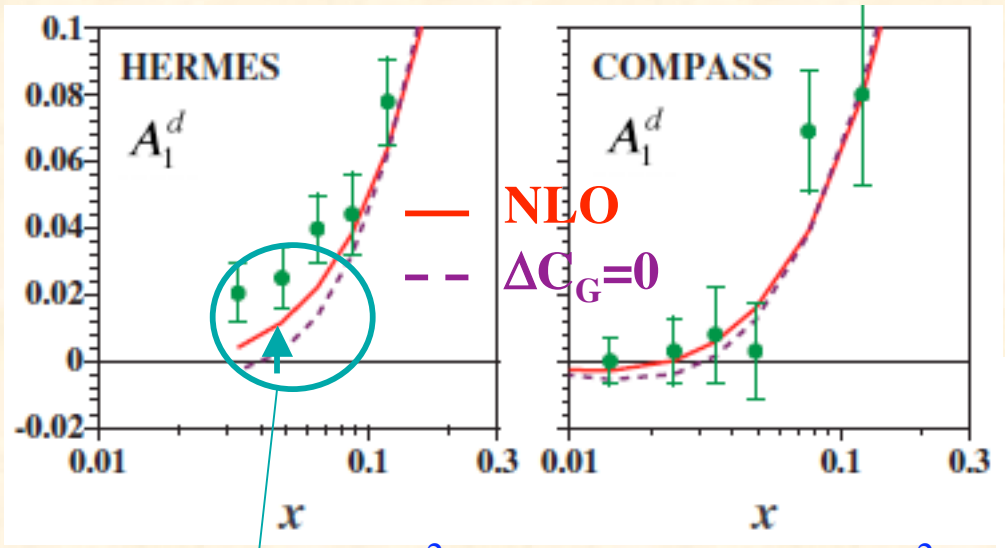
$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \int_x^1 \frac{dz}{z}\left[\Delta q(x/z,Q^2) + \Delta\bar{q}(x/z,Q^2)\right]$$

$$\times\left[\delta(1-z) + \frac{\alpha_s(Q^2)}{2\pi}\Delta C_q(z) + \cdots\right]$$

$$+\frac{1}{2}\left\langle e_q^2\right\rangle \int_x^1 \frac{dz}{z}\Delta g(x/z,Q^2)\left[n_f \frac{\alpha_s(Q^2)}{2\pi}\Delta C_g(z) + \cdots\right]$$

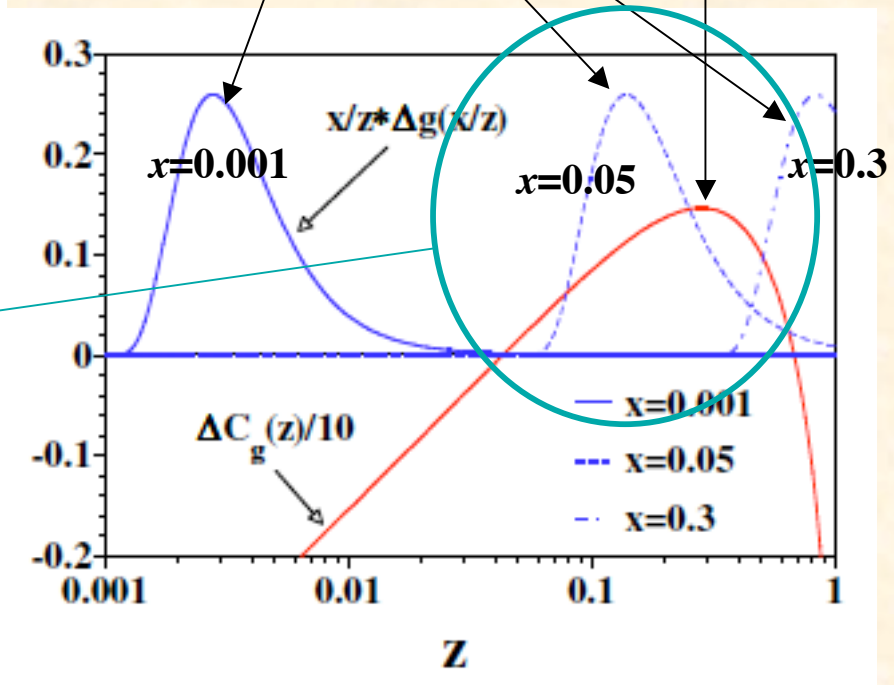
**AAC, PRD74 (2006) 014015:**

**Analysis without higher-twist effects**




$Q^2_{HERMES} \sim 1$ GeV$^2$ $Q^2_{COMPASS} \sim 6$ GeV$^2$

This term is terminated.

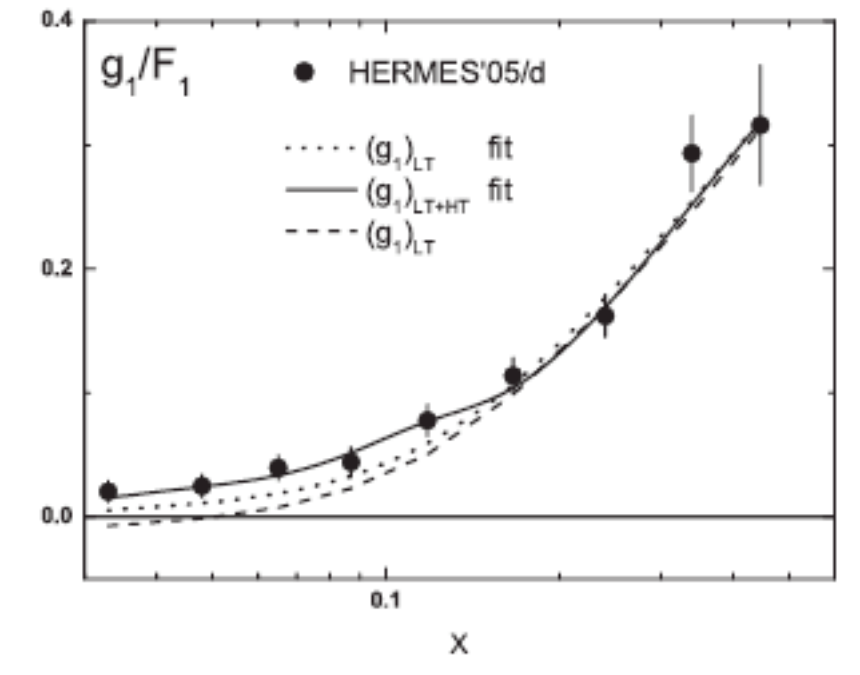



Positive contribution to $A_1$ comes from $\Delta C_G \otimes \Delta g$ at $x \sim 0.05$.

