

Odderon, Redone¹

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Abstract. We discuss the “odderon” exchange at high energy within the framework of the Color Glass Condensate (CGC). We explicitly construct gauge-invariant amplitudes for multiple odderon exchanges in the scattering between the CGC and two types of color-singlet projectiles: a ‘color dipole’ and three quarks.

Keywords: High energy scattering in QCD, Color Glass Condensate, Odderon

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1. INTRODUCTION

Recent years have seen a remarkable progress in understanding the high-energy scattering in QCD. The main trigger of this progress is the observation that, when the scattering energy is very large, the hadron or nucleus exhibits a *high density gluonic matter* which is now called the Color Glass Condensate (CGC) [1]. In short words, the CGC emerges as follows: When the scattering energy is increased, soft gluon emission ($g \rightarrow gg$) is induced, and its successive occurrence leads to multiple production of “small- x gluons” that have small fractions $x \ll 1$ of the total momentum ($x \propto 1/\sqrt{s}$ with \sqrt{s} being the scattering energy). However, as the density of gluons becomes high, “*gluon recombination*” ($gg \rightarrow g$) starts to contribute and eventually leads to *saturation* where both processes are balanced. This state is the CGC. Technically, all these are described by a weak-coupling method ($\alpha_s \ll 1$ due to the presence of a hard scale $Q_s \gg \Lambda_{\text{QCD}}$, see below) which is powered by resummation schemes with respect to $(\alpha_s \ln 1/x)^n$ and strong gauge fields.

The properties of CGC are specified by correlation functions of gluons, and the change of CGC with increasing energy is determined by “evolution” equations for these correlation functions. In particular, the *Balitsky-Kovchegov (BK) equation* is a nonlinear evolution equation for a *2-point correlation function* which physically corresponds to the *scattering amplitude* of a “color dipole” off the CGC, and reduces to the gluon density in the weak-field regime. The nonlinearity comes into play due to the recombination effects, whose contribution is naively proportional to square of the gluon density. As a result of extensive investigation of this equation both in analytic and numerical methods, it turned out that there exists saturation regime whose borderline in the kinematical plain is given by the *saturation momentum* $Q_s(x)$ in such a way that gluons having transverse momenta lower than $Q_s(x)$

¹ On the first slide of the talk, I showed an *odd-looking* lithograph “Smiling spider” drawn by a French painter, Odilon Redon.

is saturated. Since the saturation momentum depends upon energy (or x) as $Q_s^2(x) \propto (1/x)^\lambda$ with $\lambda \simeq 0.3$, it grows with increasing energy (decreasing x) and the kinematical region for saturation expands. As most of the gluons have their transverse momenta around $Q_s(x)$, the weak-coupling treatment becomes better and better with increasing energy $\alpha_s(Q_s) \ll 1$, as mentioned above. Lastly, the solutions to the BK equation show new scaling phenomena called *geometric scaling* which naturally comes out due to the presence of a saturation momentum, and is also observed in experimental results in a beautiful way.

So far, studies of CGC have been focusing on the BK equation for a 2-point correlation function. However, in order to accurately understand the dynamics of CGC, we need to treat higher-point functions too. For example, the BK equation is obtained by a "mean-field" approximation of the more fundamental equation (the Balitsky equation), but this original equation involves 4-point function. Also, at high energy, we anticipate that multi-gluon exchanges between a projectile and a target, which are not expressed by multiple 2-gluon exchanges, will become important. All these should be addressed in the framework of the CGC.

The first nontrivial step beyond the BK equation in the direction of higher-point correlation functions should be the studies on "odderon" which requires an exchange of at least three reggeized gluons (in contrast to 2 reggeized gluons for a pomeron) and is odd under the charge conjugation operation (see Ref. [2] for a review of traditional description of odderon). This has been done in Ref. [3], and this talk gives a summary of the results presented there. Below, I first explain how to construct C -odd scattering amplitudes which correspond to "odderon" exchange in two different processes. Then, I show evolution equations for these amplitudes which are obtained by the application of the JIMWLK equation to them. Lastly, I compare the result with the BKP equation [4, 5] which is the evolution equation for the odderon exchanges in the traditional framework.

