Simulations with Improved Staggered Fermions

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SciDAC All Hands Meeting

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MILC Collaboration

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Dataset

coarse

fine

$am_{u,d}$ / am_s	$10/g^2$	L	lats.	a/r_1
quenched	8.00	20	408 N	0.3762(8)
0.02 / na	7.20	20	547	0.3744(11)
0.40 / 0.40	7.35	20	332 N	0.3766(10)
0.20 / 0.20	7.15	20	341 N	0.3707(10)
0.10 / 0.10	6.96	20	339 N	0.3730(14)
0.05 / 0.05	6.85	20	425 N	0.3742(15)
0.04 / 0.05	6.83	20	351 N	0.3765(14)
0.03 / 0.05	6.81	20	564 N	0.3775(12)
0.02 / 0.05	6.79	20	484 N	0.3775(12)
0.01 / 0.05	6.76	20	658 N	0.3846(12)
0.01 / 0.05	6.76	28	268	0.3801(17)
0.007 / 0.05	6.76	20	474 N	0.3782(14)
0.005 / 0.05	6.76	24	221 R	0.3794(23)
0.03 / 0.03	6.79	20	248 R	
0.01 / 0.03	6.76	24	93 R	
quenched	8.40	28	415 N	0.2686(6)
0.031 / 0.031	7.18	28	496 N	0.2613(9)
0.0124 / 0.031	7.11	28	527 N	0.2697(10)
0.0062 / 0.031	7.09	28	592 N	0.2712(8)
0.0031 / 0.031	7.08	40	48 R	

$$r_1 = 0.317\,(7)(3)\text{fm}$$

Tests of Simulation Parameters

- Precision (double/single)
- RMD step size
- Finite volume
- MD autocorrelation

Precision

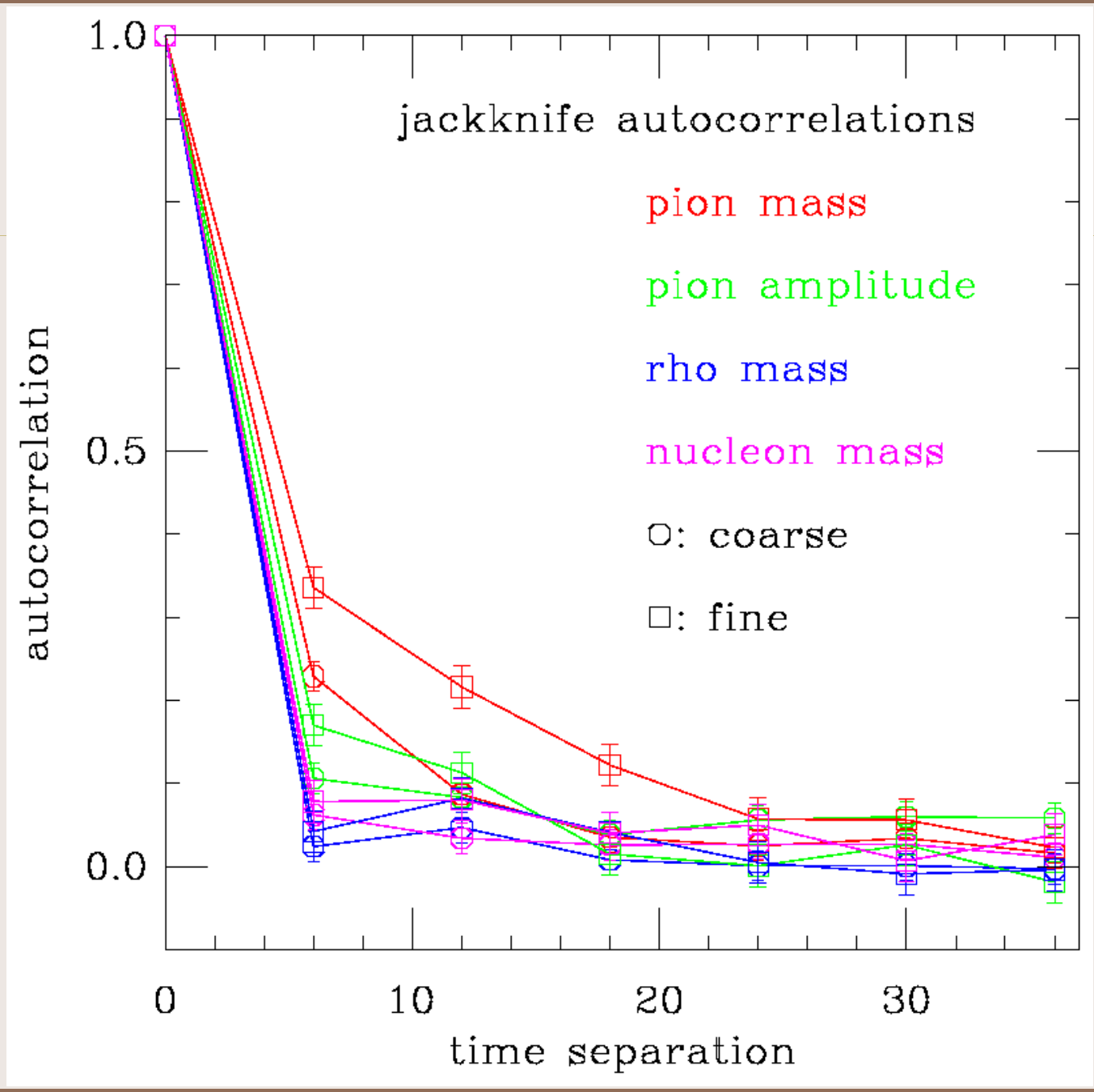
Quantity	Double	Single	Comment
$V(2,0,0)$	0.829883(852)	0.829888(853)	potential at r=(2,0,0)
$V(2,2,2)$	1.05426(503)	1.05451(502)	
$V(3,3,3)$	1.2511(194)	1.2511(194)	
r_1/a	2.63933(1679)	2.63915(1678)	t=4–5, block=5
r_0/r_1	1.4566(64)	1.4566(64)	t=4–5, block=5
$\pi(20)$	411.53(1.55)	411.44(1.55)	prop. at d=20
$\rho(6)$	143.76(1.78)	143.73(1.78)	
aM_π	0.15965(22)	0.15966(21)	d=20–31, $\chi^2/D = 0.60$
aM_K	0.36519(34)	0.36519(34)	d=20–32, $\chi^2/D = 0.82$
aM_ρ	0.5330(83)	0.5330(83)	d=6–14, $\chi^2/D = 0.85$
aM_N	0.7311(84)	0.7312(84)	d=6–14, $\chi^2/D = 0.50$

MD Step Size

Q.	$L = 20$ $\epsilon = 0.0067$	$L = 28$ $\epsilon = 0.0067$	$L = 20$ $\epsilon = 0.0100$	$L = 20$ $\epsilon = 0.013$
$\Box$	1.70092(2)	1.70094(3)	1.70096(7)	1.70066(7)
$\bar{\psi}\psi$	0.07421(10)	0.07420(13)	0.07374(37)	0.07488(35)
r_1/a	2.598(8)	2.621(9)	2.649(29)	2.619(28)
aM_π	0.22439(20)	0.22421(12)	0.22500(73)	0.22554(70)
aM_ρ	0.569(5)	0.568(3)	0.557(18)	0.558(18)
aM_N	0.771(4)	0.767(3)	0.785(15)	0.753(14)

Finite Volume

Quantity	$L = 20$	$L = 28$	Δ
r_1/a	2.598(8)	2.621(9)	-0.023(12)
r_0/r_1	1.4461(36)	1.4533(34)	-0.0072(50)
M_π	0.22439(20)	0.22421(12)	0.00018(23)
M_ρ	0.569(5)	0.568(3)	0.001(6)
M_N	0.771(4)	0.767(3)	0.004(5)


jackknife autocorrelations
pion mass
pion amplitude
rho mass
nucleon mass
○: coarse
□: fine
autocorrelation
1.0
0.5
0.0
0
10
20
30
time separation

