Monte Carlo study of SU(3) gauge theory on a 12^4 lattice

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We calculate Wilson loops for pure SU(3) gauge theory on a 12^4 lattice by Monte Carlo simulations. All the Wilson loops up to size 3×3, previously calculated on a 6^4 lattice, are reproduced on the 12^4 lattice. In addition we calculate all additional Wilson loops up to size 6×6. Using all of these Wilson loops, we estimate the asymptotic-freedom scale parameter Λ_0.

In a series of recent papers, we studied Wilson loops for pure SU(3) gauge theory on 6^4 (Ref. 1) and 8^4 (Ref. 2) lattices. Because of finite-size effects, we can only study Wilson loops of size as large as $L\times L$ on an L^4 lattice, so we have been able to measure 4×4 Wilson loops on our 8^4 lattice. However, a lattice of this size does not appear[3] to be large enough to accommodate an elementary particle such as a proton. It therefore seems reasonable to extend our previous analysis to a larger lattice. The largest lattice which will fit comfortably in real memory in the $2m$-word 2-pipe CDC CYBER 205, operating in 32-bit mode, is a 12^4 lattice. As a result, we can measure all Wilson loops up to size 6×6, can test asymptotic freedom up to this level, and improve our previous measurements of the asymptotic-freedom scale parameter Λ_0.

Our theory is formulated on a hypercubical lattice in four Euclidean space-time dimensions. The sites of our lattice are denoted by $n=(n_1,n_2,n_3,n_4)$. The link joining any pair of nearest-neighbor sites is labeled by (n,μ). The degrees of freedom of our pure gauge field are defined on the links of the lattice. $U_\mu(n)$ is a matrix-valued field in the fundamental representation of the gauge group SU(3) so that

$$U_\mu(n)=\exp[iB_\mu(n)] ,$$

where

$$B_\mu(n)=\tfrac{1}{2}ag_0\lambda_\alpha A_\mu^\alpha(n) , \quad \alpha=1,2,3,\ldots,8 ,$$

where B_μ has compact support, the reverse path gives the inverse group element, i.e.,

$$U_{-\mu}(n+\mu)=U_\mu{}^{-1}(n) ,$$

a is the lattice spacing, g_0 is the bare coupling constant, λ_α are SU(3) matrices, and $A_\mu^\alpha(n)$ are the vector potentials of the gauge field. The Euclidean action of the theory is given by

$$S[U]=\sum_{\Box}S_\Box=\sum_{n,\mu>\nu}\{1-\tfrac{1}{3}\operatorname{Re}\operatorname{Tr}[U_\mu(n)U_\nu(n+\mu)U_{-\mu}(n+\mu+\nu)U_{-\nu}(n+\nu)]\} ,$$

where the sum is taken around all unoriented plaquettes $\Box$ and μ and ν are the two positive directions in a plane. The integral we are simulating is the partition function defined by

$$Z(\beta)=\int\left[\prod_{\text{links}}dU_\mu(n)\right]\exp(-\beta S[U]) ,$$

where β is the inverse coupling constant squared given by $\beta=6/g_0{}^2$ and the measure in this integral is the normalized invariant Haar measure for SU(3).

The contour C is a rectangular contour of length I and width J. We define the Wilson loops[4] by the expectation value

$$W(I,J)=\tfrac{1}{3}\langle\operatorname{Re}\operatorname{Tr}U_C\rangle ,$$

where U_C is the parallel transporter around the contour C. The leading-order strong- and weak-coupling expansions are

$$W(I,J)=\left[\frac{\beta}{18}\right]^{IJ}[1+O(\beta)]$$

and

$$W(1,1)=1-\frac{2}{\beta}+O(\beta^{-2}) ,$$

TABLE I. Data for the SU(3) Wilson loops. The first number in each column is the average value for the given loop. The second is the standard deviation over the 50 measurements. As discussed in the text, this number is approximately five times the error in the loop measurement.