Note: $\Delta C_G \otimes \Delta g > 0$
if $\Delta g(0.05/0.2 = 0.25) > 0$
Gluon polarization is positive at large x.

# However, it may be higher-twist effect.

**LSS, PRD73 (2006) 034023.**




**Leading Twist (LT)**
**Higher Twist (HT)**

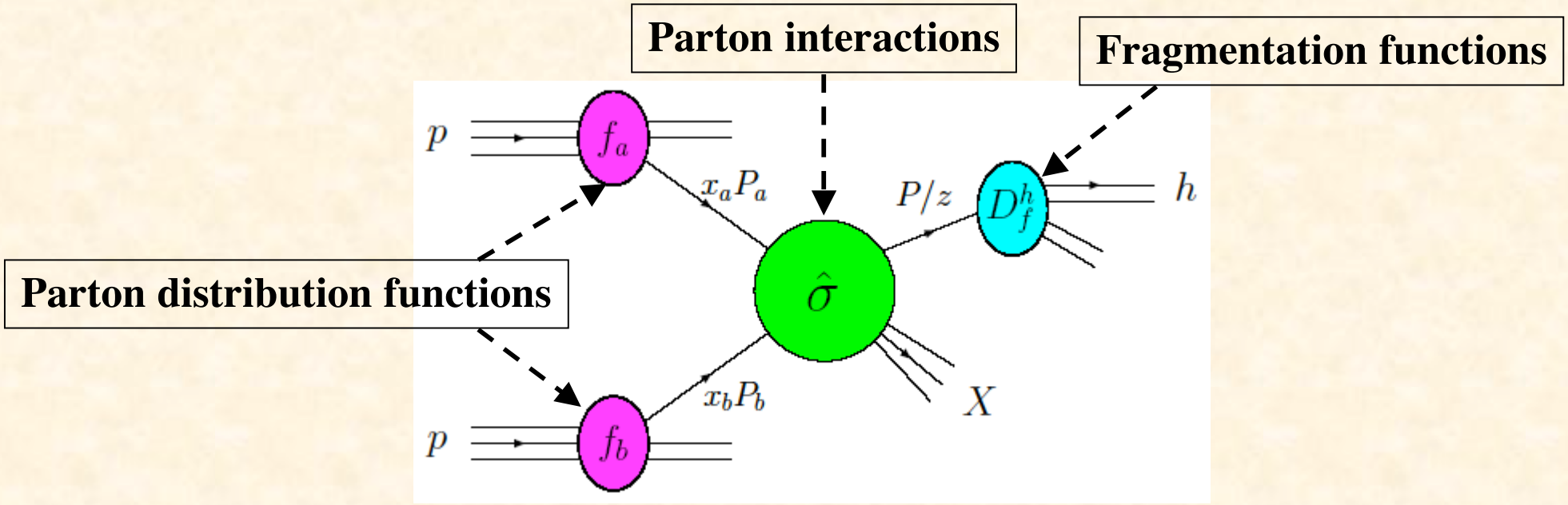
..... **LT fit**

——— **LT+HT fit**

- - - - - - - - **LT+HT fit, only LT term is shown**

**At this stage, we cannot conclude that the difference between the HERMES and COMPASS data should be 100% HT or HT+ΔG(large x)>0, or 100% ΔG(large x)>0 effects.**

# Description of $\pi^0$ production




$$\Delta\sigma \sim \sum_{a,b,c} \Delta f_a(x_a,Q^2) \otimes \Delta f_b(x_b,Q^2) \otimes \Delta\hat{\sigma}(ab \to cX) \otimes D_c^{\pi}(z,Q^2)$$

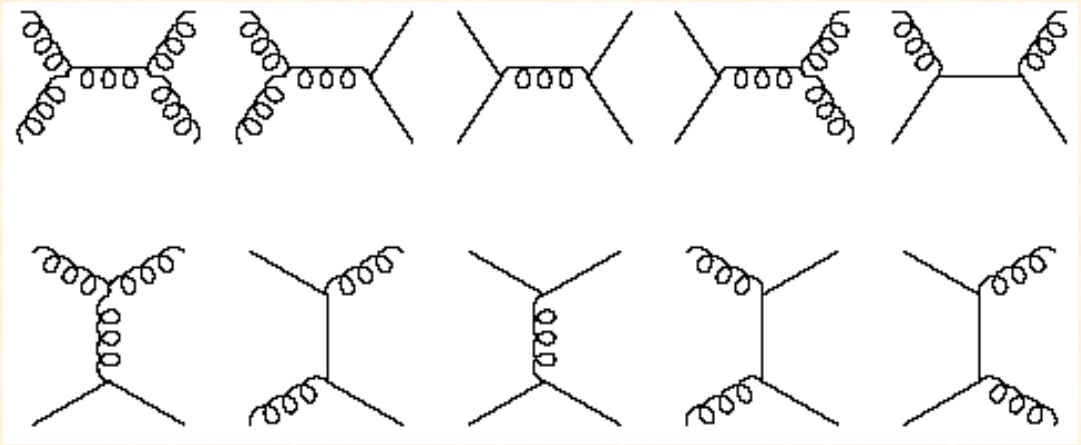
$$\frac{d\Delta\sigma}{dp_T} = \sum_{a,b,c} \int_{\eta_{\min}}^{\eta_{\max}} d\eta \int_{x_a^{\min}}^{1} dx_a \Delta f_a(x_a) \int_{x_b^{\min}}^{1} dx_b \Delta f_b(x_b) \frac{\partial(\hat{t},z_c)}{\partial(p_T,\eta)} \frac{d\Delta\hat{\sigma}}{dp_T} D_c^{\pi}(z_c)$$