Setting the Scale

- Y(2S) – Y(1S) or Y(2P) – Y(1S)

$$r_1: \quad r_1^2 F_{Q\bar{Q}}(r_1) = 1.00$$

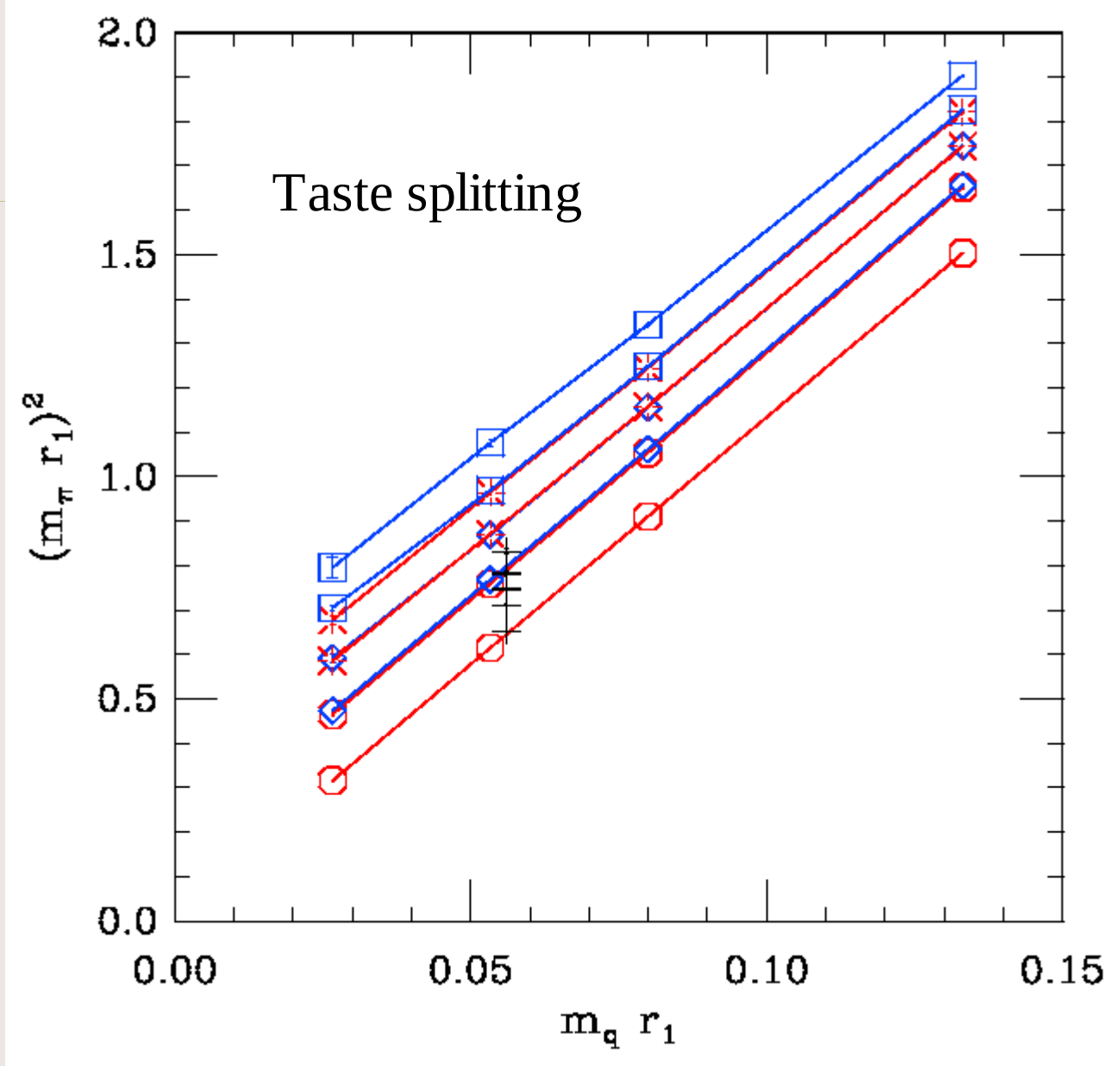
$$r_1 / a = f(am_{ud}, am_s, 10/g^2)$$

- Continuum, physical:

$$r_1 = 0.317\,(7)(3)\,\text{fm}$$

Staggered Chiral Perturbation Theory

- Taste splittings
- $m_{PS}^2/(m_1+m_2)$ and f_π
- Strange quark mass
- Topological susceptibility
- Nucleon mass


Taste splitting
2.0
1.5
1.0
0.5
0.0
0.00
0.05
0.10
0.15
$(m_\pi r_1)^2$
$m_q r_1$

Scaling of splittings

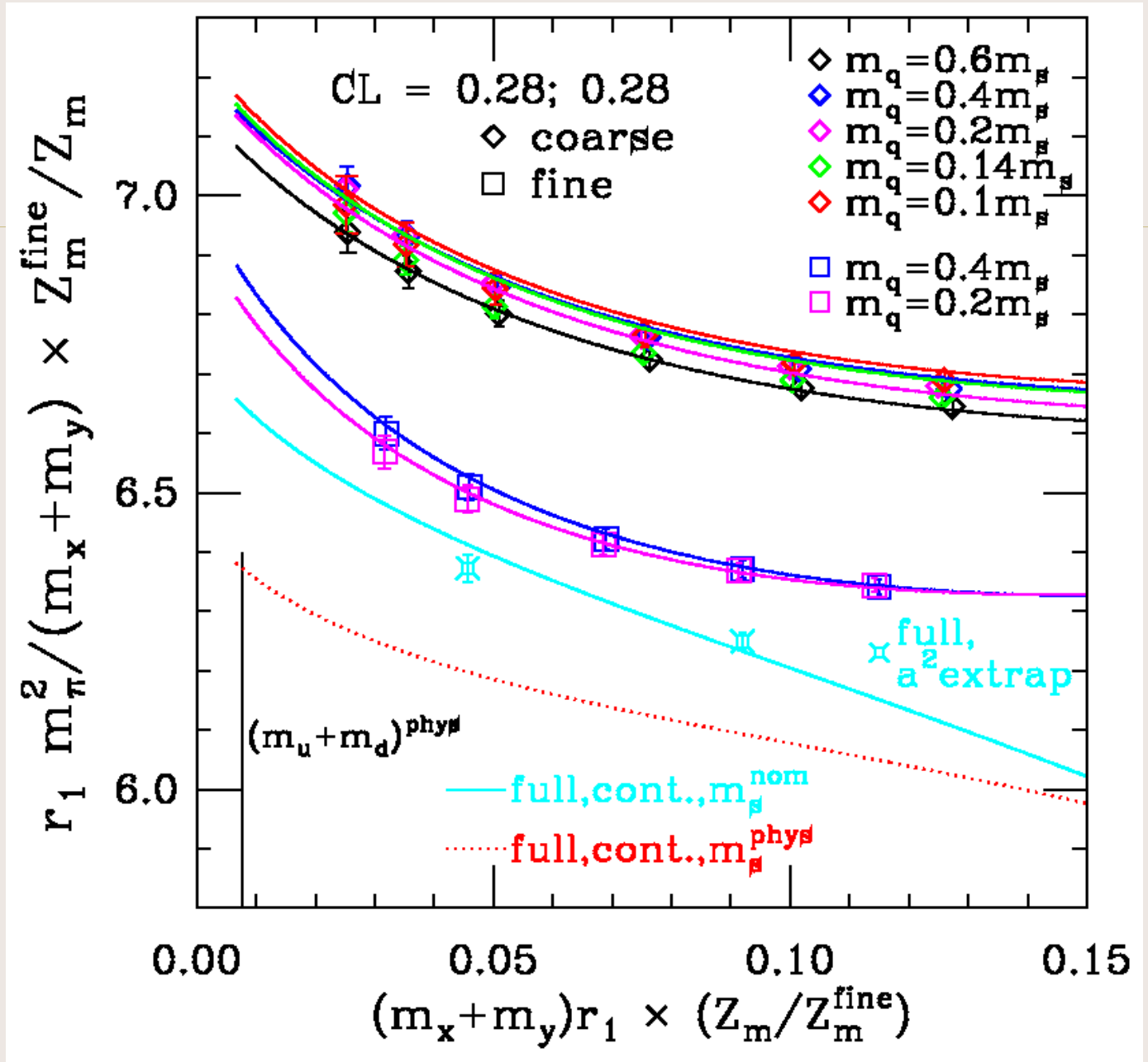
- Chiral perturbation theory prediction:

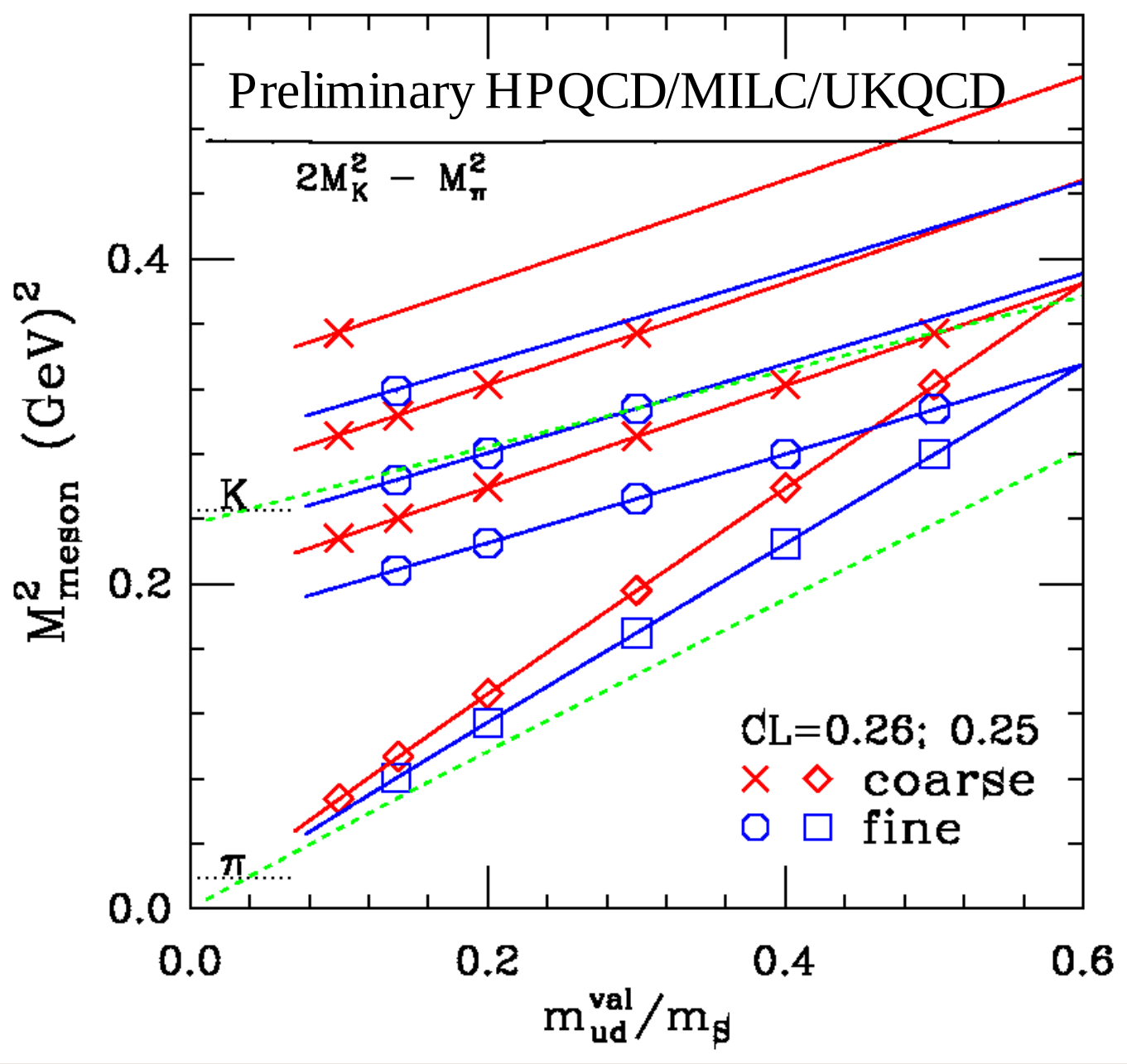
$$\Delta m_{\pi i}^2 = O(\alpha_s^2 a^2)$$