(a)

β	$\langle W(1,1)\rangle$	$\langle W(2,1)\rangle$	$\langle W(2,2)\rangle$	$\langle W(3,1)\rangle$
5.6	0.5241, 0.0016	0.2925, 0.0017	0.1038, 0.0015	0.1654, 0.0015
5.8	0.5685, 0.0014	0.3504, 0.0016	0.1571, 0.0018	0.2200, 0.0017
6.0	0.5936, 0.0014	0.3836, 0.0015	0.1901, 0.0017	0.2527, 0.0016
6.2	0.6135, 0.0013	0.4100, 0.0013	0.2170, 0.0016	0.2793, 0.0013
6.3	0.6225, 0.0011	0.4222, 0.0013	0.2300, 0.0014	0.2919, 0.0013
6.4	0.6308, 0.0013	0.4332, 0.0013	0.2413, 0.0014	0.3033, 0.0013
6.5	0.6384, 0.0011	0.4433, 0.0012	0.2517, 0.0014	0.3136, 0.0011
6.6	0.6458, 0.0012	0.4535, 0.0012	0.2628, 0.0015	0.3243, 0.0013
6.7	0.6528, 0.0012	0.4629, 0.0012	0.2729, 0.0016	0.3343, 0.0013
6.8	0.6591, 0.0011	0.4715, 0.0012	0.2824, 0.0013	0.3434, 0.0012
6.9	0.6656, 0.0013	0.4807, 0.0013	0.2926, 0.0016	0.3534, 0.0015
7.0	0.6717, 0.0013	0.4890, 0.0012	0.3018, 0.0014	0.3621, 0.0012
7.5	0.6981, 0.0012	0.5259, 0.0010	0.3435, 0.0013	0.4026, 0.0012
8.0	0.7205, 0.0013	0.5577, 0.0012	0.3806, 0.0015	0.4381, 0.0013
9.0	0.7561, 0.0009	0.6095, 0.0011	0.4436, 0.0013	0.4975, 0.0012
10.0	0.7834, 0.0012	0.6502, 0.0011	0.4951, 0.0012	0.5456, 0.0012

(b)

β	$\langle W(3,2)\rangle$	$\langle W(3,3)\rangle$	$\langle W(4,1)\rangle$	$\langle W(4,2)\rangle$	$\langle W(4,3)\rangle$
5.6	0.0390, 0.0010	0.0102, 0.0008	0.0937, 0.0012	0.0148, 0.0007	0.0028, 0.0005
5.8	0.0758, 0.0016	0.0302, 0.0011	0.1388, 0.0015	0.0372, 0.0012	0.0125, 0.0008
6.0	0.1015, 0.0014	0.0472, 0.0012	0.1673, 0.0016	0.0554, 0.0012	0.0229, 0.0009
6.2	0.1234, 0.0014	0.0627, 0.0014	0.1911, 0.0013	0.0717, 0.0012	0.0332, 0.0010
6.3	0.1346, 0.0014	0.0713, 0.0015	0.2028, 0.0014	0.0804, 0.0014	0.0395, 0.0012
6.4	0.1439, 0.0012	0.0780, 0.0012	0.2133, 0.0013	0.0876, 0.0011	0.0442, 0.0009
6.5	0.1528, 0.0012	0.0845, 0.0011	0.2229, 0.0012	0.0945, 0.0012	0.0485, 0.0010
6.6	0.1627, 0.0013	0.0928, 0.0014	0.2330, 0.0013	0.1027, 0.0012	0.0549, 0.0011
6.7	0.1716, 0.0016	0.0994, 0.0016	0.2425, 0.0013	0.1099, 0.0015	0.0598, 0.0013
6.8	0.1800, 0.0014	0.1061, 0.0014	0.2511, 0.0012	0.1167, 0.0013	0.0647, 0.0011
6.9	0.1895, 0.0015	0.1142, 0.0015	0.2609, 0.0015	0.1248, 0.0015	0.0712, 0.0012
7.0	0.1976, 0.0012	0.1206, 0.0009	0.2693, 0.0014	0.1315, 0.0012	0.0762, 0.0009
7.5	0.2365, 0.0013	0.1534, 0.0013	0.3093, 0.0013	0.1652, 0.0012	0.1023, 0.0012
8.0	0.2728, 0.0016	0.1860, 0.0017	0.3453, 0.0013	0.1980, 0.0015	0.1298, 0.0015
9.0	0.3363, 0.0014	0.2449, 0.0016	0.4073, 0.0013	0.2578, 0.0015	0.1822, 0.0016
10.0	0.3905, 0.0015	0.2979, 0.0018	0.4590, 0.0012	0.3107, 0.0017	0.2311, 0.0020