2. ODDERON OPERATORS IN THE CGC

Let us construct the relevant operators for multiple odderon exchanges in the scattering between the CGC and two types of simple projectiles: a color dipole and three quarks in a colorless state. The C -odd dipole-CGC scattering can be considered as a sub-process of the diffractive scattering of a virtual photon on the CGC into C -even mesons like η_c . The 3-quark system may be regarded as a crude model of a baryon.

2.1. The dipole-CGC scattering

Consider the high energy scattering of a $q\bar{q}$ dipole off the CGC which is treated as a random classical gauge field α . To obtain the S -matrix of this process, we first compute the S -matrix for a *fixed* configuration α , and then average over it. For the first step, we can use the eikonal approximation: $S(\mathbf{x}, \mathbf{y}; \alpha) = \langle \text{out} | \text{in} \rangle$, where the

transverse positions of the quark (\mathbf{x}) and the antiquark (\mathbf{y}) are the same in the in-coming and the out-going states:

$$|\text{in}\rangle \sim \bar{\psi}_i^{\text{in}}(\mathbf{x})\psi_i^{\text{in}}(\mathbf{y})|0\rangle, \quad |\text{out}\rangle \sim \bar{\psi}_i^{\text{out}}(\mathbf{x})\psi_i^{\text{out}}(\mathbf{y})|0\rangle.$$

The relation between the in-coming and the out-going fields is found by solving $(\partial_- - ig\alpha^a t^a)\psi = 0$ for a given gauge configuration α . Namely, $\psi_i^{\text{out}} = (V_{\mathbf{x}}^\dagger)_{ij}\psi_j^{\text{in}}$ with $V_{\mathbf{x}}^\dagger$ being the Wilson line in the fundamental representation along the trajectory of the quark:

$$V_{\mathbf{x}}^\dagger = \text{P exp} \left\{ ig \int dx^- \alpha^a(x^-, \mathbf{x}) t^a \right\}. \quad (2.1)$$

Then,

$$S(\mathbf{x}, \mathbf{y}; \alpha) = \langle \text{out} | \text{in} \rangle = \frac{1}{N_c} (V_{\mathbf{x}}^\dagger)^{ij} (V_{\mathbf{y}})^{ki} \delta^{kl} \delta^{jl} = \frac{1}{N_c} \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}}).$$

The physical S -matrix is obtained after averaging over the random classical field:

$$S_\tau(\mathbf{x}, \mathbf{y}) = \int \mathcal{D}\alpha W_\tau[\alpha] S(\mathbf{x}, \mathbf{y}; \alpha) = \frac{1}{N_c} \langle \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}}) \rangle_\tau. \quad (2.2)$$

Notice that this result depends upon the scattering energy or the rapidity τ defined by $\tau = \ln 1/x$ (x is the Bjorken variable) because the weight function $W_\tau[\alpha]$ which governs the randomness of the gauge field α changes with increasing energy. The change of $W_\tau[\alpha]$ under the change of rapidity is formulated as a renormalization group equation, which is called the JIMWLK equation.

So far, the scattering process is generic and the exchanged object can be either even, or odd, under the charge conjugation C . To single out C -even ('pomeron') or C -odd ('odderon') exchanges, one needs to project Eq. (2.2) onto in-coming and out-going states with appropriate C -parities. Since the charge conjugation for fermions is defined by $C\psi C^{-1} = -i(\bar{\psi}\gamma^0\gamma^2)^T$, and $C\bar{\psi} C^{-1} = (-i\gamma^0\gamma^2\psi)^T$, it is easy to check that the eigenstates of C in the dipole sector are given by $(\bar{\psi}(\mathbf{x})\psi(\mathbf{y}) \pm \bar{\psi}(\mathbf{y})\psi(\mathbf{x}))|0\rangle$, where $+(-)$ sign yields the C -even(odd) state. These are reasonable because the charge conjugation essentially works as the exchange of a quark and an antiquark.