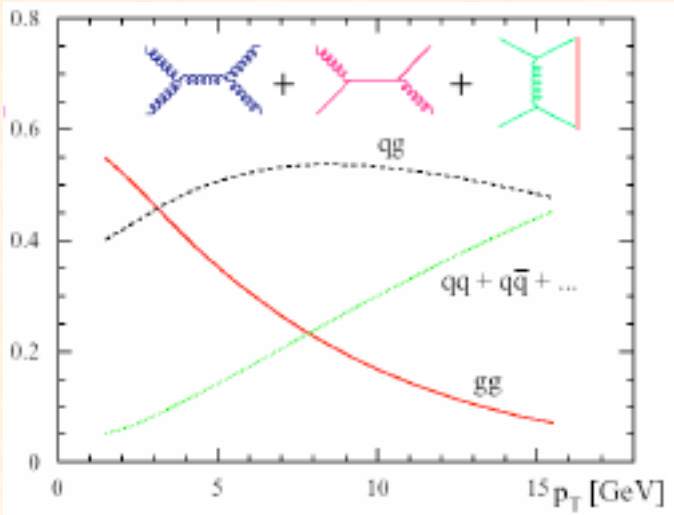
$$x_a^{\min} = \frac{x_1}{1-x_2},\quad x_b^{\min} = \frac{x_a x_2}{x_a - x_1},\quad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b},\quad x_1 = \frac{x_T}{2} e^{+\eta},\quad x_2 = \frac{x_T}{2} e^{-\eta},\quad x_T = \frac{2p_T}{\sqrt{s}},\quad \eta = -\ln\left[\tan\left(\frac{\theta_\pi}{2}\right)\right]$$

# Subprocesses

$$gg \to q(g)X, \quad qg \to q(g)X, \quad qq \to qX,$$
$$q\bar{q} \to q(g,q')X, \quad qq' \to qX, \quad q\bar{q}' \to qX$$

**(from Torii's talk at Pacific-Spin05)**



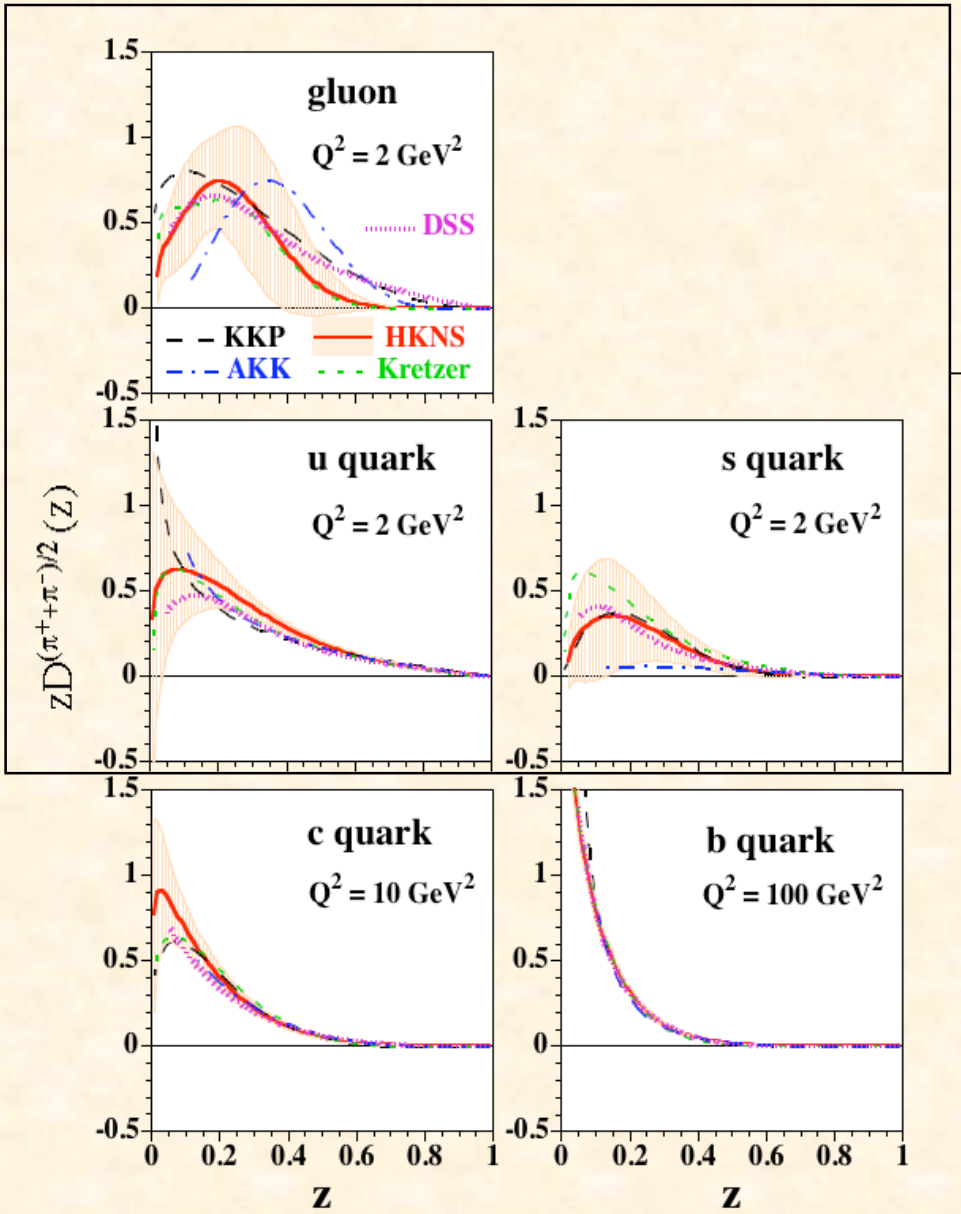



$g + g \to q(g) + X$ processes are dominant at small $p_T$

$q + g \to q(g) + X$ at large $p_T$

**The $\pi^0$ production process is suitable for finding the gluon polarization $\Delta g$.**

# Comparison of fragmentation functions in pion




(KKP) Kniehl, Kramer, Pötter

(AKK) Albino, Kniehl, Kramer

(HKNS) Hirai, Kumano, Nagai, Sudoh

(DSS) de Florian, Sassot, Stratmann

- Gluon and light-quark fragmentation functions have large uncertainties, but they are within the uncertainty bands.