(c)

β	$\langle W(4,4)\rangle$	$\langle W(5,1)\rangle$	$\langle W(5,2)\rangle$	$\langle W(5,3)\rangle$	$\langle W(5,4)\rangle$
5.6	0.0006, 0.0007	0.0531, 0.0009	0.0056, 0.0006	0.0008, 0.0005	0.0002, 0.0008
5.8	0.0044, 0.0007	0.0877, 0.0013	0.0185, 0.0009	0.0053, 0.0006	0.0018, 0.0006
6.0	0.0102, 0.0007	0.1110, 0.0015	0.0305, 0.0008	0.0113, 0.0007	0.0047, 0.0006
6.2	0.0167, 0.0009	0.1310, 0.0012	0.0419, 0.0009	0.0179, 0.0007	0.0086, 0.0006
6.3	0.0206, 0.0011	0.1411, 0.0013	0.0485, 0.0012	0.0222, 0.0009	0.0111, 0.0008
6.4	0.0237, 0.0009	0.1501, 0.0012	0.0536, 0.0009	0.0252, 0.0007	0.0129, 0.0006
6.5	0.0264, 0.0010	0.1587, 0.0012	0.0588, 0.0010	0.0282, 0.0008	0.0147, 0.0007
6.6	0.0310, 0.0010	0.1676, 0.0014	0.0652, 0.0010	0.0329, 0.0010	0.0179, 0.0008
6.7	0.0345, 0.0011	0.1762, 0.0014	0.0709, 0.0013	0.0365, 0.0010	0.0203, 0.0008
6.8	0.0378, 0.0011	0.1839, 0.0010	0.0762, 0.0011	0.0400, 0.0010	0.0227, 0.0009
6.9	0.0429, 0.0012	0.1928, 0.0015	0.0827, 0.0013	0.0450, 0.0011	0.0264, 0.0011
7.0	0.0463, 0.0010	0.2005, 0.0014	0.0880, 0.0011	0.0485, 0.0008	0.0286, 0.0007
7.5	0.0661, 0.0012	0.2379, 0.0014	0.1160, 0.0011	0.0688, 0.0009	0.0434, 0.0008
8.0	0.0884, 0.0015	0.2724, 0.0013	0.1444, 0.0014	0.0915, 0.0013	0.0611, 0.0012
9.0	0.1331, 0.0016	0.3337, 0.0014	0.1982, 0.0015	0.1364, 0.0015	0.0982, 0.0015
10.0	0.1763, 0.0023	0.3865, 0.0013	0.2480, 0.0017	0.1804, 0.0019	0.1359, 0.0020

TABLE I. (*Continued.*)

(d)