Taking the C -odd dipole state as the in-coming state (a photon is C -odd), and the C -even dipole state as the out-going state, one obtains the C -odd contribution:

$$S_\tau^{\text{odd}}(\mathbf{x}, \mathbf{y}) = \langle \text{out, even} | \text{in, odd} \rangle = \frac{1}{2N_c} \left\langle \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}}) - \text{tr}(V_{\mathbf{y}}^\dagger V_{\mathbf{x}}) \right\rangle_\tau. \quad (2.3)$$

This allows us to identify the operator for C -odd exchanges in the dipole-CGC scattering ("the dipole odderon operator") as

$$O(\mathbf{x}, \mathbf{y}) \equiv \frac{1}{2iN_c} \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}} - V_{\mathbf{y}}^\dagger V_{\mathbf{x}}) = -O(\mathbf{y}, \mathbf{x}). \quad (2.4)$$

One can check that the operator (2.4) is indeed C -odd by using the transformation property of the gauge fields, $C A_\mu C^{-1} = -(A_\mu)^T$, or, $C V C^{-1} = (V^\dagger)^T$ for a generic

Wilson line V built with A_μ . Note that the C -odd contribution (2.3) is the imaginary part of the S -matrix element:

$$\langle O(\mathbf{x}, \mathbf{y}) \rangle_\tau = \Im S_\tau(\mathbf{x}, \mathbf{y}). \quad (2.5)$$

Correspondingly, the C -even, pomeron exchange, amplitude, that we shall denote as $N(\mathbf{x}, \mathbf{y})$, is identified with the real part of the S -matrix:

$$N(\mathbf{x}, \mathbf{y}) \equiv 1 - \frac{1}{2N_c} \text{tr}(V_\mathbf{x}^\dagger V_\mathbf{y} + V_\mathbf{y}^\dagger V_\mathbf{x}), \quad (2.6)$$

$$\langle N(\mathbf{x}, \mathbf{y}) \rangle_\tau = 1 - \Re S_\tau(\mathbf{x}, \mathbf{y}). \quad (2.7)$$

From perturbative QCD, we expect that the lowest order contribution to the odderon exchange is of the form $d^{abc} A_\mu^a(\mathbf{x}) A_\nu^b(\mathbf{y}) A_\rho^c(\mathbf{z})$ with $d^{abc} = 2\text{tr}(\{t^a, t^b\}t^c)$ being a totally symmetric tensor. A similar structure indeed emerges from the CGC operator (2.4) in the weak-field limit. By expanding the Wilson lines (2.1) up to *cubic* order in the field α in the exponent

$$\begin{aligned} V_\mathbf{x}^\dagger[\alpha] &\approx 1 + ig \int dx^- \alpha^a(x^-, \mathbf{x}) t^a \\ &- \frac{g^2}{2} \int dx^- \int dy^- \alpha^a(x^-, \mathbf{x}) \alpha^b(y^-, \mathbf{x}) [\theta(x^- - y^-) t^a t^b + \theta(y^- - x^-) t^b t^a] \\ &+ \{ \text{cubic term in } \alpha \}, \end{aligned} \quad (2.8)$$

one finds the lowest non-trivial contribution to Eq. (2.4):

$$O(\mathbf{x}, \mathbf{y}) \simeq \frac{-g^3}{24N_c} d^{abc} \left\{ 3(\alpha_\mathbf{x}^a \alpha_\mathbf{y}^b \alpha_\mathbf{y}^c - \alpha_\mathbf{x}^a \alpha_\mathbf{x}^b \alpha_\mathbf{y}^c) + (\alpha_\mathbf{x}^a \alpha_\mathbf{x}^b \alpha_\mathbf{x}^c - \alpha_\mathbf{y}^a \alpha_\mathbf{y}^b \alpha_\mathbf{y}^c) \right\}, \quad (2.9)$$

where $\alpha_\mathbf{x}^a = \int dx^- \alpha^a(x^-, \mathbf{x})$. As expected, this expression is cubic in α^a with the color indices contracted symmetrically by the d -symbol. Note that this combination of trilinear field operators is gauge invariant by construction.