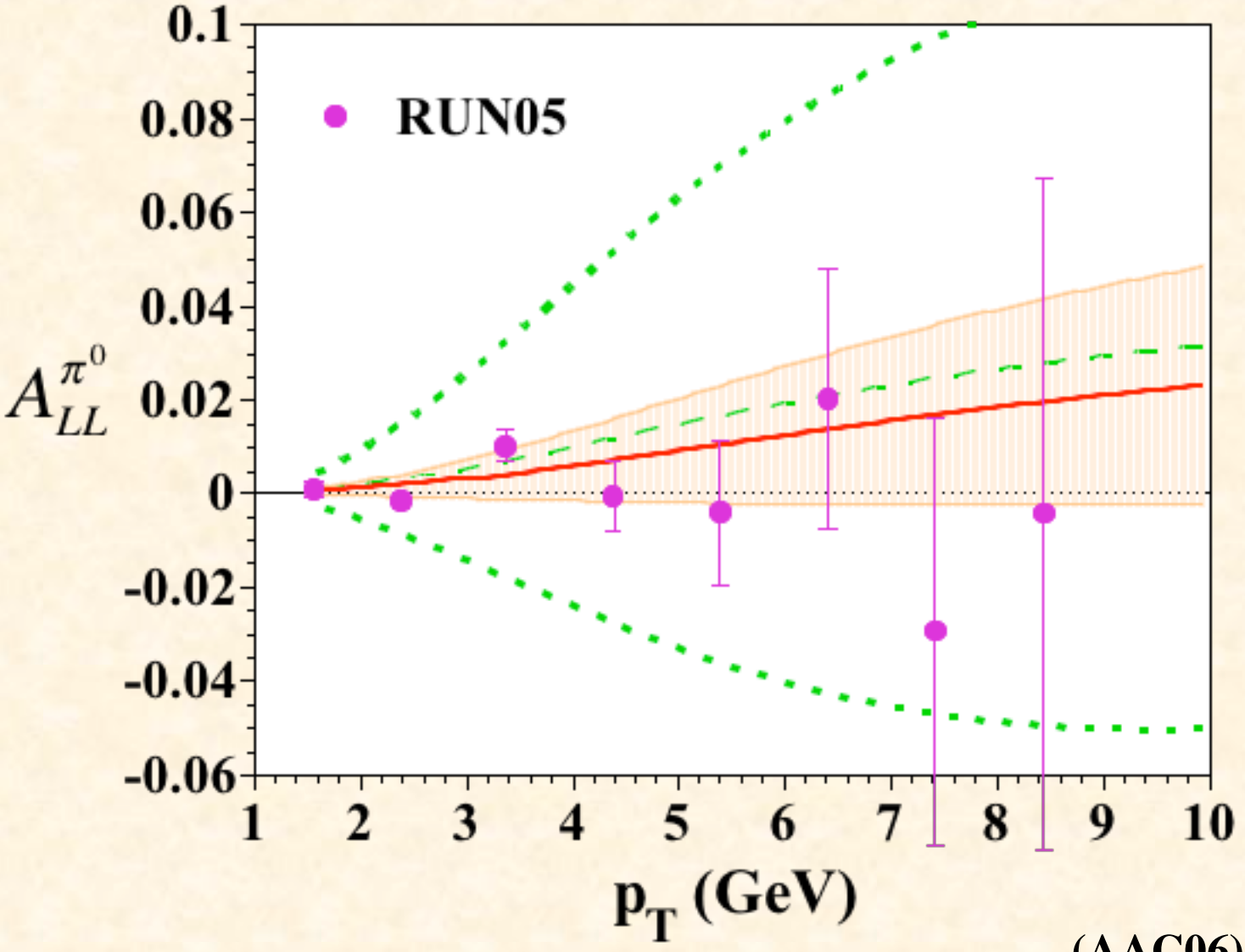
→ The functions of KKP, Kretzer, AKK, DSS, and HKNS are consistent with each other.

All the parametrizations agree in charm and bottom functions.