β	$\langle W(5,5)\rangle$	$\langle W(6,1)\rangle$	$\langle W(6,2)\rangle$	$\langle W(6,3)\rangle$	$\langle W(6,4)\rangle$
5.6		0.0301, 0.0008	0.0020, 0.0004	0.0004, 0.0005	
5.8	0.0006, 0.0008	0.0553, 0.0011	0.0092, 0.0007	0.0023, 0.0005	0.0007, 0.0005
6.0	0.0020, 0.0007	0.0735, 0.0013	0.0168, 0.0007	0.0057, 0.0005	0.0022, 0.0005
6.2	0.0043, 0.0007	0.0898, 0.0011	0.0246, 0.0008	0.0098, 0.0006	0.0045, 0.0005
6.3	0.0058, 0.0009	0.0982, 0.0013	0.0293, 0.0011	0.0125, 0.0008	0.0060, 0.0007
6.4	0.0068, 0.0008	0.1058, 0.0011	0.0328, 0.0008	0.0146, 0.0006	0.0072, 0.0007
6.5	0.0076, 0.0009	0.1130, 0.0011	0.0368, 0.0009	0.0166, 0.0006	0.0083, 0.0006
6.6	0.0101, 0.0007	0.1205, 0.0013	0.0414, 0.0011	0.0199, 0.0009	0.0106, 0.0005
6.7	0.0116, 0.0007	0.1281, 0.0014	0.0458, 0.0012	0.0224, 0.0008	0.0121, 0.0007
6.8	0.0131, 0.0009	0.1348, 0.0009	0.0498, 0.0010	0.0248, 0.0007	0.0136, 0.0007
6.9	0.0158, 0.0010	0.1426, 0.0015	0.0549, 0.0011	0.0286, 0.0009	0.0164, 0.0008
7.0	0.0174, 0.0009	0.1493, 0.0014	0.0590, 0.0010	0.0310, 0.0008	0.0179, 0.0006
7.5	0.0280, 0.0008	0.1830, 0.0014	0.0816, 0.0010	0.0465, 0.0009	0.0288, 0.0007
8.0	0.0416, 0.0012	0.2150, 0.0013	0.1056, 0.0013	0.0648, 0.0012	0.0426, 0.0012
9.0	0.0717, 0.0015	0.2735, 0.0016	0.1526, 0.0015	0.1025, 0.0013	0.0728, 0.0014
10.0	0.1040, 0.0019	0.3255, 0.0014	0.1982, 0.0018	0.1413, 0.0018	0.1053, 0.0019

(e)

β	$\langle W(6,5)\rangle$	$\langle W(6,6)\rangle$
5.6		0.0001, 0.0006
5.8	0.0003, 0.0005	0.0002, 0.0008
6.0	0.0009, 0.0005	0.0003, 0.0006
6.2	0.0021, 0.0005	0.0010, 0.0007
6.3	0.0031, 0.0006	0.0016, 0.0008
6.4	0.0038, 0.0006	0.0019, 0.0007
6.5	0.0042, 0.0005	0.0023, 0.0007
6.6	0.0058, 0.0006	0.0032, 0.0006
6.7	0.0069, 0.0005	0.0040, 0.0007
6.8	0.0077, 0.0006	0.0045, 0.0007
6.9	0.0097, 0.0007	0.0058, 0.0009
7.0	0.0107, 0.0007	0.0064, 0.0007
7.5	0.0183, 0.0007	0.0119, 0.0008
8.0	0.0287, 0.0010	0.0196, 0.0010
9.0	0.0527, 0.0012	0.0385, 0.0013
10.0	0.0801, 0.0017	0.0613, 0.0018

respectively. The string tension, the coefficient of the area term in the asymptotic Wilson loops, is obtained by forming the logarithmic ratios

$$\chi(I,J)=-\ln\left[\frac{W(I,J)W(I-1,J-1)}{W(I,J-1)W(I-1,J)}\right].$$

The leading-order strong-coupling expansion for the string tension is given by

$$\chi(I,J)=-\ln\left[\frac{\beta}{18}\right]+O(\beta)\ . \tag{1}$$

In the region where asymptotic-freedom scaling sets in, we have an asymptotic-freedom scale parameter Λ_0 which sets the scale for our theory, e.g., mass ratios are defined in terms of Λ_0. This parameter is defined by

$$\Lambda_0=\lim_{a\to 0}\frac{1}{a}\left[\frac{33}{8\pi^2\beta}\right]^{-51/121}\exp\left[-\frac{4\pi^2\beta}{33}\right]. \tag{2}$$

Because of the difficulty of graphical representation, we present our Wilson loops up to size 6×6 in Table I. This table should make this paper more useful to the reader. In some previous calculations, it was found that, for instance, the full 64-bit word length of the CDC CYBER 205 was not necessary to retain the accuracy of our results. The CDC CYBER 205 has a facility for working in half-word length, i.e., 32-bit word length, which effectively doubles the amount of memory available and doubles the result rate for arithmetic vector operations. Working in the 32-bit mode is found to be, in general, accurate enough for our needs. However, for reasons of numerical stability and greater confidence in the values associated with the larger loops, it is advisable to carry out the renormalization using 64-bit arithmetic. This was done for all the results presented here. Our calculations were performed by first carrying out 600 iterations through the 12^4 lattice with 9 Monte Carlo updates per link. These iterations were used to equilibriate our lattice and no averaging took place during these 600 iterations. The Wilson-loop averages were then obtained from the next 200 iterations through the lattice. However, there may be correlations

TABLE II. Error estimates for the SU(3) Wilson loops. The first number in each column is the average value for the given loop. The second is the error estimate obtained as discussed in the text.