2.2. The 3-quark–CGC scattering

We now turn to the 3-quark–CGC scattering at high energies. The 3-quark colorless state may be given by the "baryonic" operator $\epsilon^{ijk} \psi^i(\mathbf{x}) \psi^j(\mathbf{y}) \psi^k(\mathbf{z})$, where ϵ^{ijk} is the complete antisymmetric symbol ($i, j, k=1,2,3$). For our present purpose, flavor dependences are irrelevant. By using the same eikonal approximation as for the dipole-CGC scattering, one obtains the following S -matrix:

$$S_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{1}{3!} \epsilon^{ijk} \epsilon^{lmn} \left\langle V_{il}^\dagger(\mathbf{x}) V_{jm}^\dagger(\mathbf{y}) V_{kn}^\dagger(\mathbf{z}) \right\rangle_\tau, \quad (2.10)$$

where \mathbf{x} , \mathbf{y} and \mathbf{z} are transverse positions of the three quarks. The odderon contribution is given again by the imaginary part of the S -matrix :

$$\langle O(\mathbf{x}, \mathbf{y}, \mathbf{z}) \rangle_\tau = \Im S_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}), \quad (2.11)$$

where the "3-quark odderon operator" $O(\mathbf{x}, \mathbf{y}, \mathbf{z})$ has been introduced as

$$O(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{1}{3!2i} \left(\epsilon^{ijk} \epsilon^{lmn} V_{il}^\dagger(\mathbf{x}) V_{jm}^\dagger(\mathbf{y}) V_{kn}^\dagger(\mathbf{z}) - \text{c.c.} \right). \quad (2.12)$$

Gauge invariance of this operator becomes manifest if one rewrites this as

$$\begin{aligned} O(\mathbf{x}, \mathbf{y}, \mathbf{z}) = & \frac{1}{3!2i} \left[\text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}}) \text{tr}(V_{\mathbf{y}}^\dagger V_{\mathbf{w}}) \text{tr}(V_{\mathbf{z}}^\dagger V_{\mathbf{w}}) - \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}}) \text{tr}(V_{\mathbf{y}}^\dagger V_{\mathbf{w}} V_{\mathbf{z}}^\dagger V_{\mathbf{w}}) \right. \\ & - \text{tr}(V_{\mathbf{y}}^\dagger V_{\mathbf{w}}) \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}} V_{\mathbf{z}}^\dagger V_{\mathbf{w}}) - \text{tr}(V_{\mathbf{z}}^\dagger V_{\mathbf{w}}) \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}} V_{\mathbf{y}}^\dagger V_{\mathbf{w}}) \\ & \left. + \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}} V_{\mathbf{y}}^\dagger V_{\mathbf{w}} V_{\mathbf{z}}^\dagger V_{\mathbf{w}}) + \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{w}} V_{\mathbf{z}}^\dagger V_{\mathbf{w}} V_{\mathbf{y}}^\dagger V_{\mathbf{w}}) - \text{c.c.} \right], \quad (2.13) \end{aligned}$$

which is easily done with the help of the identity related to the definition of the determinant for SU(3) matrices: $\epsilon^{ijk} \epsilon^{lmn} V_{il}(\mathbf{w}) V_{jm}(\mathbf{w}) V_{kn}(\mathbf{w}) = 3! \det V(\mathbf{w}) = 3!$ (\mathbf{w} is an arbitrary transverse coordinate). By construction, this expression is independent of \mathbf{w} when $N_c = 3$. Note that it can be simplified by choosing \mathbf{w} to be one of the quark coordinates, say $\mathbf{w} = \mathbf{z}$:

$$O(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{1}{3!2i} \left[\text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{z}}) \text{tr}(V_{\mathbf{y}}^\dagger V_{\mathbf{z}}) - \text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{z}} V_{\mathbf{y}}^\dagger V_{\mathbf{z}}) - \text{c.c.} \right]. \quad (2.14)$$

Furthermore, when two of the coordinates are the same, the 3-quark odderon operator reduces to the dipole odderon operator, Eq. (2.4):

$$O(\mathbf{x}, \mathbf{z}, \mathbf{z}) = O(\mathbf{x}, \mathbf{z}) = -O(\mathbf{x}, \mathbf{x}, \mathbf{z}). \quad (N_c = 3) \quad (2.15)$$

This is physically reasonable because the diquark state is equivalent to an antiquark as far as color degrees of freedom are concerned.

