Lattice gauge theory: A retrospective

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I discuss some of the historical circumstances that drove us to use the lattice as a non-perturbative regulator. This approach has had immense success, convincingly demonstrating quark confinement and obtaining crucial properties of the strong interactions from first principles. I wrap up with some challenges for the future.

1. Introduction

I am honored to have this opportunity to give the final talk at this wonderful meeting. I was originally asked to talk about "Lattice gauge theory: past, present, and future." Well, I really don't know much about the future, and if I did, I would be out trying to develop the ideas. As for the present, well you have for the last few days been hearing all about it, and there is too much for me to usefully review anyway. Thus the majority of this talk will refer back to the early days. I will try to recall the conditions that drove us to this peculiar approach to regularizing quantum field theory.

2. Particle physics in the late 60's

I begin by summarizing the situation in particle physics in the late 60's, when I was a graduate student. Quantum-electrodynamics had already been immensely successful, but the basic theory was in some sense "done." While hard calculations remained, and indeed still remain, there was no major conceptual advance remaining.

These were the years when the "eightfold way" for describing multiplets of particles had recently gained widespread acceptance. The idea of "quarks" was around, but with considerable caution about assigning them any physical reality; maybe they were nothing but a useful mathematical construct. A few insightful theorists were working on the weak interactions, and the basic electroweak unification was beginning to emerge. The SLAC experiments were observing substantial inelastic electron-proton scattering at large angles, and this was interpreted as evidence for substructure, with the term "parton" coming into play. While occasionally there were speculations relating quarks and partons, people tended to be rather cautious about pushing this too hard.

A crucial feature at the time was that the extension of quantum electrodynamics to a meson-nucleon field theory was failing miserably. The analog of the electromagnetic coupling had a value about 15, in comparison with the 1/137 of QED. This meant that higher order corrections to perturbative processes were substantially larger than the initial calculations. There was no known small parameter in which to expand.

In frustration over this situation, much of the particle theory community set aside traditional quantum field theoretical methods and explored the possibility that particle interactions might be completely determined by fundamental postulates such as analyticity and unitarity. This "S-matrix" approach raised the deep question of just "what is elementary." A delta meson might be regarded as a combination of a proton and a pion, but it would be just as correct to regard the proton as a bound state of a pion with a delta. Furthermore, the particles were all bound together by exchanging themselves. These "dual" views of

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the basic objects of the theory persist today in string theory.

3. The early 70's

As we entered the 1970's, partons were increasingly identified with quarks. This shift was pushed by two dramatic theoretical accomplishments. First was the proof of renormalizability for non-Abelian gauge theories [1], giving confidence that these elegant mathematical structures [2] might have something to do with reality. Second was the discovery of asymptotic freedom, the fact that interactions in non-Abelian theories become weaker at short distances [3]. Indeed, this was quickly connected with the pointlike structures hinted at in the SLAC experiments. Out of these ideas evolved QCD, the theory of quark confining dynamics.

The viability of this picture depended upon the concept of "confinement." While there was strong evidence for quark substructure, no free quarks were ever observed. This was particularly puzzling given the nearly free nature of their apparent interactions inside the nucleon. This returns us to the question of just "what is elementary." Are the physical particles we see in the laboratory the fundamental objects or are they these postulated quarks and gluons?

Struggling with this paradox led to the now standard flux tube picture of confinement. The gluonic fields are analogues of photons except that they carry "charge" with respect to eachother. Massless charged particles are rather singular objects, leading to a conjectured instability that removes zero mass gluons from the spectrum, but does not violate Gauss's law. A Coulombic $1/r^2$ field is a solution of the equations of a massless field, but, without massless particles, such a spreading of the gluonic flux is not allowed. The field lines from a quark cannot end, nor can they spread in the inverse square law manner. Instead, as in Fig. 1, the flux lines cluster together, forming a tube of flux emanating from the quark and ultimately ending on an antiquark. This tube is a real physical object, and grows in length as the quark and antiquark are pulled apart. The resulting force is constant at long distance, and can be determined from the slope of the famous "Regge trajectories." In physical units, it has a strength of about 14 tons.

