CP symmetry and the strong interactions

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Two topics

- For what quark masses is CP spontaneously broken?
- Why is $m_u = 0$ not a physically meaningful concept?

Tools

- effective chiral Lagrangian
- defining the continuum from the lattice

The effective meson theory

Setup

- three flavors
- SU(3) octet of mesons π_α
- effective matrix valued field

$$\Sigma = \exp(i\pi_\alpha \lambda_\alpha / f_\pi)$$

- λ_α: generators of SU(3)

To lowest order

$$L_0 = \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$$

Parity and Charge conjugation

$$P: \ \Sigma \ \to \ \Sigma^{-1}$$
$$CP: \ \Sigma \ \to \ \Sigma^*$$

- mesons are pseudoscalars
- $\pi^0 \equiv \pi_3$ and $\eta \equiv \pi_8$ even under C

Chiral symmetry

$$\Sigma \to g_L^\dagger \Sigma g_R$$

- (g_L, g_L) in $(SU(3) \times SU(3))$
- Shadow from quark level of

$$\psi_L \to \psi_L \ g_L$$
$$\psi_R \to \psi_R \ g_R$$

- $\langle 0|\overline{\psi}_L \psi_R|0\rangle \leftrightarrow v \ \langle \Sigma \rangle$

Spontaneous chiral symmetry breaking

- $\langle \Sigma \rangle \neq 0$

Quark masses

$$L = \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - v \,\mathrm{Re}\ \mathrm{Tr}(\Sigma M)$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$M = \frac{m_u + m_d + m_s}{3} + \frac{m_u - m_d}{2}\ \lambda_3 + \frac{m_u + m_d - 2m_s}{2\sqrt{3}}\ \lambda_8$$

- both singlet and octet pieces under flavor $SU(3)$

Expand to quadratic order in meson fields

- up-down mass difference mixes π^0 and η
- diagonalize to find meson masses

$$m^2_{\pi^\pm} \propto m_u + m_d$$
$$m^2_{K^\pm} \propto m_u + m_s$$
$$m^2_{K^0,\overline{K}^0} \propto m_d + m_s$$
$$m^2_{\pi^0} \propto$$
$$\frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$
$$m^2_{\eta} \propto$$
$$\frac{2}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

Quark mass ratios

$$\frac{m_u}{m_d} = \frac{m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}}{m^2_{\pi^+} - m^2_{K^+} + m^2_{K^0}} \sim 0.66$$