In the weak-field approximation, as obtained after expanding to lowest non-trivial order (i.e., to cubic order in α) the Wilson lines, one finds again a gauge invariant linear combination of trilinear field operators with the d -symbol:

$$\begin{aligned} O(\mathbf{x}, \mathbf{y}, \mathbf{z}) \simeq & \frac{g^3}{144} d^{abc} \quad (2.16) \\ & \times \left\{ (\alpha_{\mathbf{x}}^a - \alpha_{\mathbf{z}}^a) + (\alpha_{\mathbf{y}}^a - \alpha_{\mathbf{z}}^a) \right\} \left\{ (\alpha_{\mathbf{y}}^b - \alpha_{\mathbf{x}}^b) + (\alpha_{\mathbf{z}}^b - \alpha_{\mathbf{x}}^b) \right\} \left\{ (\alpha_{\mathbf{z}}^c - \alpha_{\mathbf{y}}^c) + (\alpha_{\mathbf{x}}^c - \alpha_{\mathbf{y}}^c) \right\}. \end{aligned}$$

Lastly, one can similarly introduce the "3-quark pomeron operator" $N(\mathbf{x}, \mathbf{y}, \mathbf{z})$ by the real part of the scattering matrix:

$$\langle N(\mathbf{x}, \mathbf{y}, \mathbf{z}) \rangle_\tau = 1 - \Re S_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}), \quad (2.17)$$

$$N(\mathbf{x}, \mathbf{y}, \mathbf{z}) = 1 - \frac{1}{3!2} \left(\epsilon^{ijk} \epsilon^{lmn} V_{il}^\dagger(\mathbf{x}) V_{jm}^\dagger(\mathbf{y}) V_{kn}^\dagger(\mathbf{z}) + \text{c.c.} \right). \quad (2.18)$$

This operator should be important in describing the high-energy behavior of the proton-nucleus collisions, which is, however, out of the scope of the present paper.

3. ODDERON EVOLUTION

Once we know the relevant operators for the C -odd scattering amplitudes, we can apply the JIMWLK equation (or its simplified version proposed in Ref. [3]) to the operators to derive the evolution equations for them. We consider the two cases which were discussed in the previous section: the dipole-CGC scattering and the 3-quark-CGC scattering, and then discuss the relation between our result and the BKP equation.

3.1. The dipole-CGC scattering

For the dipole-CGC scattering, the evolution equations obeyed by the average amplitudes $\langle N(\mathbf{x}, \mathbf{y}) \rangle_\tau$ and $\langle O(\mathbf{x}, \mathbf{y}) \rangle_\tau$ can be easily derived from the first Balitsky equation because the operators $N(\mathbf{x}, \mathbf{y})$ and $O(\mathbf{x}, \mathbf{y})$ are, respectively, the real part and the imaginary part of the dipole-CGC scattering operator $(1/N_c)\text{tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}})$ which satisfies the Balitsky equation. Therefore, the respective equations can be simply obtained by separating the real part and the imaginary part in the Balitsky equation. The result is

$$\frac{\partial}{\partial \tau} \langle O(\mathbf{x}, \mathbf{y}) \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z} \mathcal{M}_{\mathbf{xyz}} \left\langle O(\mathbf{x}, \mathbf{z}) + O(\mathbf{z}, \mathbf{y}) - O(\mathbf{x}, \mathbf{y}) - O(\mathbf{x}, \mathbf{z})N(\mathbf{z}, \mathbf{y}) - N(\mathbf{x}, \mathbf{z})O(\mathbf{z}, \mathbf{y}) \right\rangle_\tau, \quad (3.1)$$

$$\frac{\partial}{\partial \tau} \langle N(\mathbf{x}, \mathbf{y}) \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z} \mathcal{M}_{\mathbf{xyz}} \left\langle N(\mathbf{x}, \mathbf{z}) + N(\mathbf{z}, \mathbf{y}) - N(\mathbf{x}, \mathbf{y}) - N(\mathbf{x}, \mathbf{z})N(\mathbf{z}, \mathbf{y}) + O(\mathbf{x}, \mathbf{z})O(\mathbf{z}, \mathbf{y}) \right\rangle_\tau, \quad (3.2)$$

where we have defined the dipole kernel

$$\mathcal{M}_{\mathbf{xyz}} = \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2(\mathbf{z} - \mathbf{y})^2}. \quad (3.3)$$

Several comments are in order about these equations:

- As is the case with the Balitsky equations, the equations above do not close by themselves (since they contain both two-point and four-point functions, as mentioned before), but rather belong to an infinite hierarchy.
- In the weak-field limit, both of the evolution equations reduce to the (linear) BFKL equation. However, the BFKL equation for the odderon exchange must be solved with the antisymmetric condition (2.4). Therefore, even if the evolution equations are the same, the respective solutions behave differently. In particular, it is known that the highest intercept of the BFKL solution with C being odd is given by 1 which is smaller than the (hard) pomeron intercept [6].