$$(\alpha_s^2 a^2)_{\text{fine}} / (\alpha_s^2 a^2)_{\text{coarse}} = 0.324 - 0.398$$

for $q^* = \pi / a - 2\pi / a$

- Observed: 0.35


CL = 0.28; 0.28
coarse
fine
$m_q=0.6m_s$
$m_q=0.4m_s$
$m_q=0.2m_s$
$m_q=0.14m_s$
$m_q=0.1m_s$
$m_q=0.4m_s$
$m_q=0.2m_s$
full, a^2 extrap
$(m_u+m_d)^{phys}$
full,cont.,m_s^{nom}
full,cont.,m_s^{phys}
$r_1\ m_\pi^2/(m_x+m_y) \times Z_m^{fine}/Z_m$
$(m_x+m_y)r_1 \times (Z_m/Z_m^{fine})$
7.0
6.5
6.0
0.00
0.05
0.10
0.15


Preliminary HPQCD/MILC/UKQCD
$2M_K^2 - M_\pi^2$
M^2_{meson} (GeV)^2
0.0
0.2
0.4
K
π
0.0
0.2
0.4
0.6
m_{ud}^{val}/m_s
CL=0.26; 0.25
coarse
fine

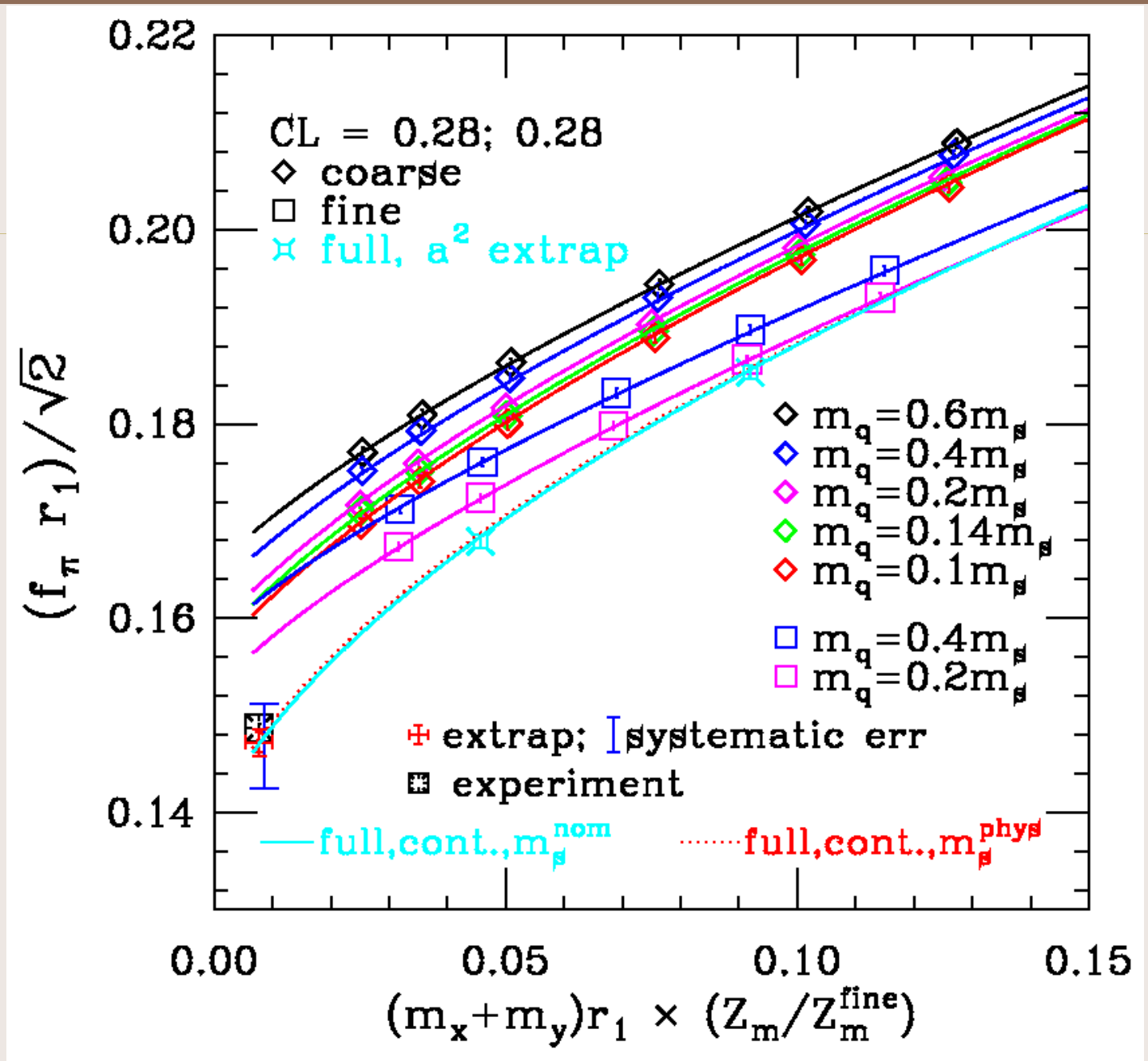
Strange quark mass HPQCD/MILC/UKQCD

$$m_s^{\overline{MS}}(2GeV) = 76(0)(3)(6)(0)MeV$$

$$m_{ud}^{\overline{MS}}(2GeV) = 2.9(0)(1)(2)(1)MeV$$

$$m_s / m_{u,d} = 26.3(1)(3)(0)(11)$$

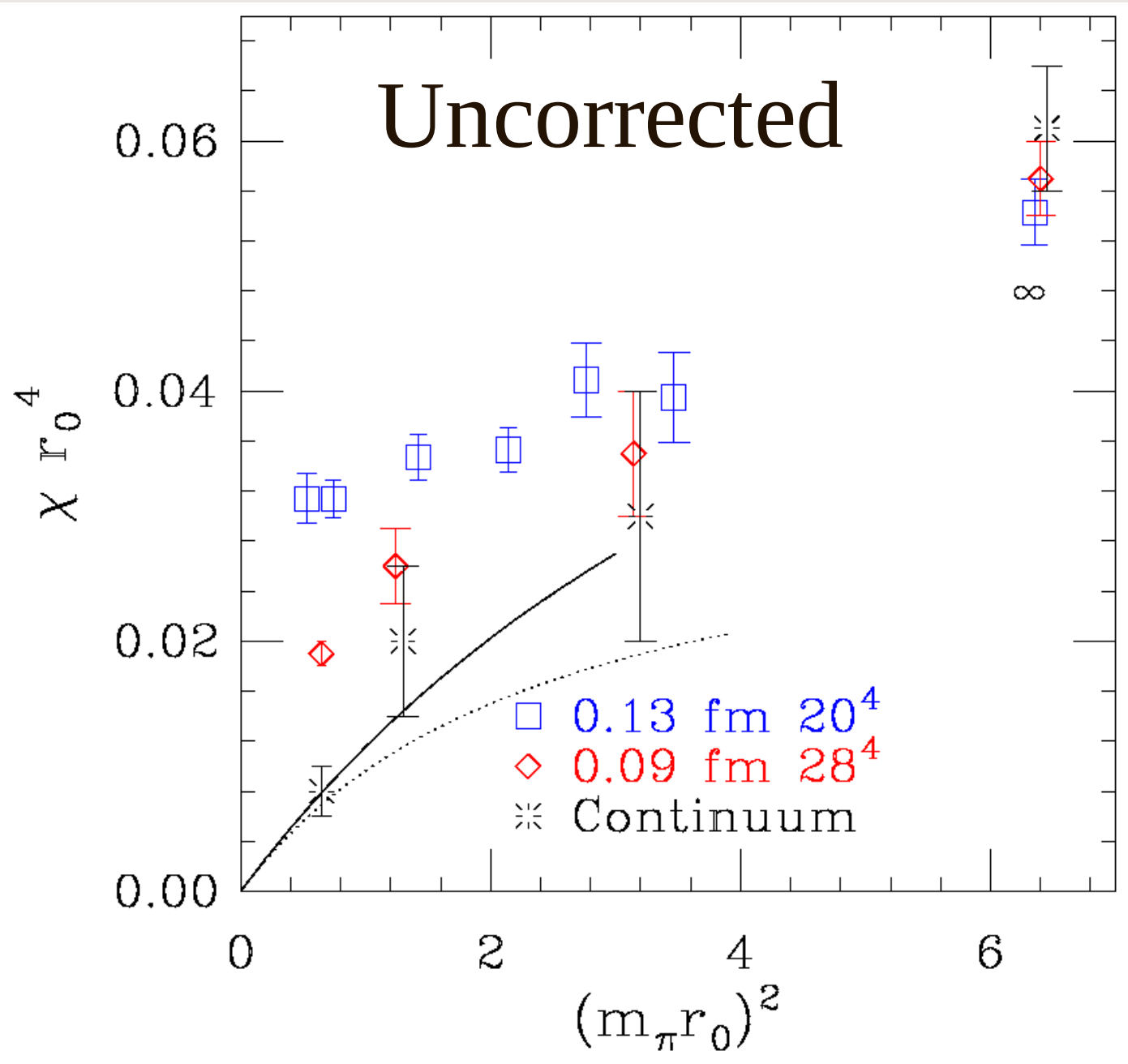
- Errors: statistical, simulation systematics, perturbation theory, e.m. effects
- Improvement helps convergence in Z_m