M. Hirai, SK, T.-H. Nagai, K. Sudoh, PRD75 (2007) 094009.

A code is available at http://research.kek.jp/people/kumanos/ffs.html

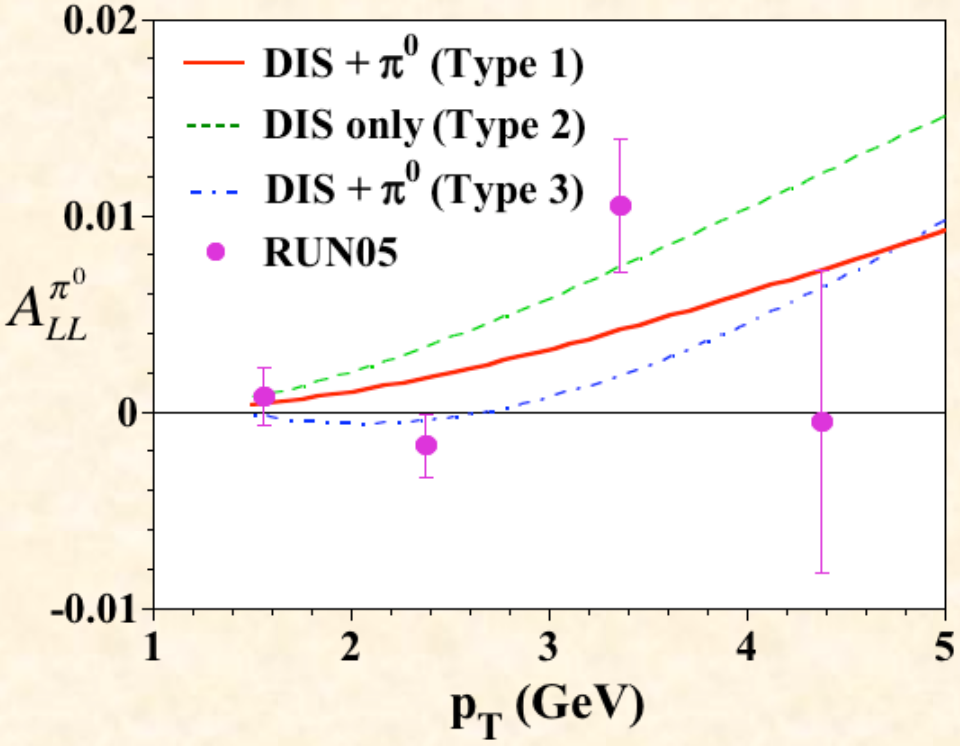
# Analysis with RHIC pion data

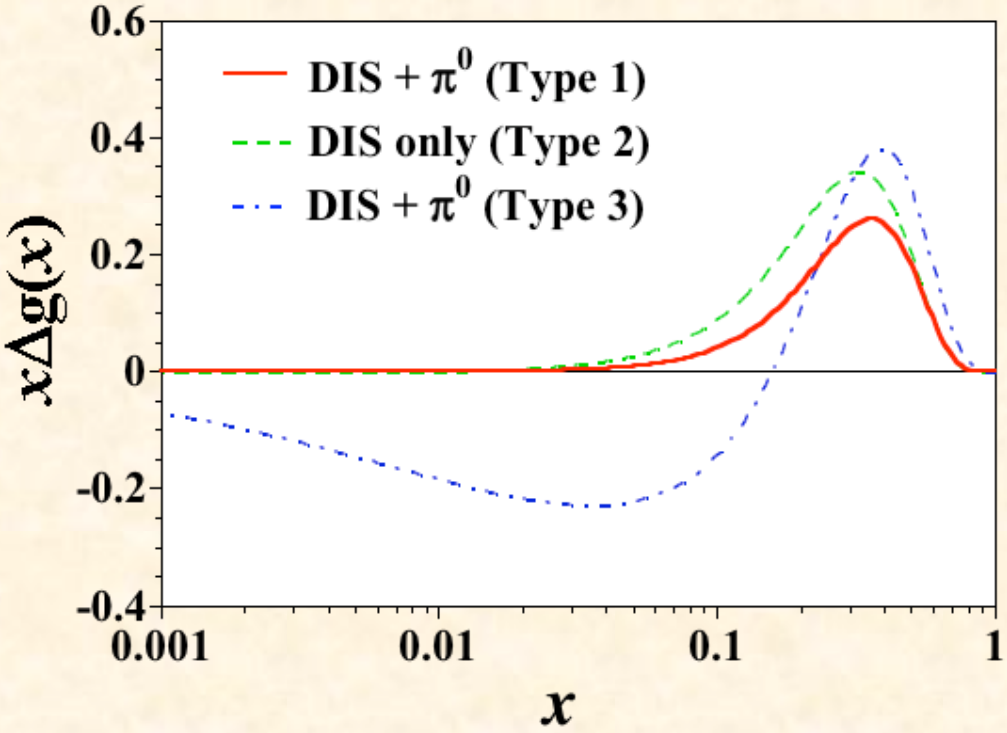



(AAC06)

# Possibility of negative gluon polarization $\Delta g < 0$

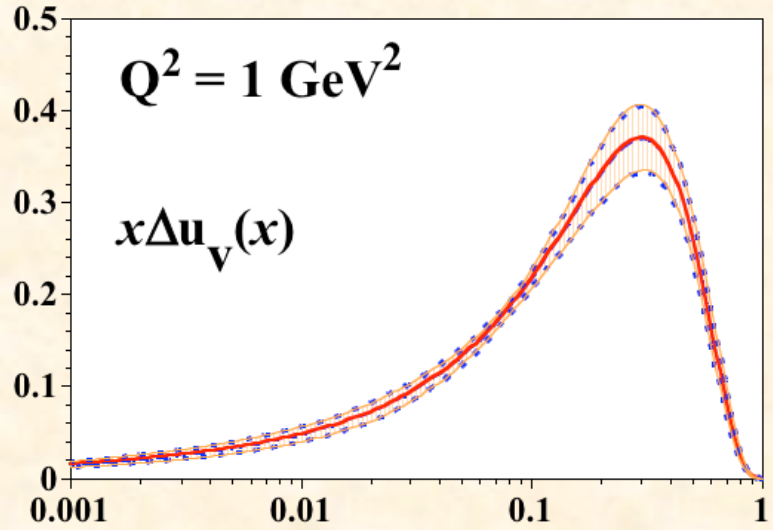
**$\pi^0$ production data do not distinguish between $\Delta g < 0$ and $\Delta g > 0$**