- The non-linear terms in these equations couple the evolution of C -odd and C -even operators. For instance, the last term, quadratic in O , in the r.h.s. of Eq. (3.2) for $\langle N \rangle_\tau$ describes the merging of two odderons into one pomeron ('merging' from the target point of view). This has not been discussed before in connection with the Balitsky hierarchy.
- In the mean-field approximation, Eqs. (3.1)–(3.2) reduce to a closed system of coupled, non-linear, equations for $\langle N \rangle_\tau$ and $\langle O \rangle_\tau$:

$$\frac{\partial}{\partial \tau} \langle O(\mathbf{x}, \mathbf{y}) \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z} \mathcal{M}_{\mathbf{xyz}} \left[\langle O(\mathbf{x}, \mathbf{z}) \rangle_\tau + \langle O(\mathbf{z}, \mathbf{y}) \rangle_\tau - \langle O(\mathbf{x}, \mathbf{y}) \rangle_\tau \right. \\ \left. - \langle O(\mathbf{x}, \mathbf{z}) \rangle_\tau \langle N(\mathbf{z}, \mathbf{y}) \rangle_\tau - \langle N(\mathbf{x}, \mathbf{z}) \rangle_\tau \langle O(\mathbf{z}, \mathbf{y}) \rangle_\tau \right], \quad (3.4)$$

$$\frac{\partial}{\partial \tau} \langle N(\mathbf{x}, \mathbf{y}) \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \mathbf{z} \mathcal{M}_{\mathbf{xyz}} \left[\langle N(\mathbf{x}, \mathbf{z}) \rangle_\tau + \langle N(\mathbf{z}, \mathbf{y}) \rangle_\tau - \langle N(\mathbf{x}, \mathbf{y}) \rangle_\tau \right. \\ \left. - \langle N(\mathbf{x}, \mathbf{z}) \rangle_\tau \langle N(\mathbf{z}, \mathbf{y}) \rangle_\tau + \langle O(\mathbf{x}, \mathbf{z}) \rangle_\tau \langle O(\mathbf{z}, \mathbf{y}) \rangle_\tau \right]. \quad (3.5)$$

The first of these equations has been already proposed in Ref. [6], as a plausible non-linear generalization of the BFKL equation in the C -odd channel. As for Eq. (3.5), this is the BK equation supplemented by a new term describing the merging of two odderons.

- One of the significant consequences of the nonlinear effects in the factorized evolution equation (3.4) is that the odderon amplitude $\langle O \rangle_\tau$ will decay into zero with increasing energy. This is most easily seen by noting that when the pomeron amplitude $\langle N \rangle_\tau$ is close to 1 (deeply in saturation regime which is expected to realize at high energy), the nonlinear terms in Eq. (3.4) cancel the first two terms on the r.h.s. and the resulting equation for $\langle O \rangle_\tau$ simply implies decrease of the solution. Therefore, as one goes to higher energies, the odderon contribution becomes less and less important.

3.2. The 3-quark–CGC scattering

Since we know the full non-linear expression of the relevant operator (2.12) for the 3-quark–CGC scattering, there is no difficulty at conceptual level in deriving the evolution equation. A straightforward application of the JIMWLK equation (or its simplified version) to this operator automatically leads to the result. However, the resulting equation turned out to be complicated and not very illuminating: Through their non-linear terms, they couple the 3-quark odderon operator to other types of operators with different color structures. Therefore, in this talk, we rather show the evolution equation for the weak-field version of the 3-quark odderon operator, Eq. (2.16). This is indeed sufficient to discuss the correspondence with the BKP equation [4, 5], which is a *linear* evolution equation for the odderon exchange.