Spontaneous CP violation

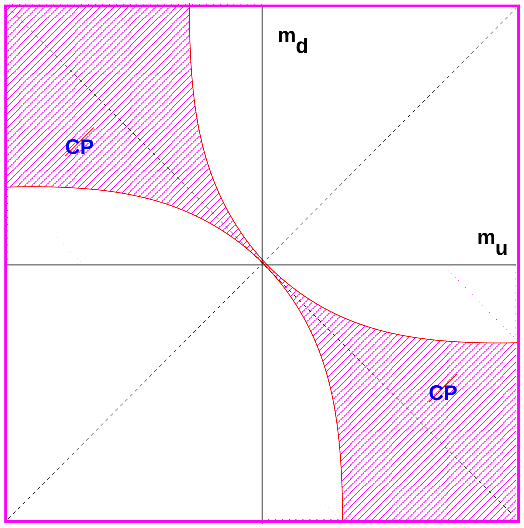
Mass of π^0 can go negative

$$\frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

Vanishes at

$$m_u = \frac{-m_s m_d}{m_s + m_d}$$

- Boundary for pion condensed phase $\langle \pi^0 \rangle \neq 0$



New vacuum state

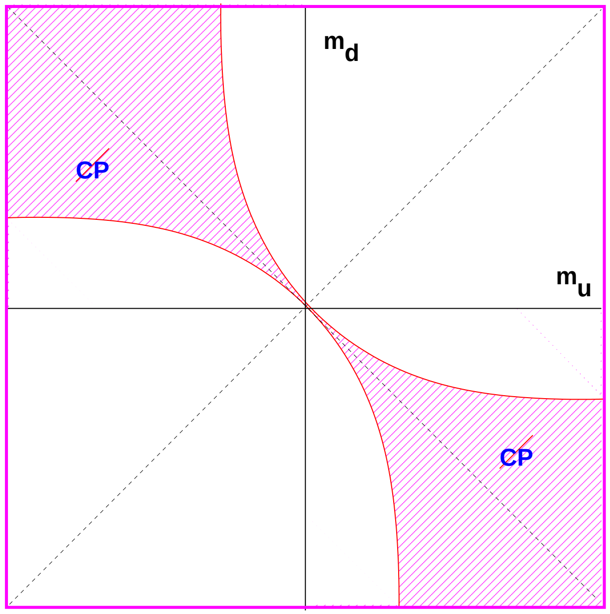
$$\Sigma = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i\phi_1 - i\phi_2} \end{pmatrix}$$

$$m_u \sin(\phi_1) = m_d \sin(\phi_2) = -m_s \sin(\phi_1 + \phi_2)$$

- second order transition at $m_{\pi^0} = 0$
- two degenerate vacua related by $\phi_i \leftrightarrow -\phi_i$
- CP violation appears in three-pseudoscalar couplings

Vafa and Witten: No spontaneous $\not{P}$ in the strong interactions?

- assumes fermion determinant positive
- not true for negative quark masses



Non perturbative

- sign of quark masses significant
- negative $|M|$ corresponds to $\theta = \pi$
- a finite region with $\theta = \pi$ has CP symmetry

Including the η'

- Shifts π^0 and η masses down slightly
- No qualitative change in phase structure

Nothing significant occurs at $m_u = 0$ when $m_d \neq 0$

Can the up quark be massless?

Not a well posed question

Concept of an ‘‘underlying basic Lagrangian’’ does not exist

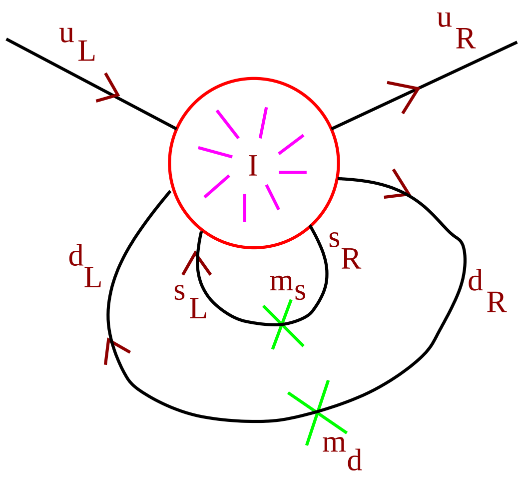
- must regulate divergences
- only underlying symmetries significant
- a single massless quark gives no special symmetry

Continuum theory must be defined as a limit

- need non-perturbative regulator
- lattice

’t Hooft vertex generates additive mass renormalization

- all species flip spin
- tie up heavy quark lines with masses
- $\delta m_u \sim m_d m_s / \Lambda_{qcd}$
- scale and scheme dependent
- non-perturbative



Consider one flavor theory

- integrate out heavier quarks
- anomaly removes all Goldstone bosons
- theory develops a mass gap

Regulated theory depends on two bare parameters

- coupling constant, controls lattice spacing a
- quark mass, use lattice units am_q

Continuum limit

- $a \to 0$
- $am_q \to 0$
- divergent mass renormalization suggests $\frac{m_q}{a} \to 0$