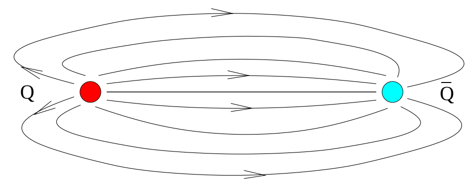

Figure 1. A tube of gluonic flux connects quarks and antiquarks. The strength of this string is 14 tons.

The reason a quark cannot be isolated is similar to the reason that a piece of string cannot have just one end. Of course one can't have a piece of string with three ends either, but this is the reason for the underlying $SU(3)$ group theory. The confinement phenomenon cannot be seen in perturbation theory; when the coupling is turned off, the spectrum becomes free quarks and gluons, dramatically different than the pions and protons of the interacting theory.

4. The mid 70's revolution

We then entered a particularly exciting time for particle physics, with a series of dramatic events revolutionizing the field. First was the discovery of the J/ψ particle [4]. The interpretation of this object and its partners as bound states of heavy quarks provided the hydrogen atom of QCD. The idea of quarks became inescapable; field theory was reborn. The $SU(3)$ non-Abelian gauge theory of the strong interactions combined with the electroweak theory became the durable "standard model."

This same period also witnessed several remarkable realizations on a more theoretical front. Non-linear behaviors in various classical field theories were shown to have deep consequences for their quantum counterparts. Classical "lumps" represented a new way to get particles out of a quantum field theory [5]. Much of the progress here was in two dimensions, where techniques such as "bosonization" showed equivalences between theories of drastically different appearance. A boson in one approach might appear as a bound state of fermions in another, but in terms of the respective Lagrangian approaches, they were equally fundamental. Again, we were faced with the question "what is elementary?" Of course modern string theory is discovering multitudes of "dualities" that continue to raise this same question.

The discovery of such phenomena had deep implications: field theory can have much more structure than seen from a Feynman diagram analysis alone. But this in turn had crucial consequences for practical calculations. Field theory is notorious for divergences requiring regularization. The bare mass and charge entering the theory are infinite quantites. They are not the physical observables, which must be defined in terms of physical processes. To carry out calculations, a "regulator" is required to tame the divergences, and when physical quantities are related to each other, any regulator dependence should drop out.

The need for controlling infinities had, of course, been known since the early days of QED. But all regulators in common use were based on Feynman diagrams; the theorist would calculate diagrams until one diverged, and that diagram was then cut off. Numerous schemes were devised for this purpose, ranging from the Pauli-Villars approach to forest formulae to dimensional regularization. But with the increasing realization that non-perturbative phenomena were crucial, it was becoming clear that we needed a "non-perturbative" regulator, independent of diagrams.

5. The lattice

The necessary tool appeared through Wilson's lattice theory. Wilson presented this as an example of a model with confinement. The strong coupling expansion had a finite radius of convergence, allowing a rigorous demonstration of confinement, albeit in an unphysical limit. The resulting spectrum had exactly the desired properties; only gauge singlet bound states of quarks and gluons could move.

This was not the first time that the basic structure of lattice gauge theory had been written down. A few years earlier, Wegner [6] presented a Z_2 lattice gauge model as an example of a system posessing a phase transistion but not exhibiting any local order parameter. In his thesis, Jan Smit [7] described using a lattice regulator to formulate gauge theories outside of perturbation theory. The time was clearly ripe for the development of such a regulator. Very quickly after Wilson's suggestion, Balian, Drouffe, and Itzykson [8] explored an amazingly wide variety of aspects of these models.

To reiterate, the primary role of the lattice is to provide a non-perturbative cutoff. Space is not really meant to be a crystal, the lattice is a mathematical trick. It provides a minimum wavelength through the lattice spacing a, *i.e.* a maximum momentum of π/a. Path summations become well defined ordinary integrals. By avoiding the convergence difficulties of perturbation theory, the lattice provides a rigorous route to the definition of quantum field theory.

The approach, however, had a marvelous side effect. By discreetly making the system discrete, it becomes sufficiently well defined to be placed on a computer. This was fairly straightforward, and came at the same time that computers were rapidly growing in power. Indeed, numerical simulations and computer capabilities have continued to grow together, making these efforts the mainstay of lattice gauge theory.

