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LETTER TO THE EDITOR

U(N) and SU(N) lattice gauge theories, in the weak-coupling region, in four dimensions

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Abstract. Monte-Carlo simulations on a 6^4 lattice are employed to calculate the 1×1 Wilson loops $W(1,1)$ and the associated string tension for pure U(N) and SU(N), $N=2, 3, 4, 5$ and 6, gauge theories in the weak-coupling region. It is shown that $\mathrm{U}(N)\rightarrow\mathrm{SU}(N)$ as N increases, with $\mathrm{U}(6)\simeq\mathrm{SU}(6)$.

In some recent papers, we studied pure U(N) gauge theories (Creutz and Moriarty 1982a) and found phase transitions for $N=2$, 3, 4, 5 and 6 and studied pure U(2) (Creutz and Moriarty 1982b), U(3) (Creutz and Moriarty 1982c) and U(4) (Barkai *et al* 1982) gauge theories and found the phase transitions to be colour confining. The U(N) gauge group contains SU(N) and U(1) components which should decouple at weak coupling leaving an effective SU(N) theory. Thus, the U(N) results should duplicate the SU(N) results when shifted by a fixed amount in the inverse coupling constant squared. This was indeed shown to be true for $N=2$ (Creutz and Moriarty 1982b) and $N=3$ (Creutz and Moriarty 1982c) with a shift of 1.75 and 1.3 in β, respectively. The result for $N=4$ (Barkai *et al* 1982) was not very precise due to the quality of the statistics. It seems reasonable to recalculate the results for $N=4$ and extend the analysis to $N=5$ and 6. This analysis shows that $\mathrm{U}(N)\simeq\mathrm{SU}(N)$ for $N=6$ in the weak-coupling region.

Our calculational procedures for both scalar (Ardill *et al* 1983) and vector (Barkai *et al* 1983) computers are by now well known and will not be described here. Our study was carried out on a hypercubical lattice in four Euclidean space–time dimensions. Nearest-neighbour lattice sites are denoted by i and j and between them we form a link $\{i,j\}$ on which sits an $N\times N$ matrix $U_{ij}\in\mathrm{U}(N)$ or $U_{ij}\in\mathrm{SU}(N)$ such that

$$U_{ji}=(U_{ij})^{-1}.$$

Our partition function is defined by

$$Z(\beta)=\int\left(\prod_{\{i,j\}}\mathrm{d}U_{ij}\right)\exp(-\beta S[U])$$

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where β is directly proportional to the inverse coupling constant squared, $\beta = 2N/g_0^2$, where g_0 is the bare coupling constant and the measure in the above integral is the normalised invariant Haar measure for U(N) or SU(N). Our action S is the sum over all plaquettes $\square$ such that

$$S[U] = \sum_{\square} S_{\square} = \sum_{\square} \left(1 - \frac{1}{N} \operatorname{Re} \operatorname{Tr} U_{\square}\right)$$

where $U_{\square}$ is the parallel transporter around a plaquette. Periodic boundary conditions were imposed throughout our calculations and our lattice was equilibriated by the method of Metropolis *et al* (1953).

We calculated the 1×1 Wilson loop (Wilson 1974) $W(1, 1)$. To form an order parameter appropriate for comparing pure U(N) and SU(N) gauge theories in the weak-coupling region we used

$$\chi(1, 1) = -\ln W(1, 1),$$

which is related to the leading-order string tension calculation.

In figure 1 we present $\chi(1, 1)$ for pure U(N) and SU(N), for $N = 2$, 3, 4, 5 and 6, in the weak-coupling region. By using a Chebyshev polynomial fit to these data, we calculate that the difference between U(N) and SU(N) for constant $\chi(1, 1)$ is approximately 1.75, 1.3, 1.0, 1.0 and 1.0 in β. Moriarty and Samuel (1983) showed that a better variable to use to