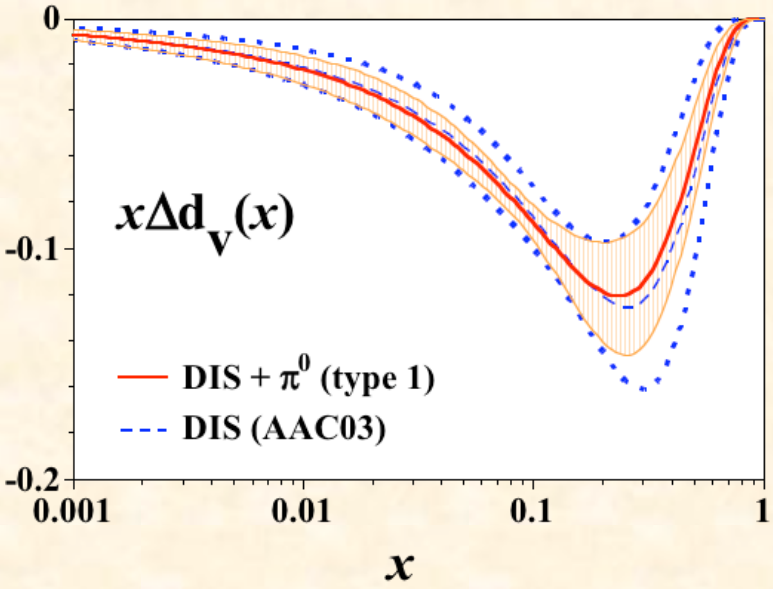


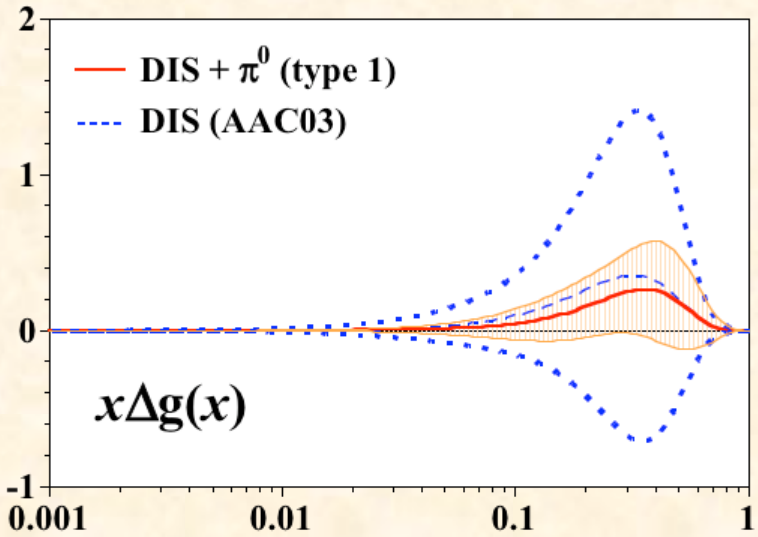



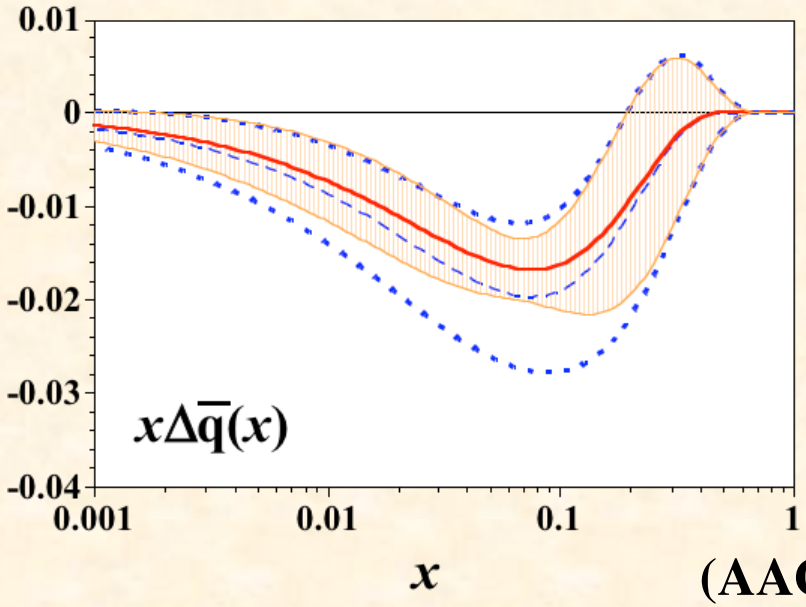
# Roles of RHIC-pion data

**Polarized PDFs, especially Δg, are better determined by the additional RHIC-$\pi^0$ data.**



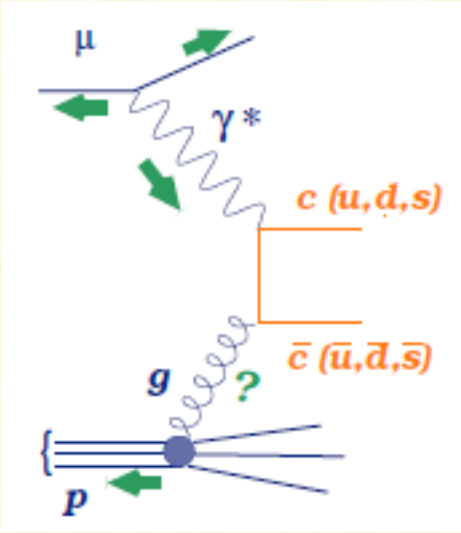



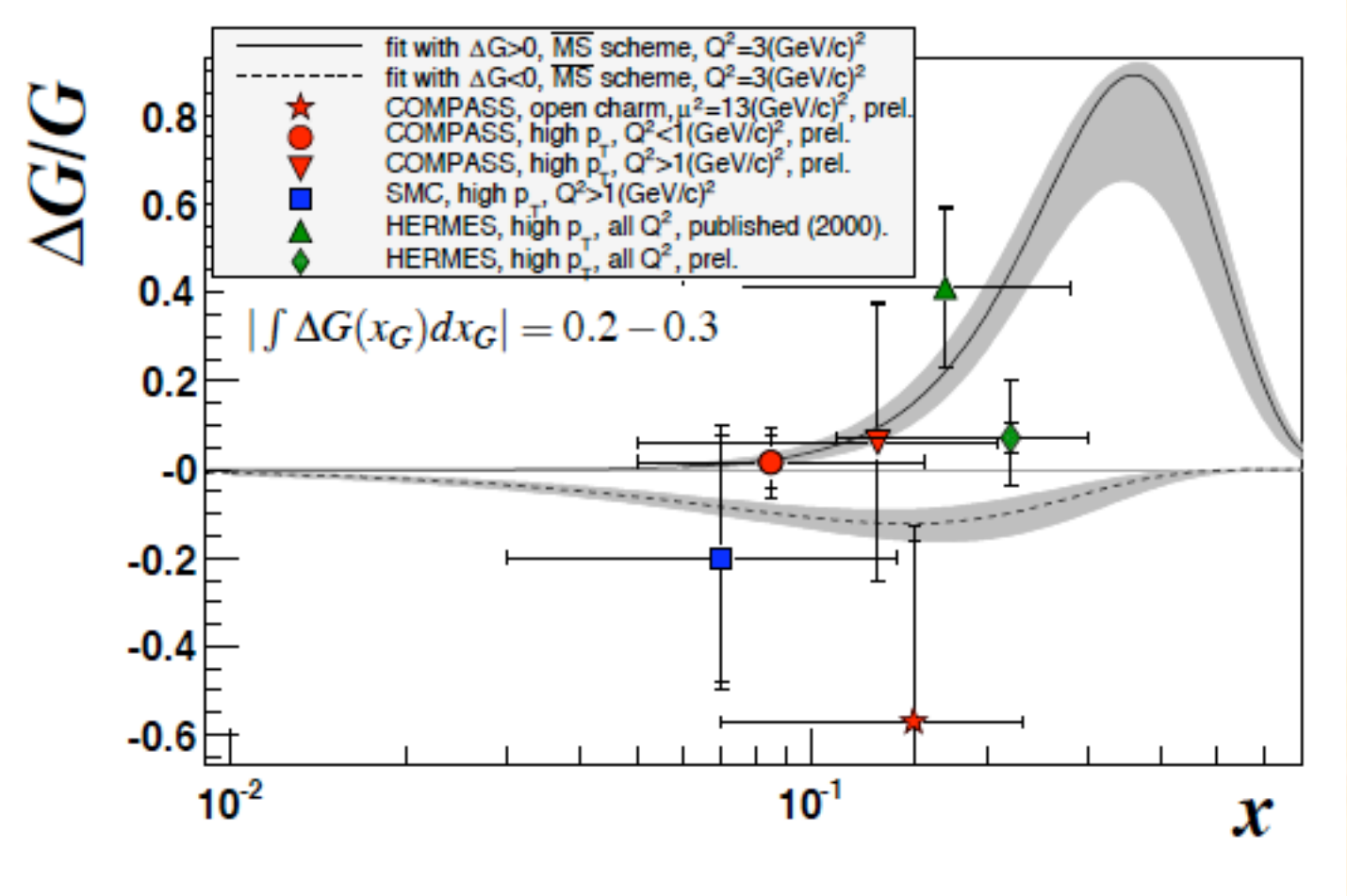




(AAC06)