6. What is a gauge theory?

The Wilson theory very naturally extends the concept of a gauge theory to the lattice. To expand on this I digress into a few of the "definitions" of a gauge theory. Indeed, there are many, and the lattice approach is closely tied to most.

At the simplest level, a gauge theory is nothing but a generalization of electromagnetism to include an internal symmetry, i.e. the gauge fields are given an "isospin-like" quantum number. Both on the lattice and in the continuum the internal symmetry of the strong gauge field is $SU(3)$. Through this extension the gluons acquire charges with respect to each other. Infrared difficulties with massless charged particles lie at the root of the confinement phenomenon.

A common interpretation of a gauge theory is as a dynamics with an exact local symmetry. The action should be invariant under a gauge transformation, such as the familiar

$$A \longrightarrow g^\dagger A g + ieg^\dagger \partial g$$

This definition is appealing in that it directly extends to gravity, where the basic equations are invariant under local coordinate changes. This symmetry also immediately applies exactly to the plaquette form of the Wilson lattice gauge action. The only difference between electromagnetism and the strong interactions is that the fundamental variables are generalized from a simple $U(1)$ for photons to the internal $SU(3)$ symmetry group with eight gluons.

Another point of view considers a gauge theory directly as a theory of phases. The interaction of an electron with the electromagnetic field appears through its wave function acquiring a phase

$$\psi \longrightarrow \exp(ie\int_x^y A^\mu dx_\mu)\psi$$

where the gauge field is integrated along the path traversed by the particle. In the rest frame of the particle, this is simply the statement that the oscillation frequency, or energy, is increased by the vector potential, *i.e.* $E \to E + eA_0$. A particularly nice thing about this interpretation is that it easily generalizes to non-Abelian theories; one just replaces the word "phase" with "matrix" taken from the appropriate group. For the strong interations this is the group $SU(3)$, where the 3 comes from the requirement of 3 quarks making a proton.

Wilson's formulation directly implements a theory of phases on the lattice. The basic variables are phases associated with the links of a lattice. One such variable is associated with every link connecting a nearest neighbor pair. For the strong interactions they are elements of $SU(3)$. The bond variables are directed, reversing the order of two neighbors gives the inverse matrix.

For completeness let me mention one elegant definition of a gauge theory that does not translate particularly well onto the lattice. This concept was pushed some time ago by Weinberg [9], and looks for interactions in quantum field theory where one is forced to involve fields that do not transform simply under Lorentz transformations. Instead, when one does such a transformation, one must allow for the possibility of a gauge change. This point of view also applies to gravity, where the local formulation requires introducing Christoffel symbols, the analogue of the gauge potentials. That these ideas do not formulate well on the lattice is a consequence of the strong breaking of Lorentz invariance by the lattice regulator.

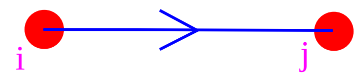

Figure 2. The fundamental variables of lattice gauge theory are matrices associated with nearest neighbor links. For the strong interactions these are $SU(3)$ matrices.

7. Parameters

The way the link matrices are combined to form the Wilson action is well known to this audience, so I won't pursue it here. I do, however, want to reiterate one of the most remarkable aspects of this theory, the paucity of adjustable parameters. To begin with, the lattice spacing itself is not an observable. We are using the lattice to define the theory, and thus for physics we are interested in the continuum limit $a \to 0$. Then there is the coupling constant, which is also not a physical parameter due to the phenomenon of asymptotic freedom. The lattice works directly with a bare coupling, and in the continuum limit this should vanish

$$e_0^2 \to 0$$

In the process, the coupling is replaced by an overall scale for this vanishing. Coleman and Weinberg [10] gave this phenomenon the marvelous name "dimensional transmutation." Of course an overall scale is not really something we should expect to calculate from first principles. Its value would depend on the units chosen, be they furlongs or light-fortnights.

