From boats to antimatter

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Two concepts crucial to particle physics

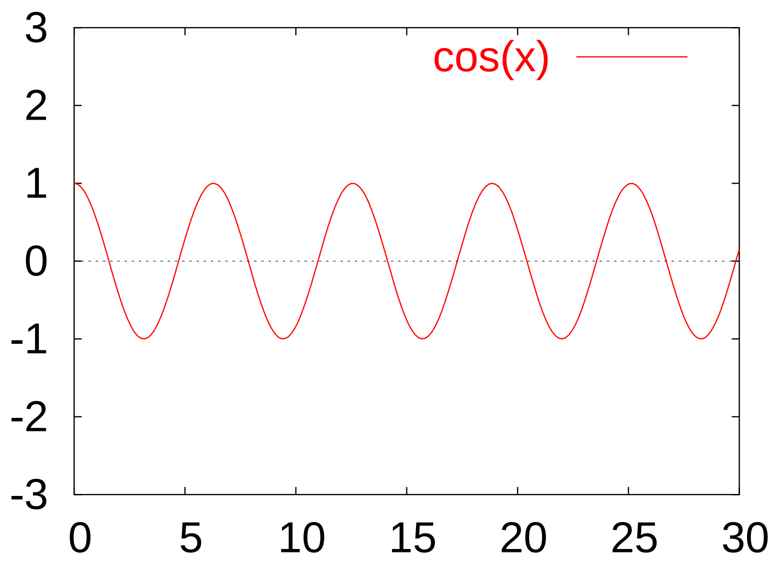
- Relativity: $v < c$
- Quantum mechanics: particles are waves

Only in particle physics do both come together with a vengeance

- “quantum field theory”
- predicts “anti-matter”

This is really a talk about some neat properties of waves

- consequences for boats as well as antimatter!
- prototype wave $\psi(x) = \cos(x)$



Examples

- water: $\psi(x)$ $=$ water height
- sound: $\psi(x)$ $=$ air pressure
- light: $\psi(x)$ $=$ electric field
- electron: $\psi(x)$ $=$ “wave function”

Quantum mechanics: probability for electron at location x

- $P(x) \sim |\psi(x)|^2$

Let the wave move

- $\cos(x) \to \cos(kx - \omega t)$
- $k =$"wavenumber" controls the wavelength ($\lambda = \frac{2\pi}{k}$)
- $\omega =$"frequency" in radians per second ($\frac{\omega}{2\pi}$ cycles per second)

Velocity

- cosine maximum when $kx - \omega t = 0$
- $x = \frac{\omega}{k} t = vt$
- $\boxed{v_p = \frac{\omega}{k} = \text{"phase velocity"}}$

Quantum mechanics

- particle of energy E and momentum p
- really a wave with given frequency and wave number

$$\omega = \frac{E}{\hbar}$$

$$k = \frac{p}{\hbar}$$

- $\hbar = \frac{h}{2\pi} =$ Planck's constant=1.055×10^{-34} Joule seconds
- electron frequency 10^{21} radians/sec
- high frequency $\longleftrightarrow$ high energy $\longleftrightarrow$ short wavelength

Need big accelerators to study small things

Relativity relates energy and momentum to velocity

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

- $E = mc^2 + \frac{1}{2}mv^2 + \frac{1}{8}m\frac{v^4}{c^2} + \ldots$
- energy $=$ Einstein rest energy $+$ Newton $+$ corrections

Put it all together

$$v_p = \frac{\omega}{k} = \frac{E}{p} = \frac{c^2}{v} = c \times \left(\frac{c}{v}\right) > c$$