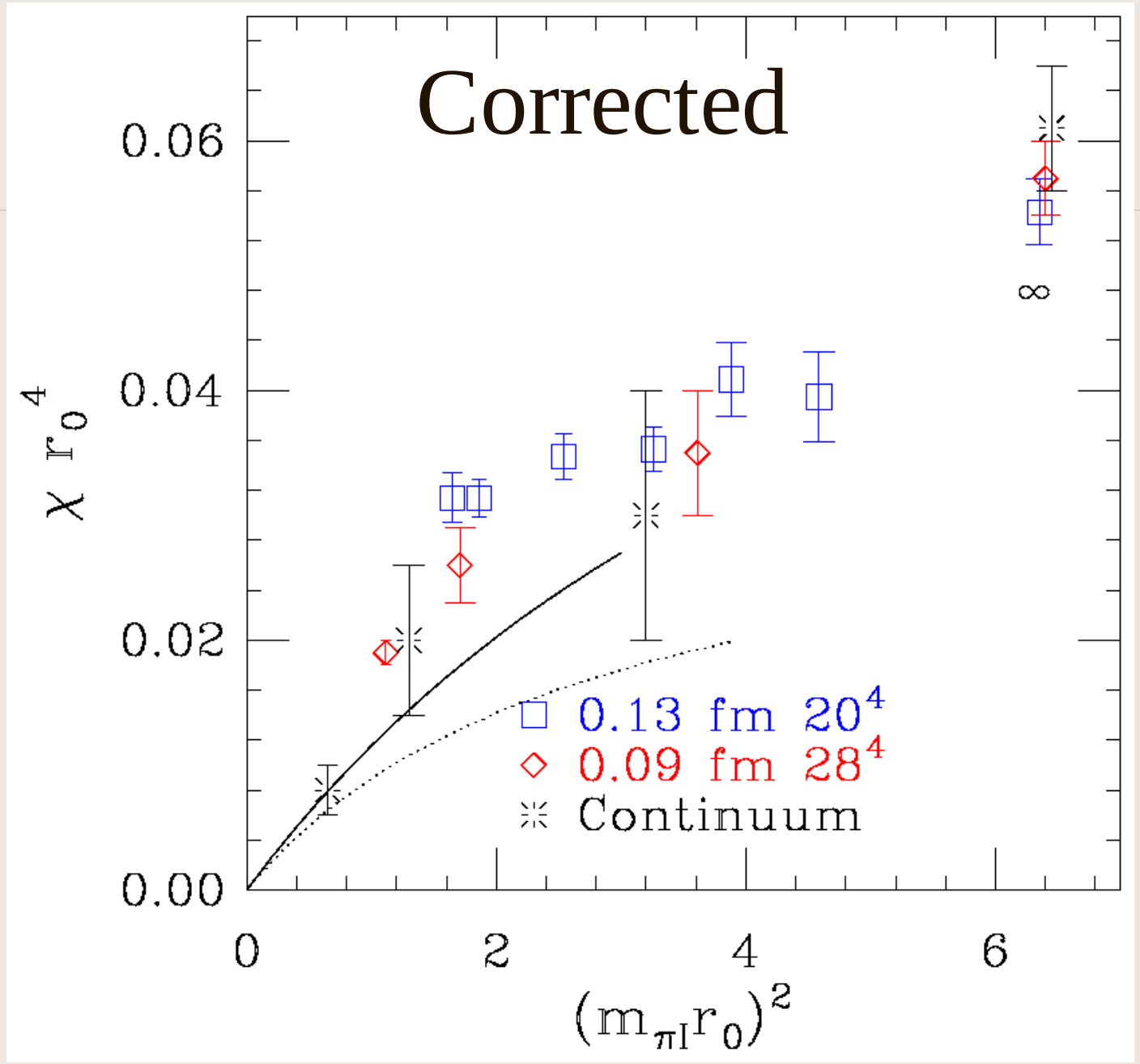

0.22
0.20
0.18
0.16
0.14
$(f_\pi\ r_1)/\sqrt{2}$
CL = 0.28; 0.28
coarse
fine
full, a^2 extrap
m_q=0.6m_s
m_q=0.4m_s
m_q=0.2m_s
m_q=0.14m_s
m_q=0.1m_s
m_q=0.4m_s
m_q=0.2m_s
extrap; systematic err
experiment
full,cont.,m_s^{nom}
full,cont.,m_s^{phys}
0.00
0.05
0.10
0.15
$(m_x+m_y)r_1 \times (Z_m/Z_m^{fine})$