Next consider the quark masses. Indeed, measured in units of the asymptotic freedom scale, these are the only free parameters in the strong interactions. Their origin remains one of the outstanding mysteries of particle physics. The massless limit gives a rather remarkable theory, one with no undetermined dimensionless parameters. This limit is not terribly far from reality; chiral symmetry breaking should give massless pions, and experimentally the pion is considerably lighter than the next non-strange hadron, the rho. A theory of two massless quarks is a fair approximation to the strong interactions at intermediate energies. In this limit all dimensionless ratios should be calculable from first principles, including quantities such as the rho to nucleon mass ratio.

The strong coupling at any physical scale is not an input parameter, but should be determined. Such a calculation has gotten lattice gauge theory into the famous particle data group tables; see Fig.3. With appropriate definition the current result is[11]

$$\alpha_s(M_Z) = 0.115 \pm 0.003$$

where the input is details of the charmonium spectrum.

8. Numerical simulation

Of course, as this audience is intimately familiar, large scale numerical simulations have come to dominate the field. They are based on attempts to evaluate the path integral

$$Z = \int dU e^{-\beta S}$$

A direct evaluation of such an integral has pitfalls. At first sight, the basic size of the calculation is overwhelming. Considering a 10^4 lattice, small by today standards, there are 40,000 links. For each is an $SU(3)$ matrix, parametrized by 8 numbers. Thus we have a $10^4 \times 4 \times 8 = 320,000$ dimensional integral. One might try to replace this with a discrete sum over values of the integrand. If we

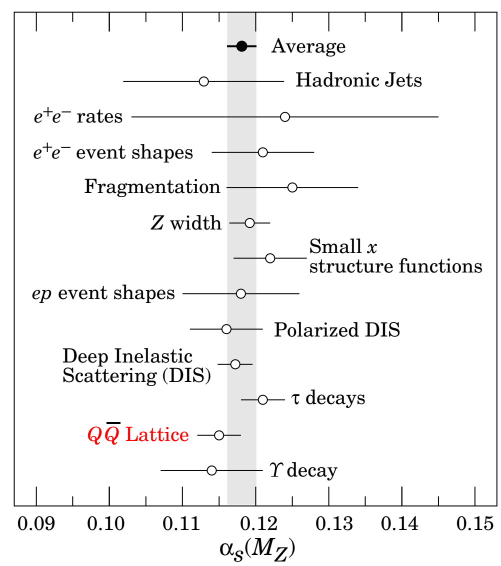

Figure 3. The strong coupling constant is not a parameter, but can be determined by lattice calculations. This figure is from the Particle Data Group summary tables [11].

make the extreme approximation of using only two points per dimension, this gives a sum with

$$2^{320,000} = 3.8 \times 10^{96,329}$$

terms! Of course, computers are getting pretty fast, but one should remember that the age of universe is only $\sim 10^{27}$ nanoseconds.

These huge numbers suggest a statistical treatment. Indeed, the integral in Eq. 8 is formally just a partition function. Consider a more familiar statistical system, such as a glass of Kingfisher. There are a huge number of ways of arranging the atoms of carbon, hydrogen, oxygen, etc. that still leaves us with a glass of Kingfisher. We don't need to know all those arrangements, we only need a dozen or so "typical" glasses to know all the important properties.

This is the basis of the Monte Carlo approach. The analogy with a partition function and the role of $\frac{1}{\beta}$ as a temperature enables the use of standard techniques to obtain "typical" equilibrium configurations, where the probability of any given configuration is given by the Boltzmann weight

$$P(C) \sim e^{-\beta S(C)}$$

For this we use a Markov process, making changes in the current configuration

$$C \to C' \to \ldots$$

biased by the desired weight.

The idea is easily demonstrated with the example of Z_2 lattice gauge theory [12]. For this toy model the links are allowed to take only two values, either plus or minus unity. One sets up a loop over the lattice variables. When looking at a particular link, calculate the probability for it to have value 1

$$P(1) = \frac{e^{-\beta S(1)}}{e^{-\beta S(1)} + e^{-\beta S(-1)}}$$

Then pull out a roulette wheel and select either 1 or -1 biased by this weight. Lattice gauge Monte-Carlo programs are by nature quite simple. They are basically a set of nested loops surrounding a random change of the fundamental variables. Fig. 4 shows the results of my first simulation of this model.