After a straightforward but lengthy calculation, the following linear evolution equation for $\langle O_{\mathbf{x}\mathbf{y}\mathbf{z}} \rangle_\tau \equiv \langle O(\mathbf{x}, \mathbf{y}, \mathbf{z}) \rangle_\tau$ is obtained

$$\begin{aligned} \frac{\partial}{\partial \tau} \langle O_{\mathbf{x}\mathbf{y}\mathbf{z}} \rangle_\tau &= \frac{3\alpha_s}{4\pi^2} \int d^2 \mathbf{w} \mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{w}} \left(\langle O_{\mathbf{x}\mathbf{w}\mathbf{z}} \rangle_\tau + \langle O_{\mathbf{w}\mathbf{y}\mathbf{z}} \rangle_\tau - \langle O_{\mathbf{x}\mathbf{y}\mathbf{z}} \rangle_\tau \right. \\ &\quad \left. - \langle O_{\mathbf{w}\mathbf{w}\mathbf{z}} \rangle_\tau - \langle O_{\mathbf{x}\mathbf{x}\mathbf{w}} \rangle_\tau - \langle O_{\mathbf{y}\mathbf{y}\mathbf{w}} \rangle_\tau - \langle O_{\mathbf{x}\mathbf{y}\mathbf{w}} \rangle_\tau \right) \\ &\quad + \left\{ 2 \text{ cyclic permutations} \right\}. \end{aligned} \quad (3.6)$$

Note that this is a *closed* equation for $\langle O_{\mathbf{x}\mathbf{y}\mathbf{z}} \rangle_\tau$, which was expected from the viewpoint of gauge invariance: the only gauge invariant C -odd operators available are $O_{\mathbf{x}\mathbf{y}\mathbf{z}}$ and $O(\mathbf{x}, \mathbf{y}) = O_{\mathbf{x}\mathbf{y}\mathbf{y}}$ (cf. Eq. (2.15)). The linear combination of O 's in the integrand vanishes at the points $\mathbf{w} = \mathbf{x}$ and $\mathbf{w} = \mathbf{y}$ where lie the poles of the dipole kernel $\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{w}}$, so the poles are harmless. Also, one can easily check that the above equation is consistent with the relation (2.15) between the dipole and the 3-quark odderon amplitudes: if one sets $\mathbf{z} = \mathbf{y}$, Eq. (3.6) reduces indeed to the BFKL equation which is the evolution equation of the dipole-CGC scattering in the weak-field regime.

Now we come to the final point: the comparison of our result (3.6) with the BKP equation. First of all, our result (3.6) does not look equivalent to the BKP equation. In fact, within our framework, the BKP equation rather appears as the evolution equation for the 3-point Green's function defined by

$$f_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}) \equiv d^{abc} \langle \alpha_{\mathbf{x}}^a \alpha_{\mathbf{y}}^b \alpha_{\mathbf{z}}^c \rangle_\tau. \quad (3.7)$$

Indeed, the evolution equation for this Green's function reads

$$\begin{aligned} \frac{\partial}{\partial \tau} f_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}) &= \frac{\bar{\alpha}_s}{4\pi} \int d^2 \mathbf{w} \mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{w}} \left(f_\tau(\mathbf{x}, \mathbf{w}, \mathbf{z}) + f_\tau(\mathbf{w}, \mathbf{y}, \mathbf{z}) - f_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z}) - f_\tau(\mathbf{w}, \mathbf{w}, \mathbf{z}) \right) \\ &\quad + \left\{ 2 \text{ cyclic permutations} \right\}. \end{aligned} \quad (3.8)$$

Notice that this equation is nothing but the Fourier transform of the BKP equation which is usually written in the momentum space. Since the 3-quark odderon operator Eq. (2.16) can be represented as a linear combination of the 3-point Green's functions, the equivalence between our result (3.6) and the BKP equation is essentially established. However, there is a caveat when we write Eq. (3.8). In fact, since the Green function $f_\tau(\mathbf{x}, \mathbf{y}, \mathbf{z})$ is not gauge invariant as it is, if one applied the original JIMWLK equation to this operator, one would obtain a result which is different from Eq. (3.8) and is even ill-defined due to infra-red divergences (the evolution equation (3.8) is finite and well-defined). Instead of doing this, we have derived Eq. (3.8) from the simplified version of the JIMWLK equation which is free of any infra-red divergences and is justified for gauge invariant operators. This means that we can use the simplified version of the JIMWLK equation to gauge *variant* operators as far as we finally consider gauge invariant quantities (infra-red divergences are canceled among themselves in the final result). In other words, the use of the simplified JIMWLK equation for the Green function corresponds to a kind of regularization of the resulting evolution equation.

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