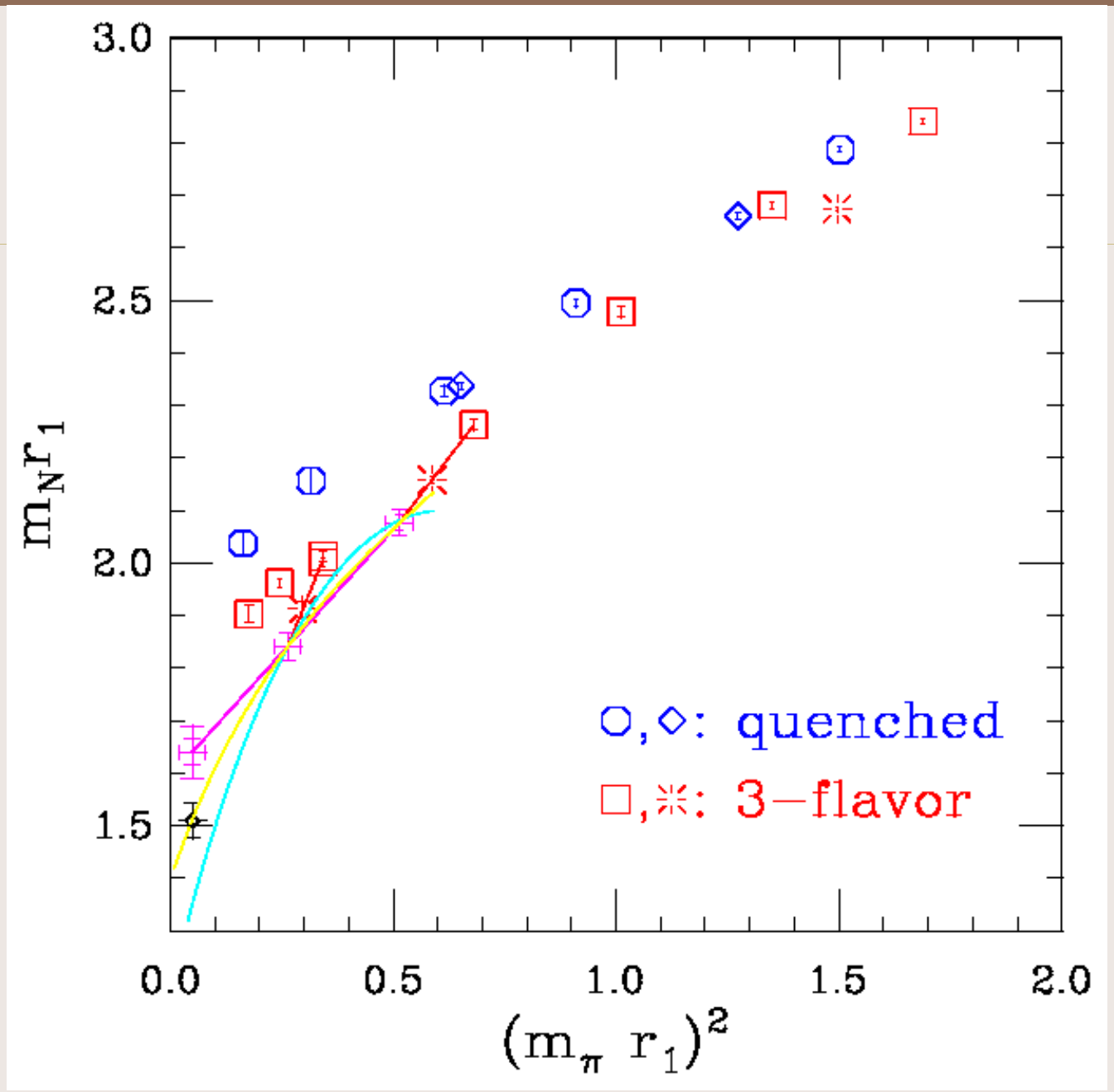
Topological Susceptibility

- Staggered chiral perturbation theory

$$\chi = \frac{f^2 m_{\pi I}^2 / 8}{1 + m_{\pi I}^2 / 2 m_{ssI}^2}$$

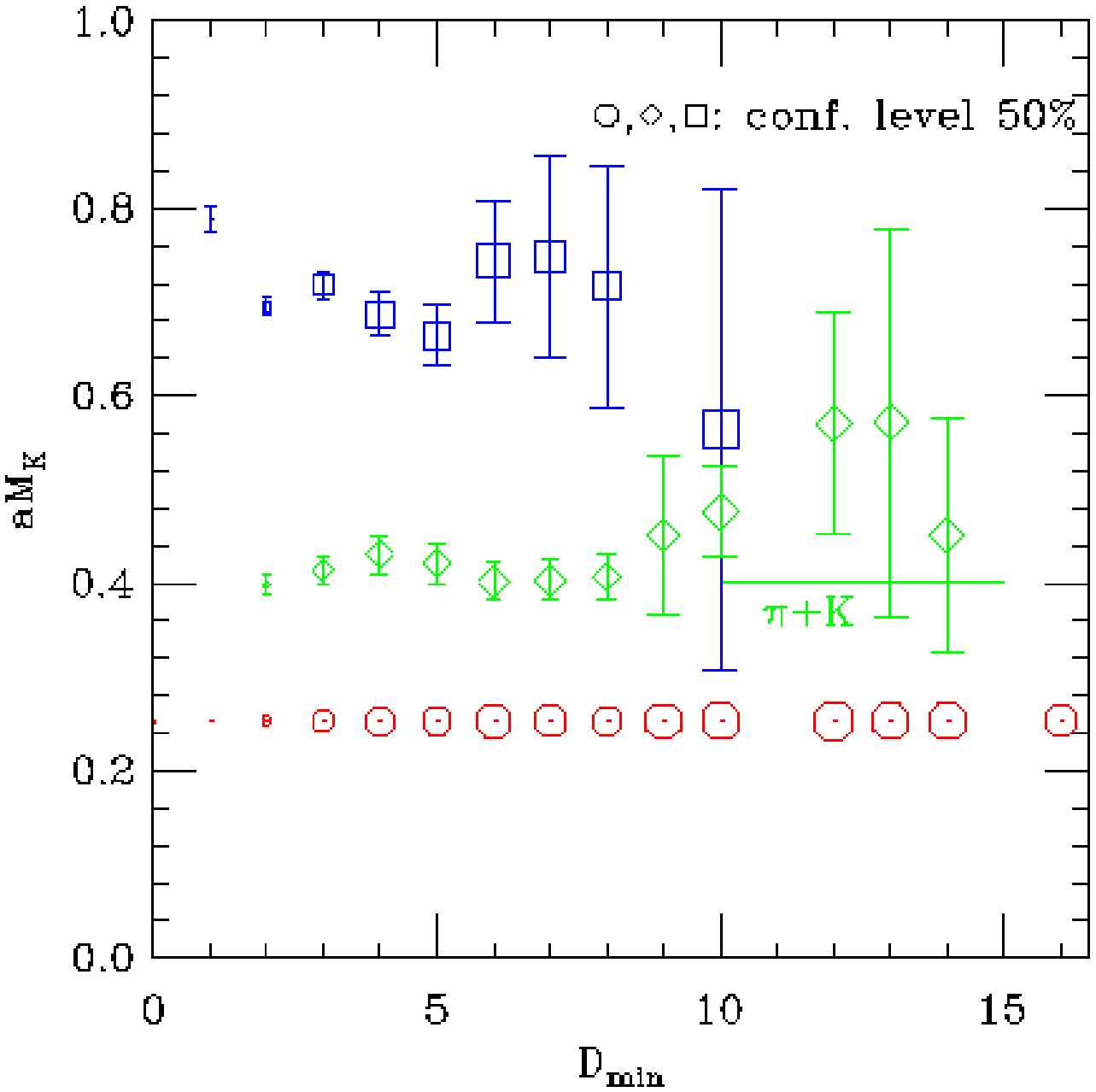

Uncorrected
0.06
0.04
0.02
0.00
χ r0 4
0
2
4
6
(mπr0)2
∞
0.13 fm 20^4
0.09 fm 28^4
Continuum

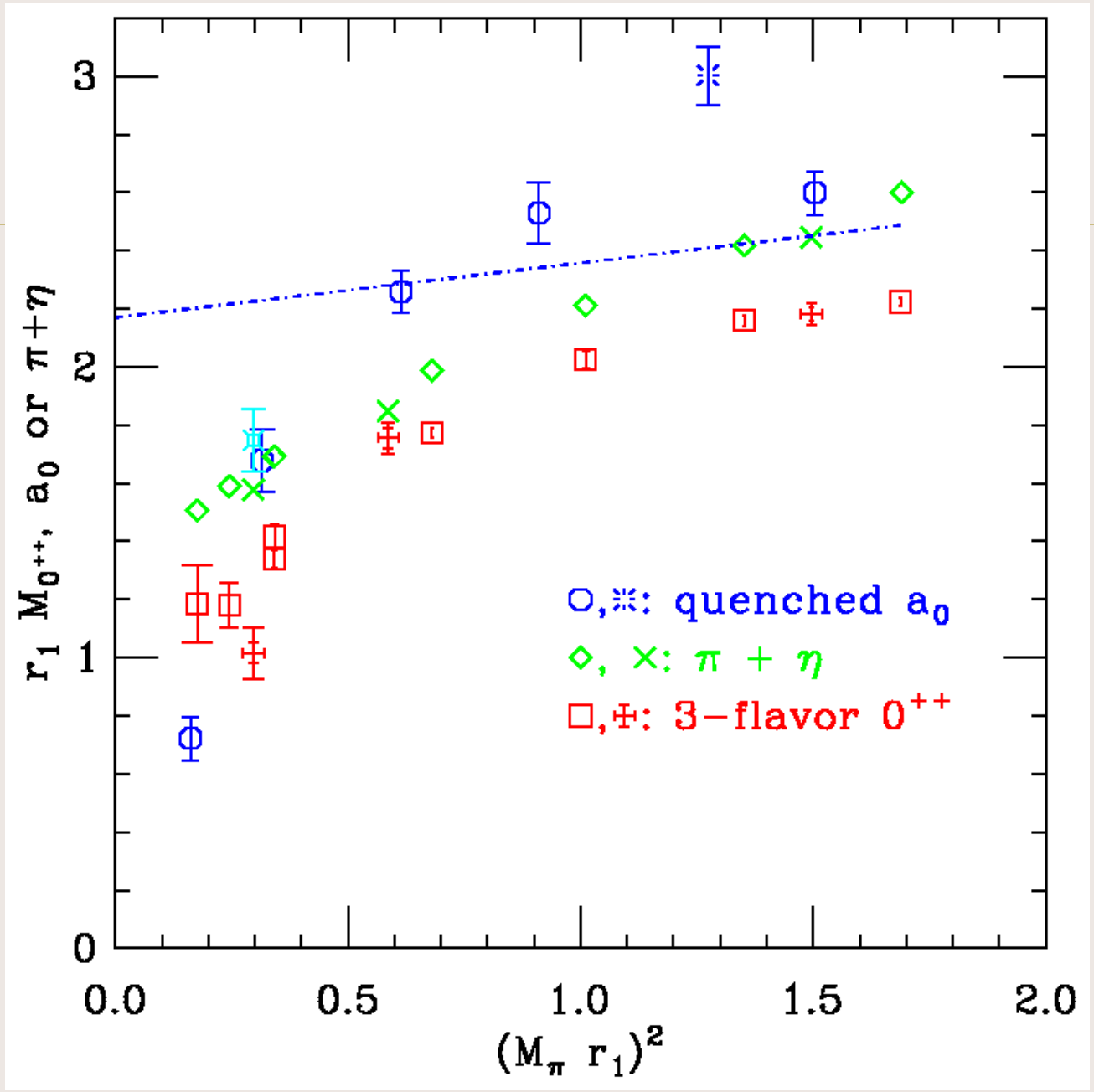

Corrected
0.06
0.04
0.02
0.00
$\chi\ r_0^4$
0
2
4
6
$(m_{\pi l} r_0)^2$
∞
0.13 fm 20^4
0.09 fm 28^4
Continuum