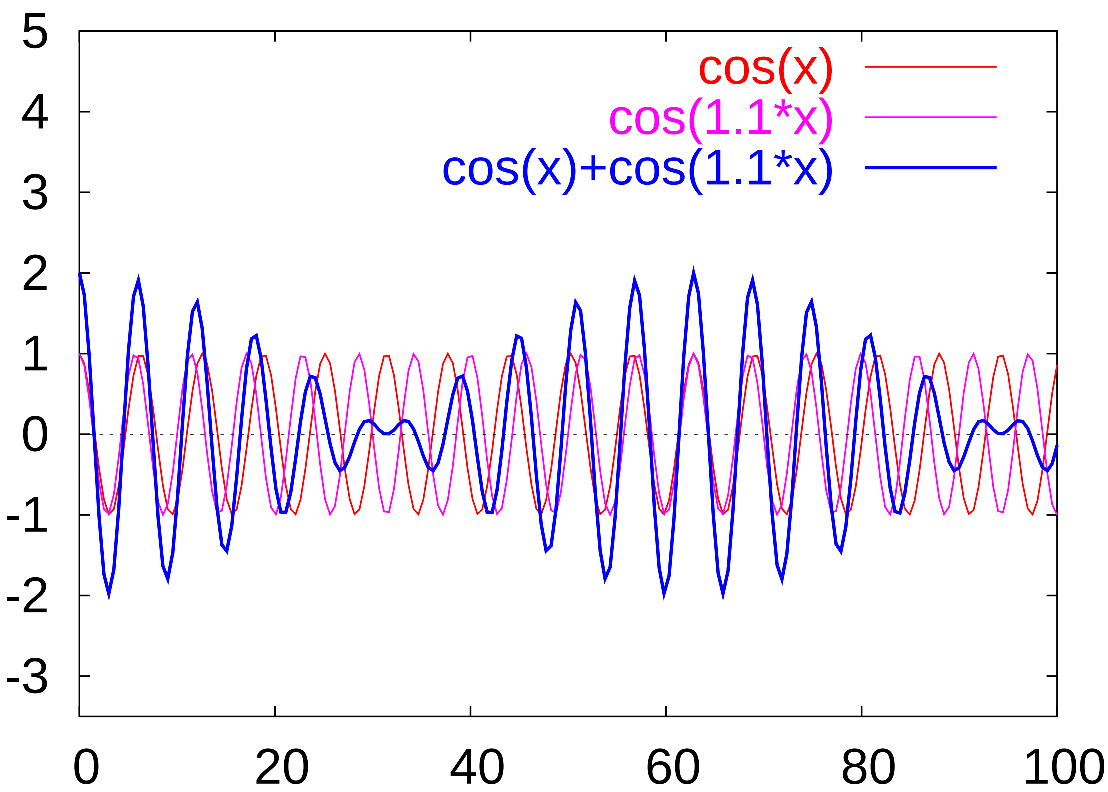
Phase moves faster than light!

- not really a problem
- phase carries no information

A signal requires “modulation”

- like AM or FM radio
- mix nearby frequencies

$$\psi = \cos(kx - \omega t) + \cos(k'x - \omega' t)$$



Waves form "packets"

- concentrated where components "in phase"
- $kx - \omega t = k'x - \omega' t$
- $x = \frac{\omega - \omega'}{k - k'} t$

$$v_g = \frac{\omega - \omega'}{k - k'} = \frac{d\omega}{dk}$$

- v_g = "group velocity"
- generally different than phase velocity: $v_p \neq v_g$

Our quantum mechanical case:

- $E = \sqrt{p^2c^2 + m^2c^4}$
- $\omega = \sqrt{c^2k^2 + m^2c^4/\hbar^2}$

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\omega} = \frac{pc^2}{E} = v$$

Particles are wave packets!!

(demo)

Note on units

- $c =$186,000 miles/sec = 3×10^{10} cm/sec = 1 foot/nanosec
- numerical value depends on units of measure
- can make $c = 1$
- reset lengths allows $\hbar = 1$

Particle physicists love to do this to keep formulas simple

- $\frac{E}{\hbar} = \omega = \sqrt{c^2k^2 + m^2c^4/\hbar^2}$ becomes $E = \omega = \sqrt{k^2 + m^2}$
- could set, say, proton mass to 1, but not usually done

Water Waves

$v_p \neq v_g$ occurs often, including with water

My favorite example of dimensional analysis: how fast are water waves

v_p might be a function of several things

- λ, wavelength, measured in some units of length, L
- g, gravitational pull, units of acceleration, L/T^2
- ρ, density, units of mass per cubic length, M/L^3