	$\beta=6.0$	$\beta=6.2$	$\beta=6.5$
$\langle W(1,1)\rangle$	0.5936 ± 0.0003	0.6135 ± 0.0002	0.6384 ± 0.0002
$\langle W(2,1)\rangle$	0.3836 ± 0.0005	0.4100 ± 0.0003	0.4433 ± 0.0003
$\langle W(2,2)\rangle$	0.1900 ± 0.0005	0.2170 ± 0.0004	0.2517 ± 0.0002
$\langle W(3,1)\rangle$	0.2528 ± 0.0005	0.2793 ± 0.0003	0.3137 ± 0.0002
$\langle W(3,2)\rangle$	0.1015 ± 0.0005	0.1234 ± 0.0004	0.1528 ± 0.0001
$\langle W(3,3)\rangle$	0.0472 ± 0.0004	0.0627 ± 0.0004	0.0845 ± 0.0001
$\langle W(4,1)\rangle$	0.1673 ± 0.0005	0.1911 ± 0.0003	0.2229 ± 0.0002
$\langle W(4,2)\rangle$	0.0554 ± 0.0004	0.0717 ± 0.0003	0.0945 ± 0.0001
$\langle W(4,3)\rangle$	0.0229 ± 0.0002	0.0332 ± 0.0002	0.0485 ± 0.0001
$\langle W(4,4)\rangle$	0.0102 ± 0.0001	0.0167 ± 0.0002	0.0265 ± 0.0001
$\langle W(5,1)\rangle$	0.1110 ± 0.0004	0.1310 ± 0.0002	0.1587 ± 0.0002
$\langle W(5,2)\rangle$	0.0305 ± 0.0002	0.0420 ± 0.0002	0.0588 ± 0.0001
$\langle W(5,3)\rangle$	0.0113 ± 0.0001	0.0179 ± 0.0002	0.0282 ± 0.0001
$\langle W(5,4)\rangle$	0.0047 ± 0.0001	0.0086 ± 0.0001	0.0147 ± 0.0001
$\langle W(5,5)\rangle$	0.0020 ± 0.0001	0.0043 ± 0.0001	0.0076 ± 0.0002
$\langle W(6,1)\rangle$	0.0736 ± 0.0003	0.0898 ± 0.0002	0.1130 ± 0.0002
$\langle W(6,2)\rangle$	0.0168 ± 0.0001	0.0246 ± 0.0002	0.0368 ± 0.0001
$\langle W(6,3)\rangle$	0.0058 ± 0.0001	0.0098 ± 0.0001	0.0166 ± 0.0001
$\langle W(6,4)\rangle$	0.0022 ± 0.0001	0.0045 ± 0.0001	0.0084 ± 0.0001
$\langle W(6,5)\rangle$	0.0009 ± 0.0001	0.0021 ± 0.0001	0.0042 ± 0.0001
$\langle W(6,6)\rangle$	0.0003 ± 0.0001	0.0010 ± 0.0001	0.0023 ± 0.0002

between iterations through the lattice. In order to reduce correlations and because measuring loops is rather time consuming, every fourth iteration was used in our average. Thus, only 50 lattice configurations were used in our averages. We renormalized our SU(3) matrices, in order to eliminate rounding errors, every four iterations through the lattice. To check if the renormalization procedure had any effect on our results, we carried out identical runs with the renormalized procedure carried out every four and eight iterations through the lattice. The results were effectively the same. In order to satisfy ourselves that the results are independent, to within reason, of the sequence of random numbers used, we repeated a run (for $\beta=5.6$) using two different sequences of random numbers. Again, the average values were in good agreement. Also, totally nonoverlapped sequences of random numbers were used for different β values. Another test was done to check if the system is indeed equilibriated and if the use of 50 configurations for calculating the Wilson-loop values is sufficient. An additional run was executed for $\beta=6.4$ as follows: The system was equilibriated using 1200 iterations, then 100 configurations taken for the average loop values where 7 configurations were discarded between each one taken. Results for the different runs were in reasonable agreement. The CDC CYBER 205, a pipelined vector processor, performs one upgrade per link in 76 μsec when operating in the 32-bit mode. The upgrade-per-link time quoted here includes the time spent in 64-bit arithmetic renormalization every four iterations. In the 64-bit mode the upgrade-per-link time is 105 μsec. It must be em-