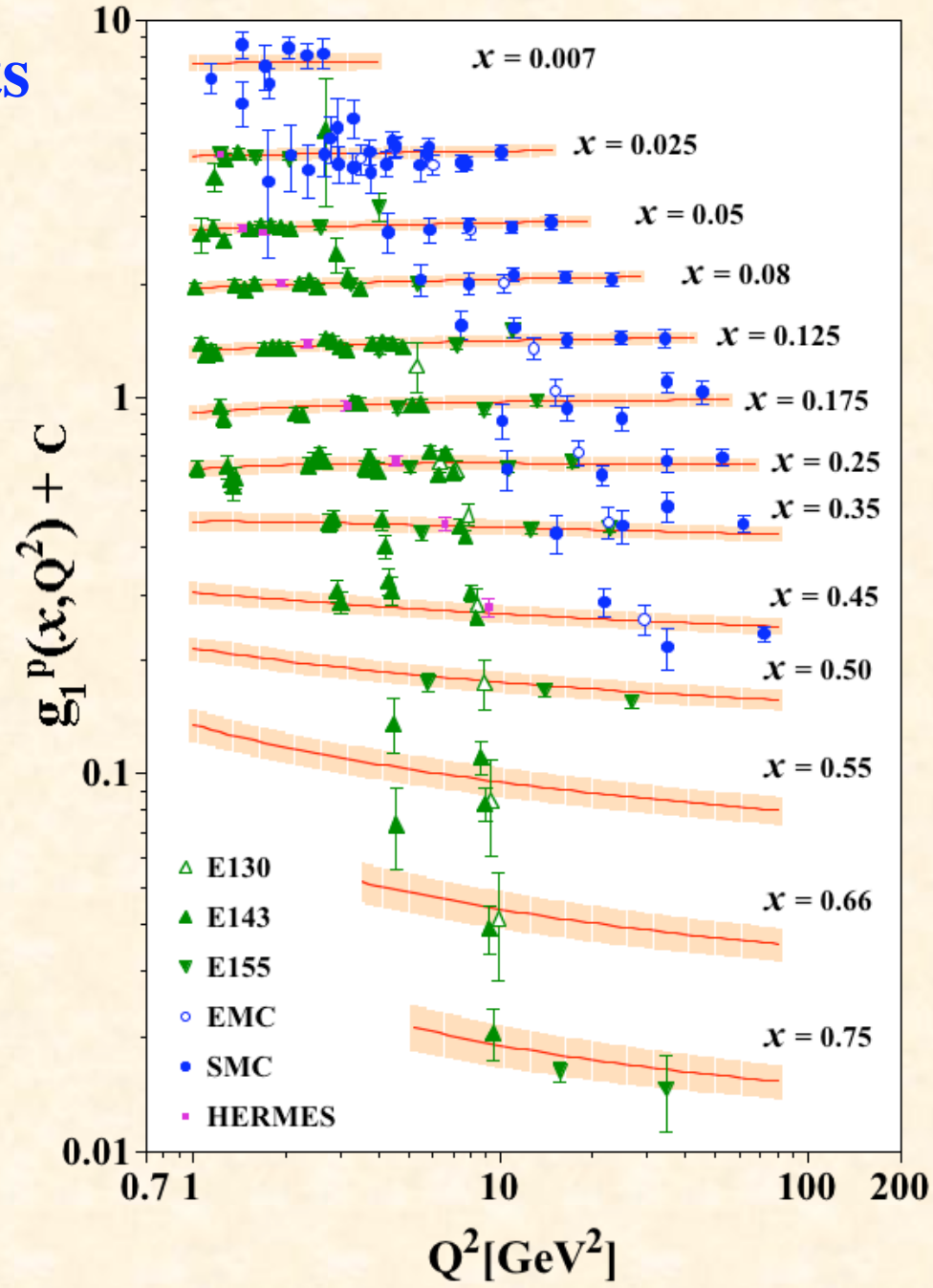
# Gluon polarization from lepton scattering