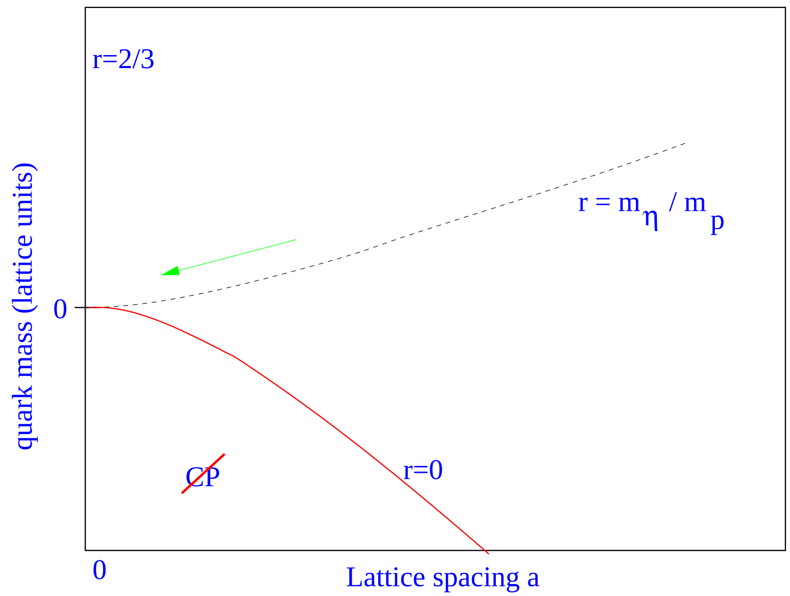
Renormalize holding physical quantities constant as $a \to 0$

- gauge coupling absorbed via dimensional transmutation
- quark mass remains as a parameter
- fix ratio of lightest boson to lightest baryon masses

$$r = \frac{m_\eta}{m_p}$$

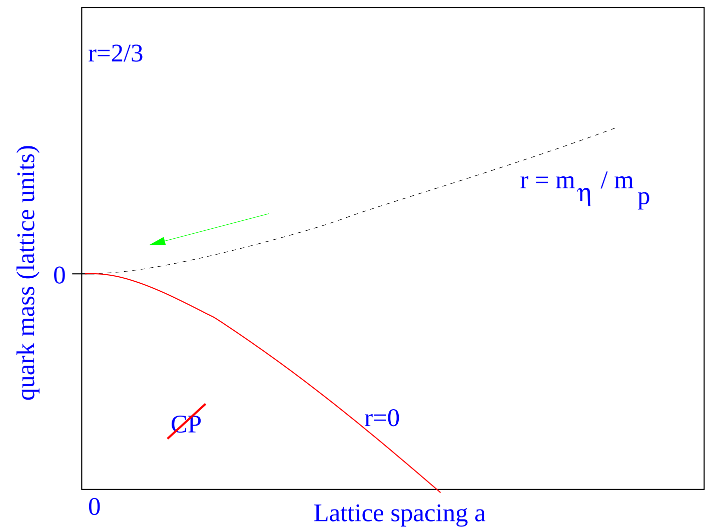
In (a, am_q) plane follow contours of constant $r = \frac{m_\eta}{m_p}$

- $r = 2/3$ represents infinite quark mass
- $r = 0$ along boundary of CP violating phase
- latter NOT $m_q = 0$



Contours depend on details of lattice action

- Wilson -- Staggered -- Domain wall -- Overlap
- $m_q = 0$ axis is not robust



Overlap not unique

- depends on Dirac operator being projected
- Starting with Wilson: input negative mass is adjustable

Can $m_q = 0$ select a unique r in the continuum limit?

- not guaranteed by the Ginsparg-Wilson condition
- continuum limit not uniquely specified

$m_q = 0$ is not well defined

Closing thought problem

$\theta = \arg(\det(M))$

- phase can be shuffled between different quarks
- put all phases into the top quark mass

How can a complex top quark mass affect low energy physics?

SUMMARY

Strong interactions can spontaneously violate CP

- large regions of parameter space
- quark masses differ in sign

$m_u = 0$ is not a physical statement

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paper: hep-th/0303254

these slides: http://thy.phy.bnl.gov/~creutz/slides/cpslides.ps