TABLE III. Data for the SU(3) loop ratios. The first number in each column is the average value of a given ratio. The second is the standard deviation on the five sets of ten average as discussed in the text.

	$\beta=6.0$	$\beta=6.2$	$\beta=6.5$
$\chi(1,1)$	0.5208 ± 0.0008	0.4881 ± 0.0006	0.4492 ± 0.0003
$\chi(2,2)$	0.2686 ± 0.0013	0.2314 ± 0.0007	0.2039 ± 0.0008
$\chi(3,2)$	0.2145 ± 0.0007	0.1809 ± 0.0008	0.1543 ± 0.0005
$\chi(3,3)$	0.1518 ± 0.0039	0.1129 ± 0.0041	0.0943 ± 0.0031
$\chi(4,2)$	0.1966 ± 0.0046	0.1625 ± 0.0028	0.1310 ± 0.0021
$\chi(4,3)$	0.1166 ± 0.0074	0.0898 ± 0.0072	0.0747 ± 0.0035
$\chi(4,4)$	0.1064 ± 0.0214	0.0661 ± 0.0149	0.0732 ± 0.0067
$\chi(5,2)$	0.1921 ± 0.0026	0.1606 ± 0.0045	0.1347 ± 0.0025
$\chi(5,3)$	0.1226 ± 0.0275	0.0949 ± 0.0116	0.0601 ± 0.0049
$\chi(5,4)$	0.0499 ± 0.0228	0.0175 ± 0.0219	0.0530 ± 0.0115
$\chi(6,2)$	0.2003 ± 0.0063	0.1582 ± 0.0039	0.1272 ± 0.0042
$\chi(6,3)$		0.1003 ± 0.0186	0.0630 ± 0.0120
$\chi(6,4)$		0.0179 ± 0.0437	0.0341 ± 0.0154