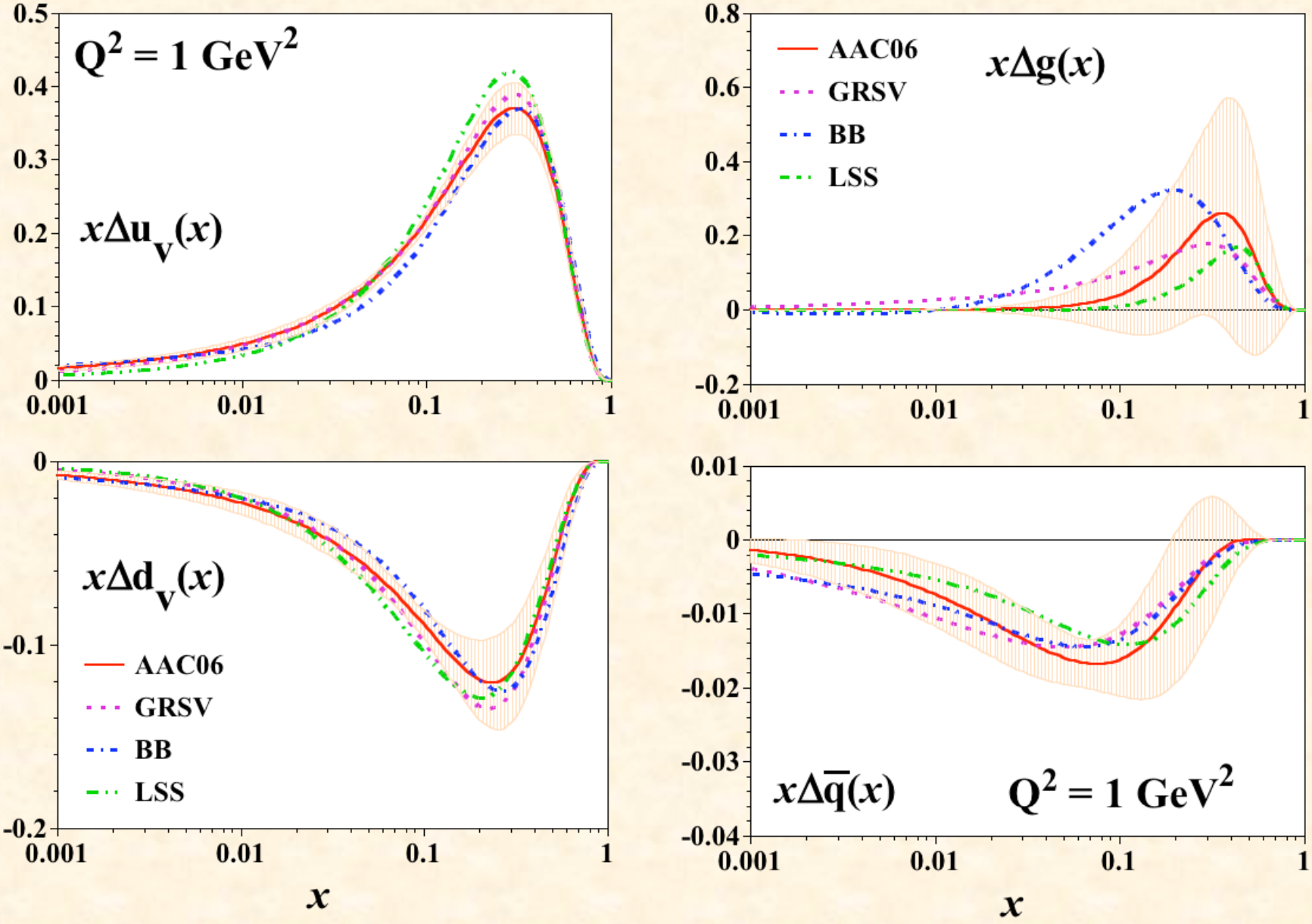




S. Koblitz@DIS07

# Situation of $g_1$ measurements and polarized PDF errors



# Comparison with other parameterizations



# 1st moments

$Q^2 = 1\,\mathrm{GeV}^2$ $\left(\Delta\Sigma = \Delta u_v + \Delta d_v + 6\Delta\bar{q}\right)$

| | Δg | ΔΣ |
|---|---|---|
| AAC06 | 0.31 ±0.32 | 0.27 ±0.07 |
| AAC03 | 0.50 ± 1.27 | 0.21 ± 0.14 |
| GRSV01 | 0.420 | 0.204 |
| LSS | 0.680 | 0.210 |
| BB | 1.026 | 0.138 |
| DNS | 0.574 | 0.311 |

- GRSV01(Sta) [ Phys. Rev. D63 (2001) 094005 ]
- LSS01 (MS) [ Eur.Phys.J. C23 (2002) 479 ]
- BB02 (SET3) [ Nucl. Phys. B636 (2002) 225 ]
- DNS05 (KKP) [ Phys. Rev. D71 (2005) 094018 ] ($Q^2$=10 $\mathrm{GeV}^2$ )

# Summary of the AAC analysis

- **Global analysis for polarized PDFs**
  - **$\Delta u_v(x)$, $\Delta d_v(x)$ are determined well**
  - **$\Delta\Sigma = 0.27 \pm 0.07$ ($Q^2 = 1$ GeV$^2$)**
  - **$\Delta g(x)$ could not be well constrained**

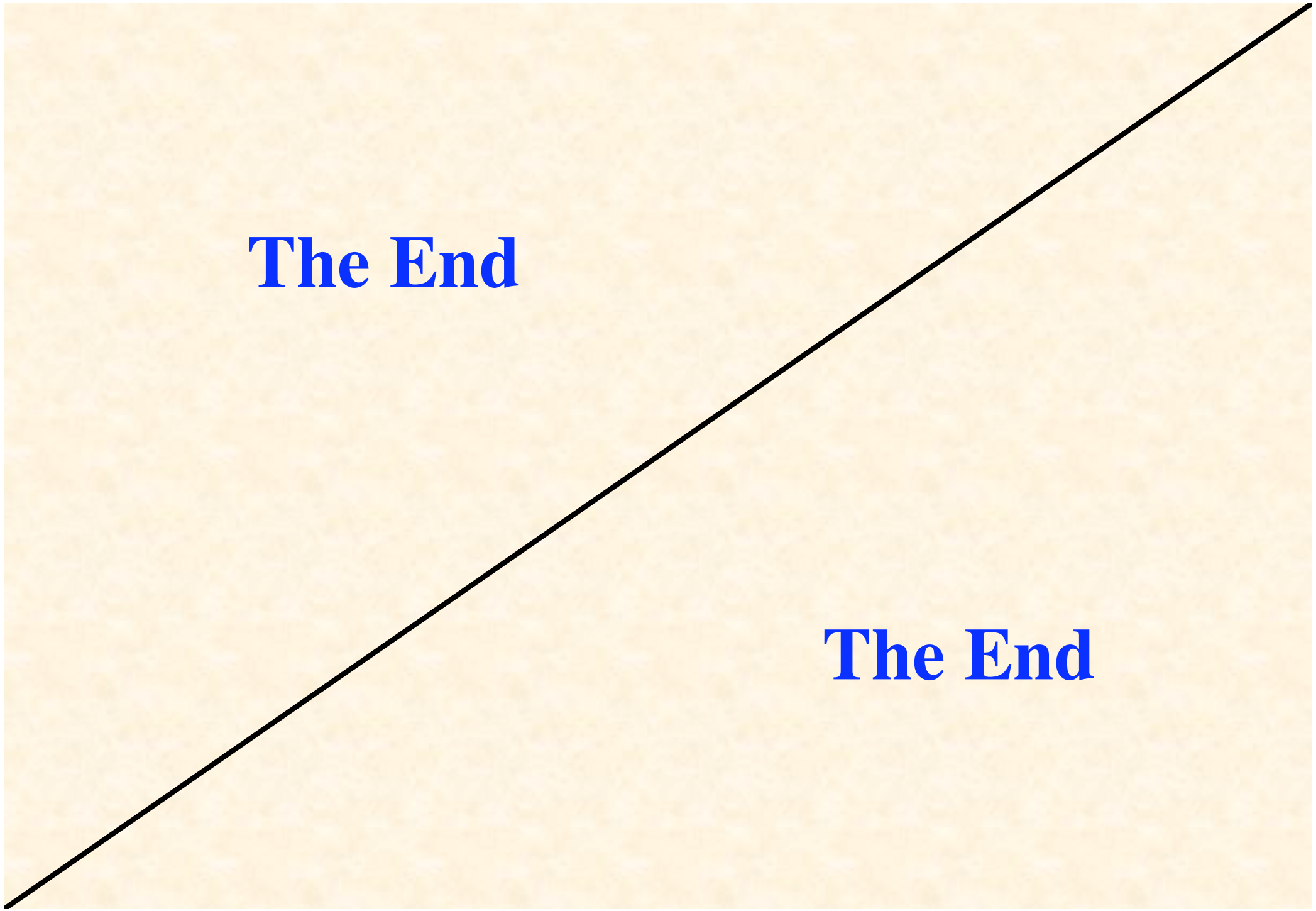
**AAC03**

- **Uncertainties of polarized PDFs**
- **Effects of E155-proton data**
- **Global analysis also with $\Delta g=0$**
- **Error correlation between $\Delta g$ and $\Delta\bar{q}$**

**AAC06**

- **Better $\Delta g$ determination by RHIC-pion**
- **$\Delta g$ determination at large x by scaling violation (HERMES, COMPASS)**

**AAC03-polarized-pdfs code could be obtained from http://spin.riken.bnl.gov/aac/**


The End
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