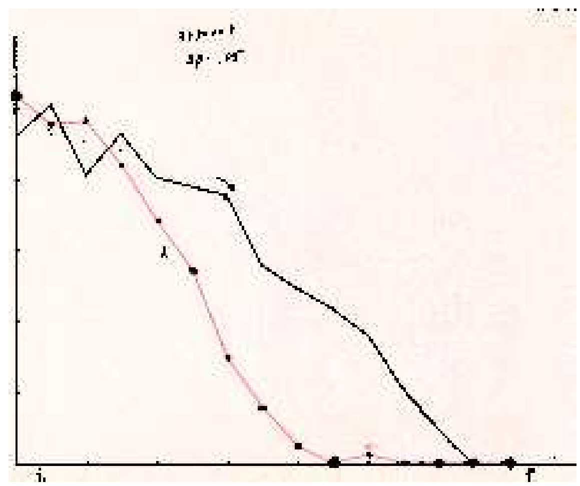

Figure 4. The results of an early lattice gauge simulation of Z_2 lattice gauge theory on a 3^4 lattice. A rapid thermal cycle shows hints of hysteresis. The "blotches" are the consequence of not raising the pen from the paper during a sweep. The program was written in basic and run on a programable Hewlett-Packard calculator.

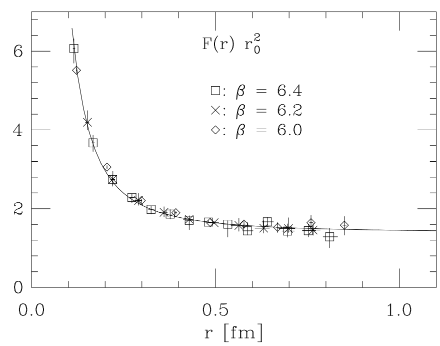

Figure 5. The force between two quarks does not fall to zero as the distance increases. This is the confinement phenomenon. (From Ref. [13]).

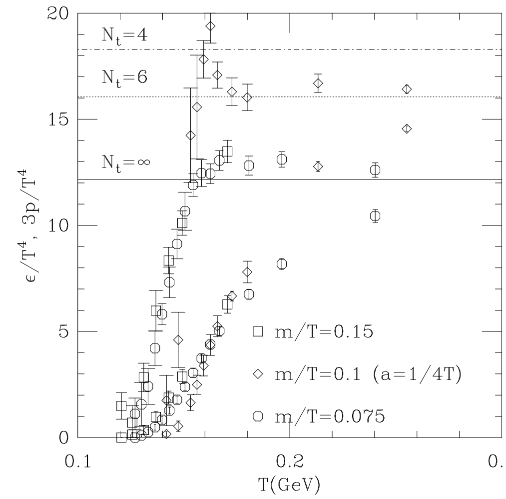

Figure 6. The energy and pressure of the æther show a dramatic structure at a temperature of about 170–190 MeV. The lattice is a unique theoretical tool for the study of this transition to a quark-gluon plasma (From Ref. [15]).

9. Selected accomplishments

Of course this entire meeting is about the accomplishments of the technique. The results have been fantastic, giving first principles calculations of interacting quantum field theory. I will just mention two examples. The early result that bolstered the lattice into mainstream particle physics was the convincing demonstration of the confinement phenomenon. The force between two quark sources indeed remains constant at large distances. A summary of this result is shown in Fig. 5, taken from Ref. [13].

Another accomplishment for which the lattice excels over all other methods has been the study the deconfinement of quarks and gluons into a plasma at a temperature of about 170–190 Mev[14]. Indeed, the lattice is a unique quantitative tool capable of making precise predictions for this temperature. The method is based on the fact that the Euclidean path integral in a finite temporal box directly gives the physical finite temperature partition function, where the size of the box is proportional to the inverse temperature. This transition represents a loss of confining flux tubes in a background plasma. Fig. 6 shows a recent calculation of this transition [15].

10. Quarks

While the gauge sector of lattice gauge theory is in good shape, from the earliest days fermionic fields have caused annoying difficulties. Actually there are several apparently unrelated fermion problems. The first is an algorithmic one. The quark operators are not ordinary numbers, but anticommuting operators in a Grassmann space. As such the exponentiated action itself is not a number but rather an operator. This makes comparison with random numbers problematic.