From these construct a velocity, with units of length per time, L/T

- only one combination has the right units $L/T = \sqrt{L \times L/T^2}$

$$v_p \sim \sqrt{\lambda g}$$

Explicit solution of $F = ma$ gives

$$v_p = \sqrt{\frac{\lambda g}{2\pi}} = \sqrt{\frac{g}{k}}$$

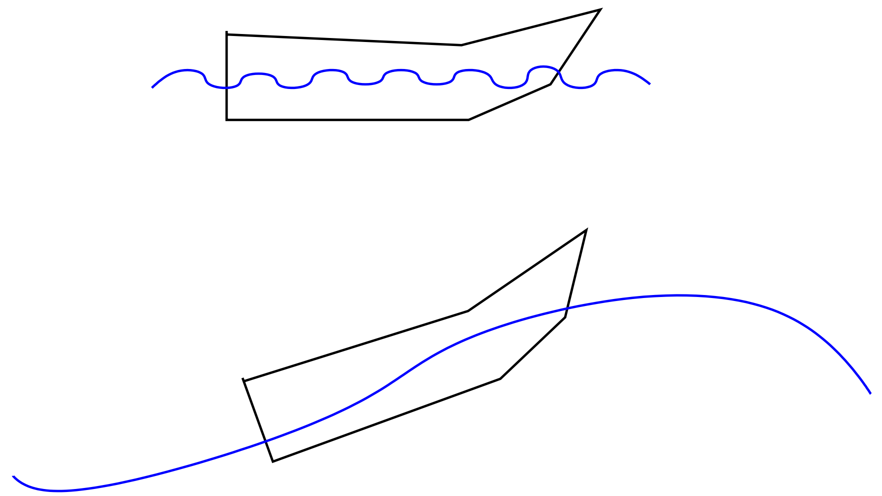
No dependence on density

- waves on mercury metal go at the same speed as on water

Long wavelengths go faster

- tsunami's can go hundreds of miles per hour

Boats have a natural "hull speed" $v_h \sim \sqrt{L}$



- short waves no problem
- at wavelength near boat length, going uphill
- at speed of wave, keep feeding energy into the wave
- a big hole just before breaking into a plane
- longer boats go faster $\sim \sqrt{L}$

Now calculate the group velocity

- $v_p = \sqrt{\frac{g}{k}} = \frac{\omega}{k}$
- $\omega = \sqrt{gk}$
- $v_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}}$

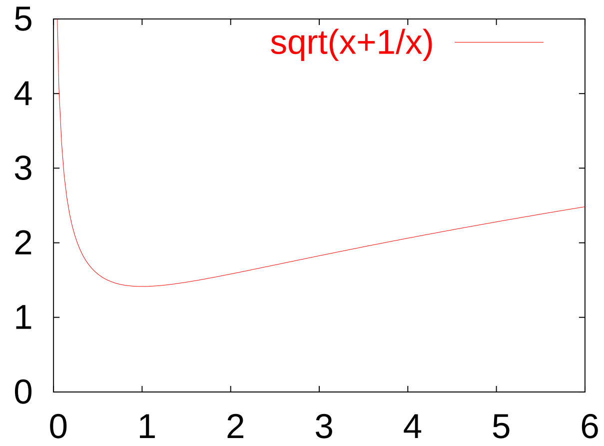
$$v_g = \frac{1}{2} v_p$$

- groups of waves move at half the speed of the wavelets
- ripples on a pond
- surf sets at the beach

(demo)

Correction for very short waves

- surface tension comes into play, $S \sim M/T^2$
- dimensional analysis gives $v_p \sim \sqrt{\frac{S}{\lambda\rho}}$
- $v_g = \frac{3}{2} v_p$
- shorter waves go faster



Water waves have a minimum velocity

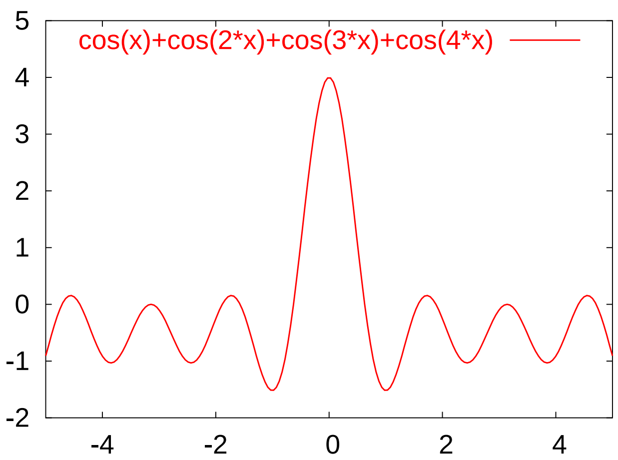
- wind below $v_{min} = 23.1$ cm/sec $\sim .5$ mile/hr cannot drive ripples
- water goes “glassy”

Back to quantum mechanics

$$\omega = \sqrt{k^2 + m^2}$$

Continue to combine many waves

$$\psi = \cos(kx) + \cos(2kx) + \cos(3kx) + \cos(4kx) + \ldots = \sum_{n=1}^{m} \cos(nkx)$$