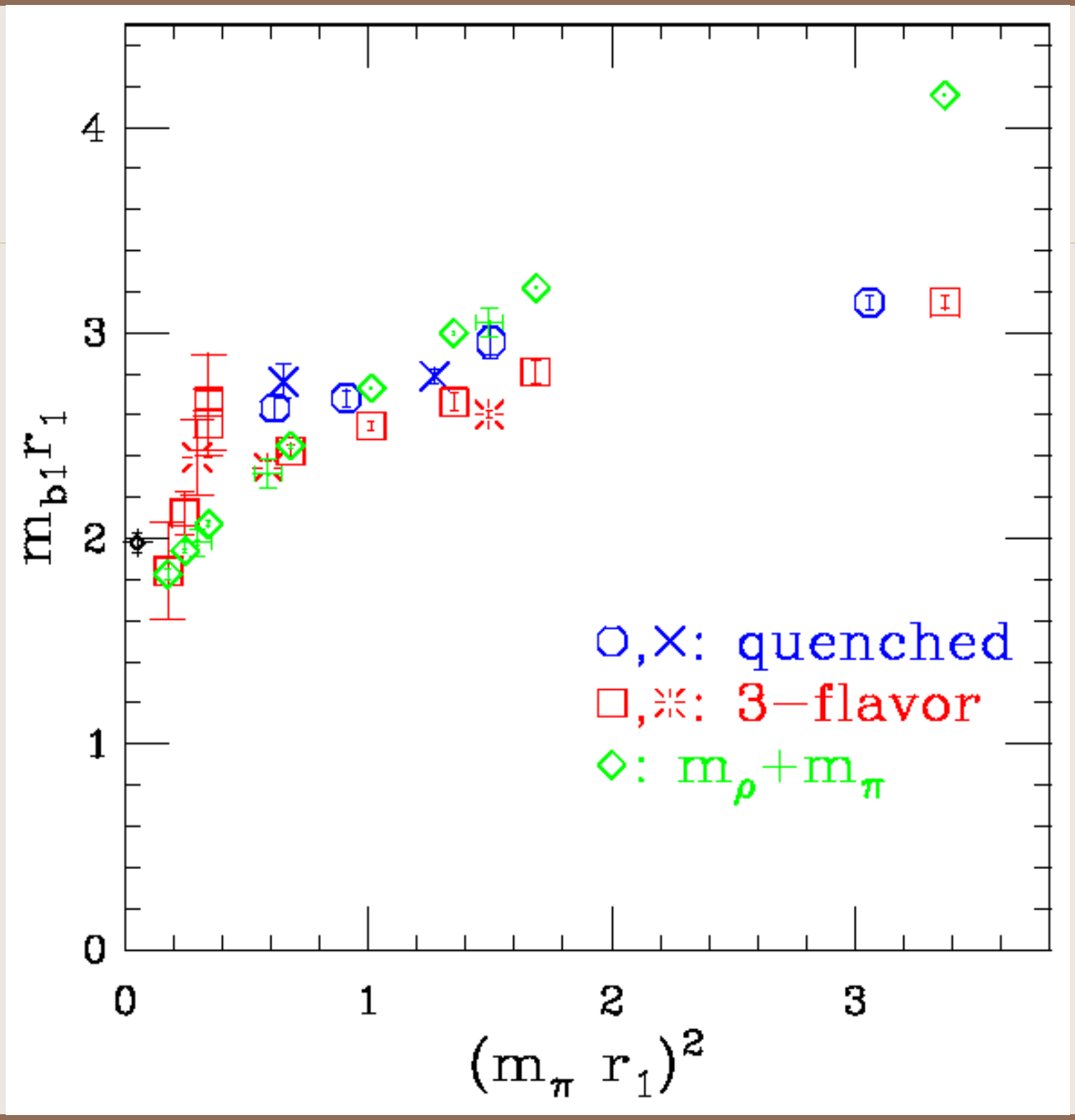

3.0
2.5
2.0
1.5
$m_N r_1$
0.0
0.5
1.0
1.5
2.0
$(m_\pi\ r_1)^2$
○,◇: quenched
□,✳: 3−flavor