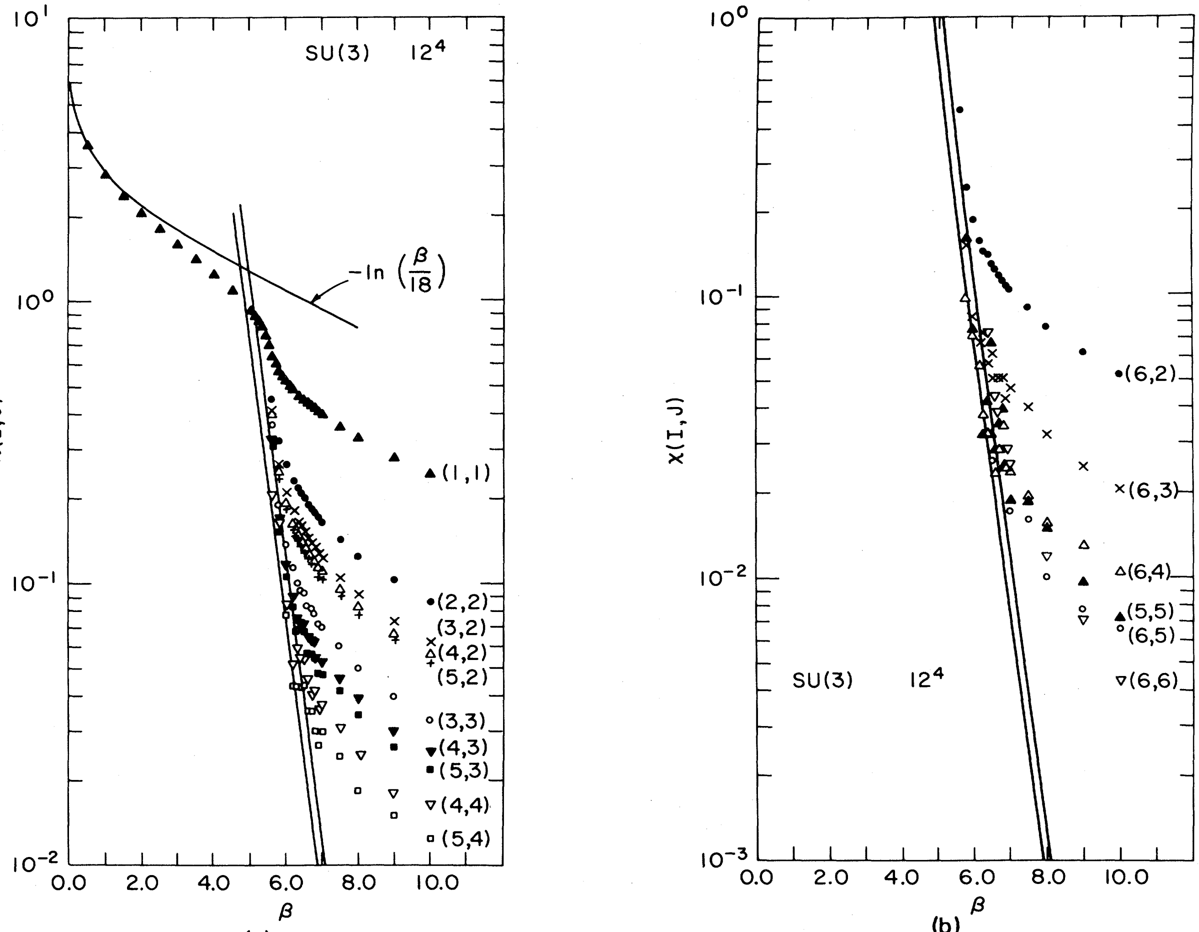

FIG. 1. The string tension $\chi(I,J)$ for pure SU(3) gauge theory on a 12^4 lattice as a function of the inverse coupling constant squared β. The leading-order strong-coupling expansion of Eq. (1) is also shown, as well as lines corresponding to the behavior of Eq. (2) with $\Lambda_0=7\times10^{-3}\sqrt{K}$ and $9\times10^{-3}\sqrt{K}$.

TABLE IV. The quantity $\lambda(I,J)$ related to the loop ratios with the scaling form removed [Eq. (3)].

	$\beta=6.0$	$\beta=6.2$	$\beta=6.5$
$\lambda(1,1)$	0.003 25	0.002 68	0.001 99
$\lambda(2,2)$	0.004 53±0.000 01	0.003 89±0.000 01	0.002 95±0.000 01
$\lambda(3,2)$	0.005 07±0.000 01	0.004 40±0.000 01	0.003 40±0.000 01
$\lambda(3,3)$	0.006 02±0.000 08	0.005 57±0.000 10	0.004 35±0.000 07
$\lambda(4,2)$	0.005 29±0.000 06	0.004 64±0.000 04	0.003 57±0.000 03
$\lambda(4,3)$	0.006 87±0.000 22	0.006 25±0.000 25	0.004 88±0.000 11
$\lambda(4,4)$	0.007 19±0.000 72	0.007 29±0.000 82	0.004 93±0.000 23
$\lambda(5,2)$	0.005 35±0.000 04	0.004 67±0.000 07	0.003 64±0.000 03
$\lambda(5,3)$	0.006 70±0.000 75	0.006 08±0.000 37	0.005 44±0.000 22
$\lambda(5,4)$			0.005 79±0.000 63
$\lambda(6,2)$	0.005 24±0.000 08	0.004 71±0.000 06	0.003 74±0.000 06
$\chi(6,3)$		0.005 91±0.000 55	0.005 31±0.000 51
$\chi(6,4)$			0.007 23±0.001 63

phasized that no shortcuts, e.g., only calculating two columns of an SU(3) matrix and reconstructing the third column by orthogonality to the first two, have been used. Using such tricks, the SU(3) upgrade time per link, when the CDC CYBER 205 is operating in the 32-bit mode, can be further reduced. The crossover point[1,2] between strong and weak coupling for SU(3) is $\beta \simeq 5.6$. Thus, for all our runs we used ordered starting lattices for $\beta \geq 5.6$ and disordered starting lattices for $\beta < 5.6$.