- all terms in phase at $x = 0$
- wave packets get very peaked

This is how you localize a particle in quantum mechanics

- combine many wavelengths
- combine many momenta
- one momentum is not localized at all

This is the famous ‘‘uncertainty principle’’

$$\Delta p \, \Delta x \geq \hbar$$

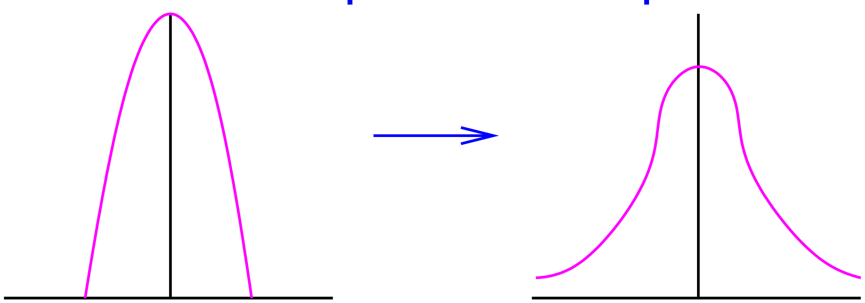
$$\Delta E \, \Delta t \geq \hbar$$

Isolate a particle and let some time pass

$$\psi = \sum \cos\left(nkx - \omega(nk)\ t\right)$$

The ω term messes up the coherence of the waves

- the wave packet will spread out



Herein lies the rub

- tail immediately spreads to all distances
- small but finite probability to go to $x > ct$
- conflicts with $v < c$

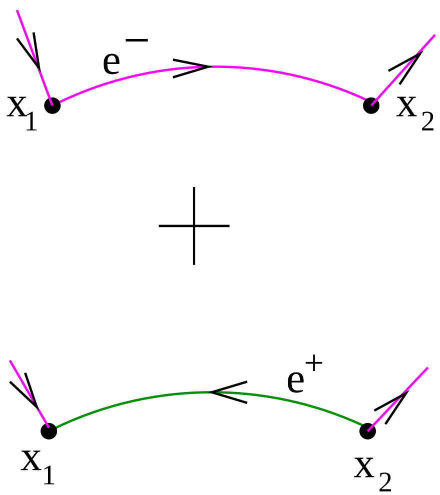
Put electron at x_1, look for it at x_2

- should not see it for distances larger than ct

(demo)

Dirac solved the problem using antimatter

- every particle has an antiparticle
- same mass
- opposite charge
- particle-antiparticle pair can annihilate to energy
- particle-antiparticle pairs can be created from energy



Solves problem by creating confusion

- did the electron at x_2 really come from x_1
- or was it part of an e^+e^- pair
- positron then annihilates the electron from x_1
- No information gets transfered!

An antiparticle is a particle going backwards in time

Mathematically

Construct "operator" $\psi^\dagger(x_1)$

- creates electron at x_1

Operator $\psi(x_2)$

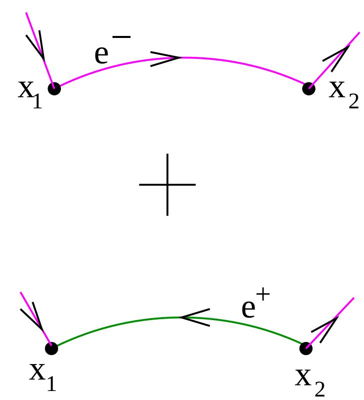
- destroys electron at x_2

If a message cannot get between the points

- order of events should not matter

$$\psi(x_2)\psi^\dagger(x_1) \ = \ \pm\, \psi^\dagger(x_1)\psi(x_2)$$

- sign ambiguity since only $|\psi|^2$ matters
- electrons use minus sign; pions plus (spin statistics relation)



Only possible if

- $\psi^\dagger(x_1)$ can also destroy a positron
- $\psi(x_2)$ can also create a positron

Closing paradox

Particle physicists bash things together

- study products for clues of composition
- a possible reaction:

$$e^- + e^- \to e^- + e^- + e^+ + e^-$$

Is the electron a component of itself??

These slides:

- http://thy.phy.bnl.gov/$\sim$creutz/slides/antimatter/antimatter.pdf

A nice discussion waves including water:

- The Feynman Lectures on Physics, Vol. 1, chapter 51

My wave program and some other toys (for the X Window System)

- http://thy.phy.bnl.gov/www/xtoys/xtoys.html