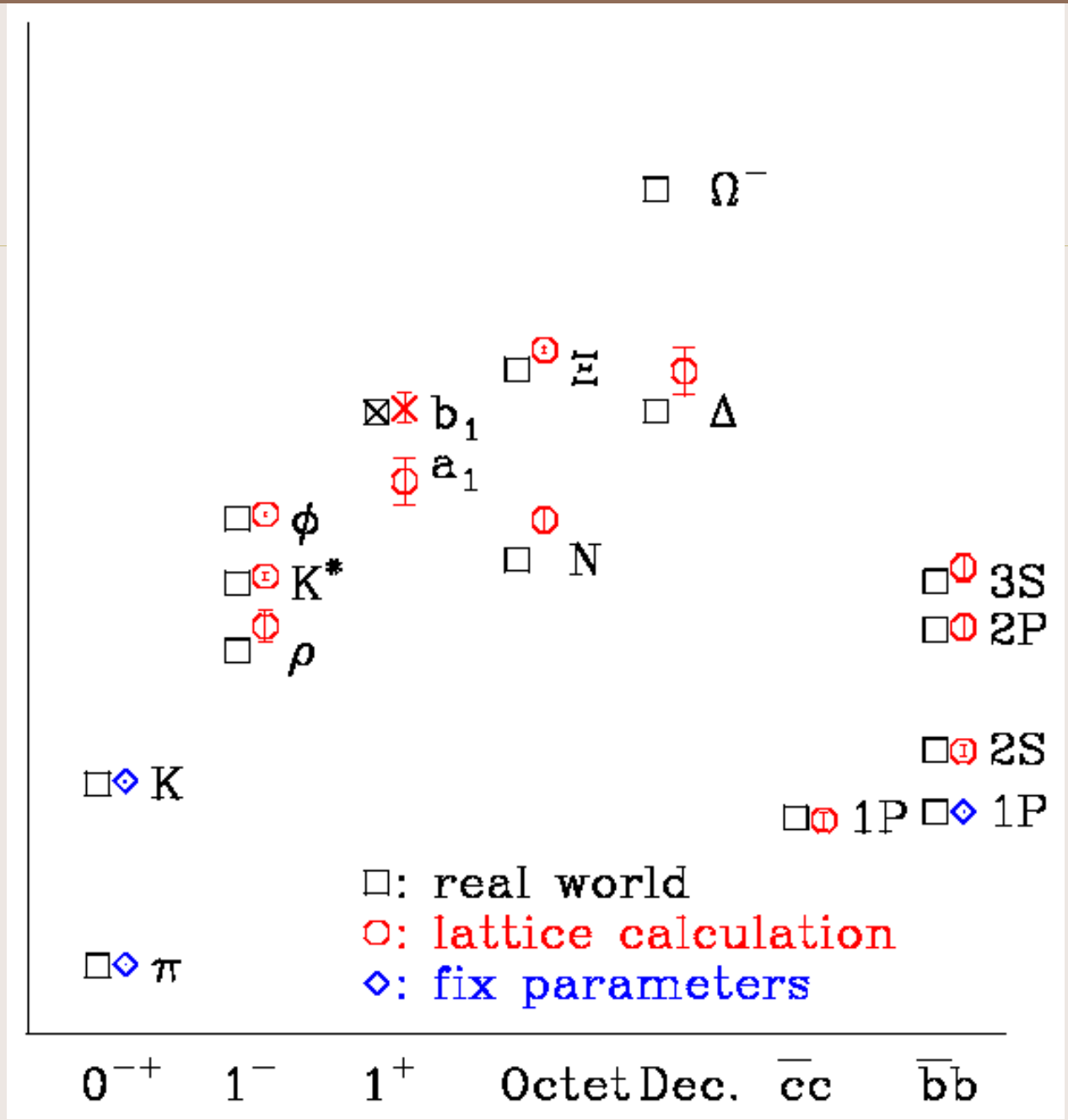
Excited States and Decays

- K
- a_0
- b_1
- Overview


○,◇,□: conf. level 50%
1.0
0.8
0.6
0.4
0.2
0.0
aM_K
0
5
10
15
D_min
π+K

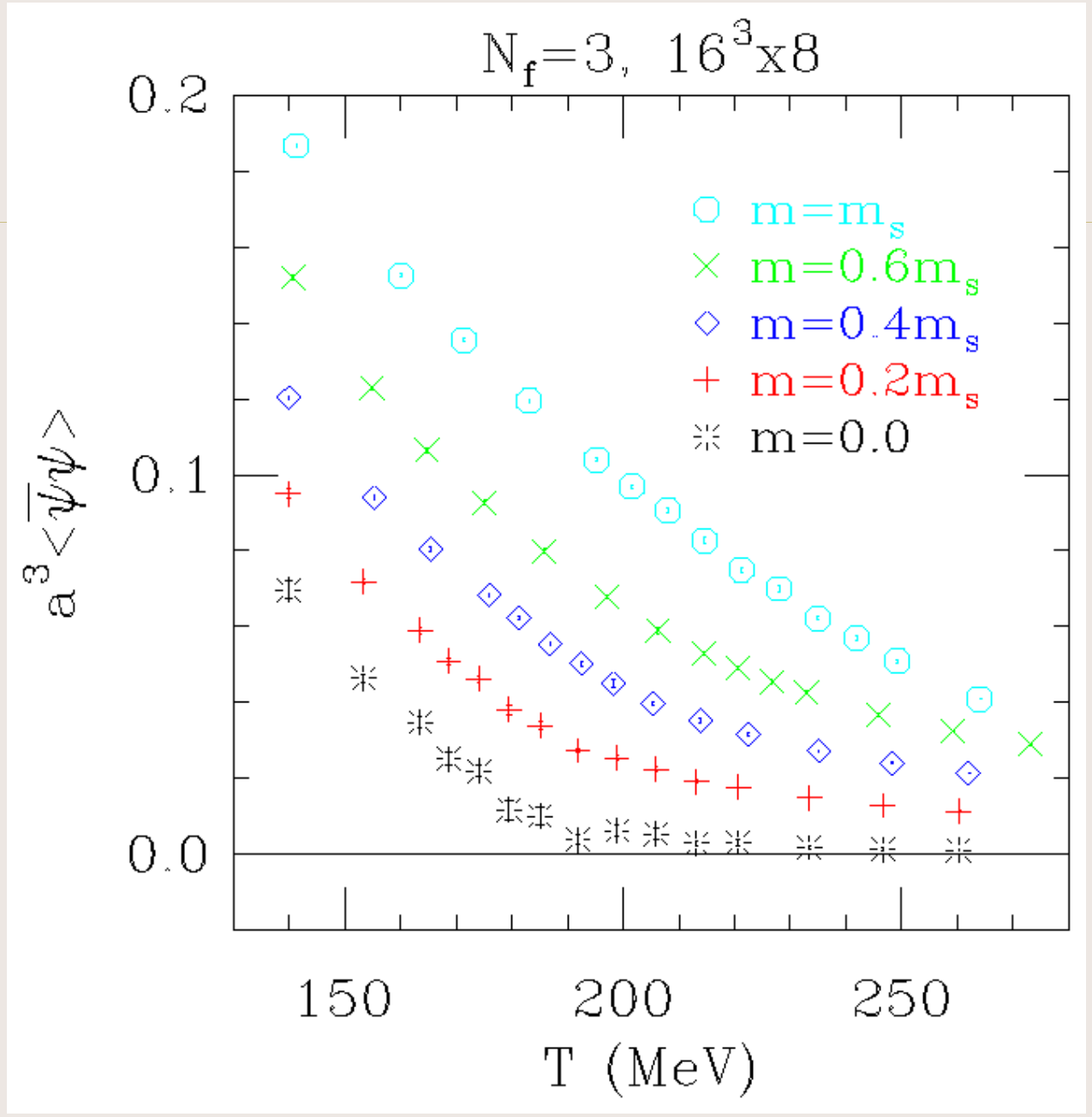

3
2
1
0
0.0
0.5
1.0
1.5
2.0
$r_1 M_{0^{++}}$, a_0 or $\pi+\eta$
$(M_\pi r_1)^2$
○,※: quenched a_0
◇, ×: $\pi + \eta$
□,✠: 3−flavor 0^{++}

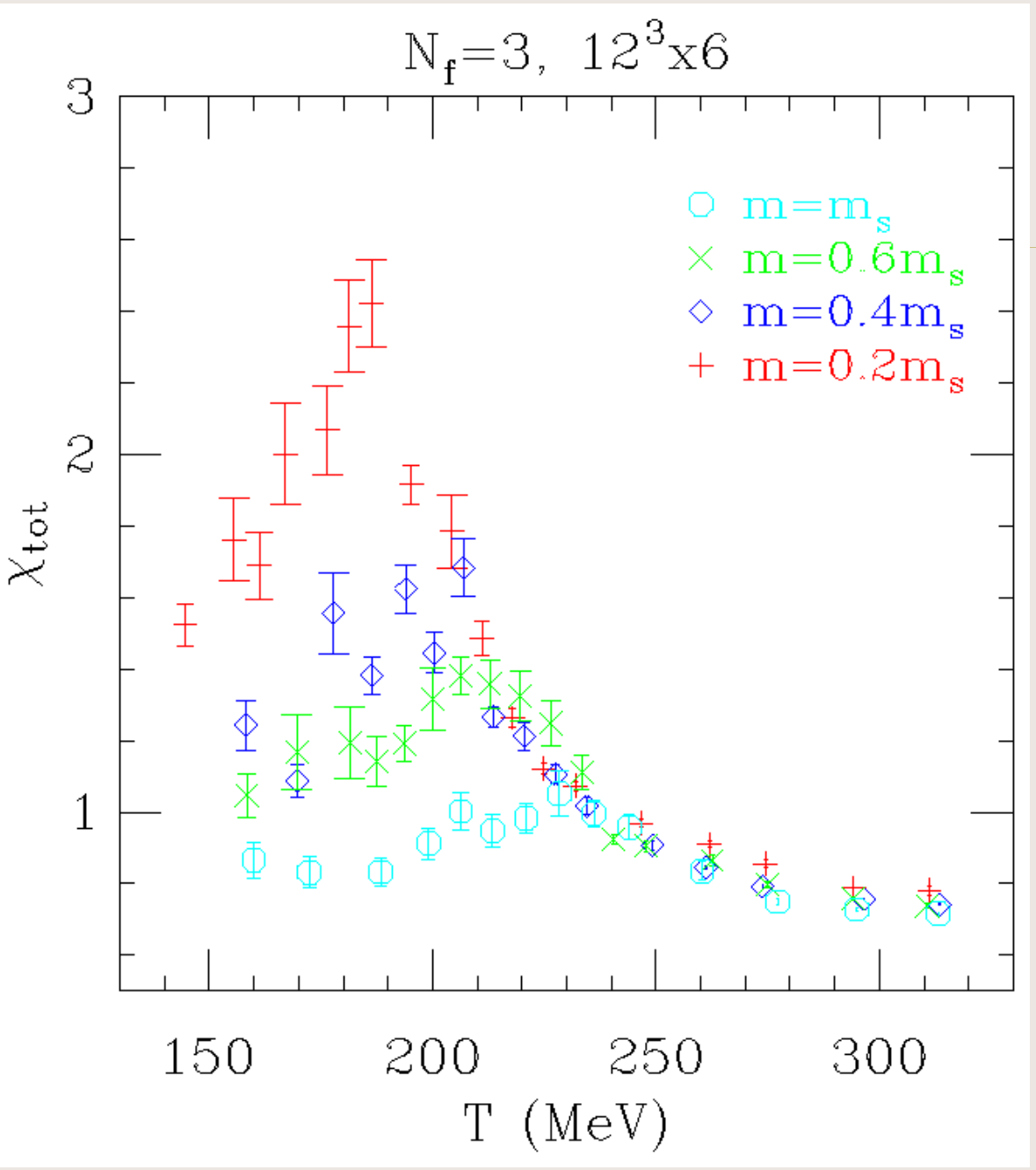

$m_{b1} r_1$
$(m_\pi\ r_1)^2$
0
1
2
3
4
○,×: quenched
□,※: 3−flavor
◇: $m_\rho + m_\pi$