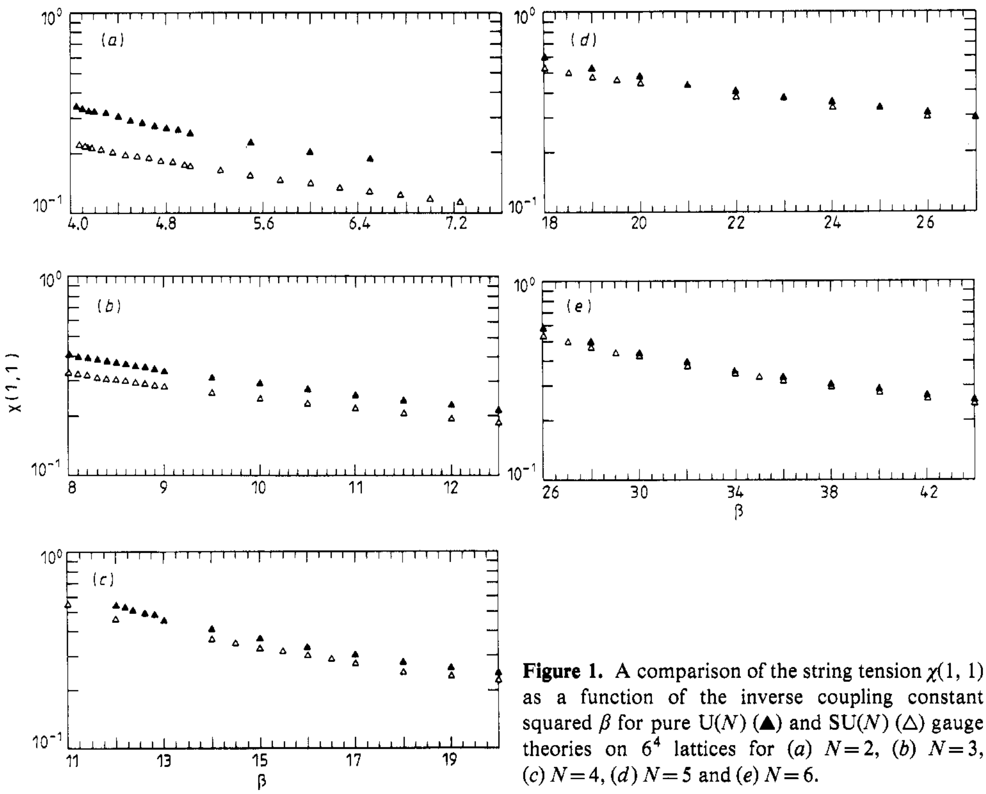

Figure 1. A comparison of the string tension $\chi(1, 1)$ as a function of the inverse coupling constant squared β for pure U(N) (▲) and SU(N) (△) gauge theories on 6^4 lattices for (*a*) $N = 2$, (*b*) $N = 3$, (*c*) $N = 4$, (*d*) $N = 5$ and (*e*) $N = 6$.

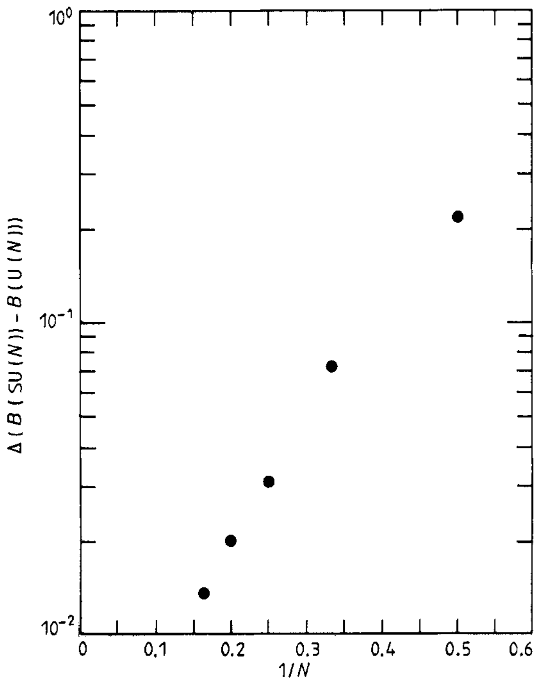

Figure 2. The difference between $\chi(1, 1)$ for pure SU(N) and U(N) gauge theories in the weak-coupling region in the units $B=1/g_0^2N$ as a function of $1/N$ for $N=2$, 3, 4, 5 and 6.

achieve a smooth transition to U(∞) and SU(∞) is

$$B=1/g_0^2N=\beta/2N^2.$$

In figure 2 we show the difference between U(N) and SU(N), in the weak-coupling region, in the variable B as a function of $1/N$ for $N=2$, 3, 4, 5 and 6. From figures 1 and 2 we can see that U(N)$\rightarrow$SU(N) as N increases with U(6)$\simeq$SU(6).

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