All the results of this paper for Wilson loops up to size 3×3 agree perfectly with the results of Ref. 1. However, some of our results for Wilson loops up to size 4×4 do not agree well with the results of Ref. 2 and when we take the logarithm of the ratio of products of these Wilson loops these discrepancies are acccentuated. These discrepancies can be clearly seen in Figs. 2 and 5 of Ref. 2 where there is a nonsmooth merger of the mixed-phase and ordered-phase starting lattice data at $\beta=7.0$. The origin of this discrepancy is not hard to find. It lies in the fact that in Ref. 2 we only used 20 iterations through the lattice to equilibriate our 8^4 lattice. This was clearly too short a time to equilibriate a 4×4 Wilson loop except for large values of β. When the results of Ref. 2 are reanalyzed by throwing away the first 200 iterations through our lattice, the results of Refs. 1 and 2 and the present calculation all agree. In the present calculation the 600 iterations used to equilibriate our lattice were clearly enough.

To obtain a good error estimate, we repeated our runs at $\beta=6.0$, 6.2, and 6.5. The Wilson-loop measurements were divided into 5 batches of 10 numbers. Assuming each batch was statistically independent, we calculated the standard deviation of the mean from these 5 averages. The results of this calculational procedure are presented in Table II. This error estimate, when compared with the standard deviations in Table I, is about five times smaller. We believe this factor can be safely applied to all the data in Table I.

The logarithmic ratios $\chi(I,J)$, for $I,J=1,2,\ldots,6$, as a function of the inverse coupling squared β are shown in Fig. 1. The leading-order strong-coupling expansion of Eq. (1) is also displayed in Fig. 1(a). Also shown in Fig. 1 are two lines corresponding to the functional form of Eq. (2) with

$$\frac{\Lambda_0}{\sqrt{K}}=7\times10^{-3} \text{ and } 9\times10^{-3}\ ,$$

where K is the string tension. In our earlier work[1,2] we estimated $(6\pm1)\times10^{-3}$ for this ratio. These new data suggest a somewhat larger value. Note that the higher statistics data presented here indicate that the string approaches the scaling behavior of Eq. (2) from above. As the errors in different loops may be correlated, we calculated the errors in the ratios, χ, at $\beta=6.2$ and 6.5 in the same way as they were calculated for the Wilson loops, i.e., by calculating the standard deviations of the means derived by dividing our measurements into 5 groups of 10 values. These results are presented in Table III.

To see how the string tension approaches the scaling behavior we form the following quantity:

$$\lambda(I,J)=\left[\frac{33}{8\pi^2\beta}\right]^{-33/121}\exp\left[\frac{4\pi^2\beta}{33}\right]\frac{1}{\sqrt{\chi(I,J)}}\ . \qquad (3)$$

This is the loop ratios with the scaling form removed. In the asymptotic-freedom region, this quantity should approach Λ_0. The results of this calculation are shown in Table IV. Note that the largest loops still show a slight trend with increasing loop size. This makes an accurate determination of the limiting value for this ratio rather subjective. The 4×4 ratio scales well from $\beta=6.0$ to 6.2 and agrees with the 6×4 ratio at $\beta=6.6$. From this we feel that a value of $\Lambda_0/\sqrt{K}=(7.5\pm0.5)\times10^{-3}$ is a reasonable estimate but conservatively this should be regarded as a lower bound.

Our results are consistent with a recent study on lattices of up to 10^4 sites,[5] where $\Lambda_0/\sqrt{K}=(7.9\pm0.5)\times10^{-3}$ was quoted. We also note that an increase in Λ_0 over old results is claimed in a limited study on a 16^4 lattice in Ref. 6.

A string tension decreasing towards its asymptotic form was previously observed for SU(4) and SU(5) gauge theories.[7] These models have a residual but not deconfining first-order phase transition. A related critical end point is present[8] for SU(3) for a more generalized action than that of Wilson. It is likely that new physics near this critical point is affecting the scaling in all these models, delaying somewhat the onset of asymptotic-freedom behavior.[9]

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