Over the years various clever tricks for dealing with this problem have been developed; we have seen numerous large scale Monte Carlo simulations involving dynamical fermions. The algorithms used are all essentially based on an initial analytic integration of the quarks to give a determinant. This, however, is the determinant of a rather large matrix, the size being the number of lattice sites times the number of fermion field components, with the latter including spinor, flavor, and color factors. In my opinion, the algorithms working directly with these large matrices remain quite awkward. I often wonder if there is some more direct way to treat fermions without the initial analytic integration.

The algorithmic problem becomes considerably more serious when a chemical potential generating a background baryon density is present. In this case the required determinant is not positive; it cannot be incorporated as a weight in a Monte Carlo procedure. This is particularly frustrating in the light of striking predictions of superconducting phases at large chemical potential [16]. This is perhaps the most serious unsolved problem in lattice gauge theory.

The other fermion problems concern chiral issues. There are a variety of reasons that such symmetries are important in physics. First is the light nature of the pion, which is traditionally related to the spontaneous breaking of a chiral symmetry expected to become exact as the quark masses go to zero. Second, the standard model itself is chiral, with the weak bosons coupling to chiral currents. Third, the idea of chiral symmetry is frequently used in the development of unified models as a tool to prevent the generation of large masses and thus avoid fine tuning.

Despite its importance, chiral symmetry and the lattice have never fit particularly well together. I regard this as evidence that the lattice is trying to tell us something deep. Indeed, the lattice fully regulates the theory, and thus all the famous anomalies must be incorporated explicitly. It is well known that the standard model is anomalous if either the quarks or leptons are left out, and this feature must appear in any valid formulation.

These issues are currently a topic with lots of activity [17]. Several schemes for making chiral symmetry more manifest have been developed, with my current favorite being the domain-wall formulation, where, as sketched in Fig. 7 our four dimensional world is an interface in an underlying five dimensional theory.

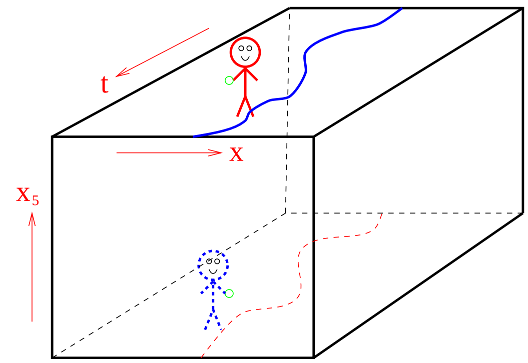

Figure 7. The domain-wall formulation of chiral symmetry regards our four dimensional world as an interface in an underlying five dimensional theory

11. My Pet Problems

As I said at the beginning, I do not know what areas will dominate lattice gauge theory in the future. Thus let me conclude by mentioning some problems that particularly interest me. These are all directly connected with the problems of quarks discussed in the previous section.

The first is the chiral symmetry problem, alluded to above. Here the recent developments have put parity conserving theories, such as the strong interactions, into quite good shape. The various schemes, including domain-wall fermions, the overlap formula, and variants on the Ginsparg-Wilson relation, all quite elegantly give the desired chiral properties. Chiral gauge theories themselves, such as the weak interactions, are not yet completely resolved, but the above techniques appear to be tantilizingly close to giving a well defined lattice regularization. It is still unclear whether the lattice regularization can simultaneously be fully finite, gauge invariant, and local. I expect these issues to be a major topic of continuing research and look forward to the final resolution.

Chiral symmetry should be expected to have a considerably broader impact on particle physics in the future. The problems encountered are closely related to similar issues with supersymmetry, another area that does not naturally fit on the lattice. This also ties in with the explosive activity in string theory and a possible regularization of gravity.

The other area in particular need of advancement lies in dynamical fermion methods. As I said earlier, I regard all existing algorithms as frustratingly awkward. This, plus the fact that sign problem with a background density remains completely unsolved, suggests that new ideas are needed. It has long bothered me that we treat fermions and bosons so differently in numerical simulations. Indeed, why is it that we have to treat them separately?

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