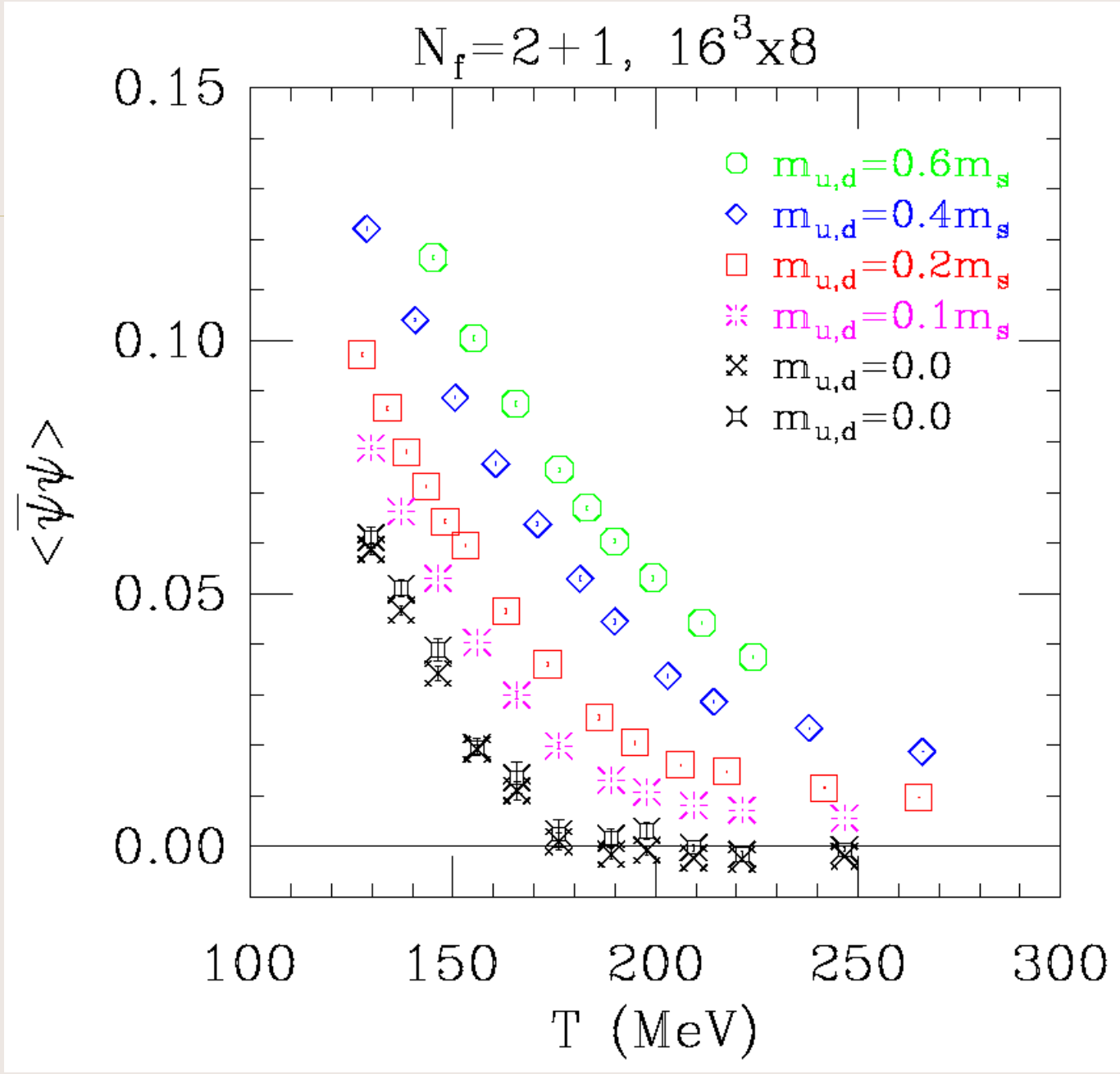

Ω⁻
Ξ
Δ
b1
a1
φ
N
K*
ρ
3S
2P
2S
K
1P
1P
π
□: real world
○: lattice calculation
◇: fix parameters
0⁻⁺
1⁻
1⁺
Octet Dec.
cc̄
bb̄

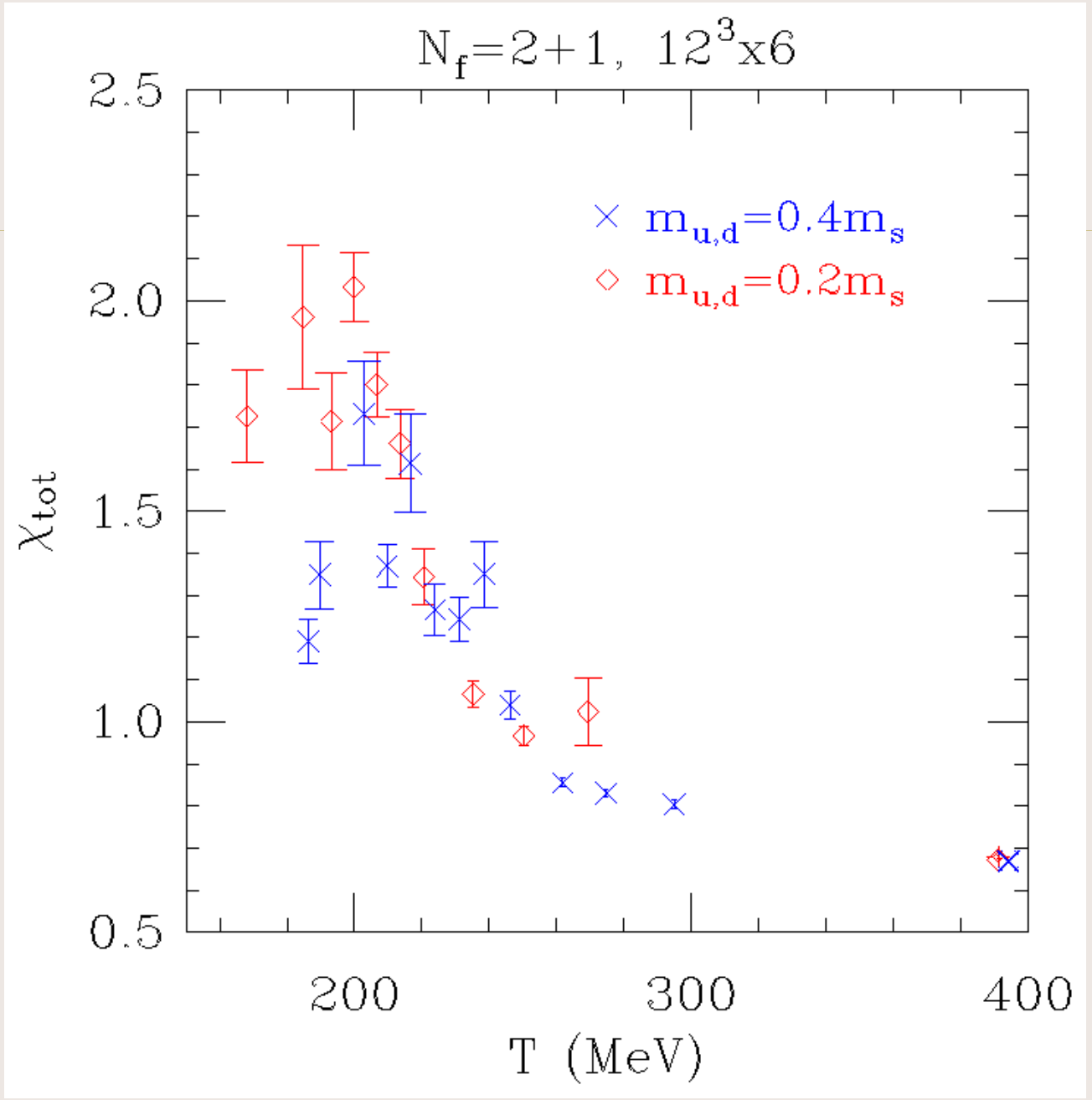
Thermodynamics

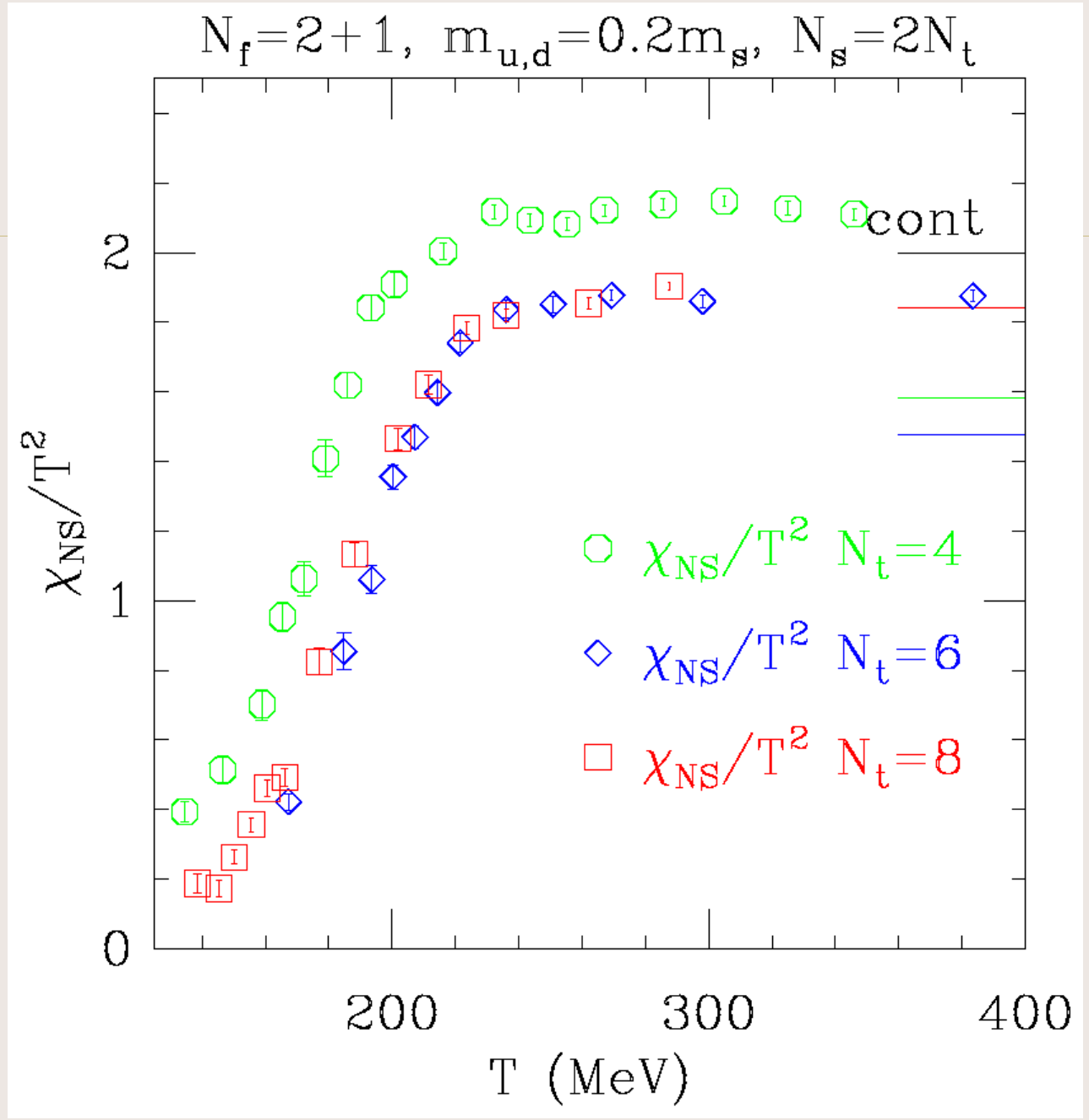
- $\langle \bar{\psi}\psi \rangle$ *vs.* T
- χ_{tot} *vs.* T
- Quark number susceptibility
- Strangeness susceptibility

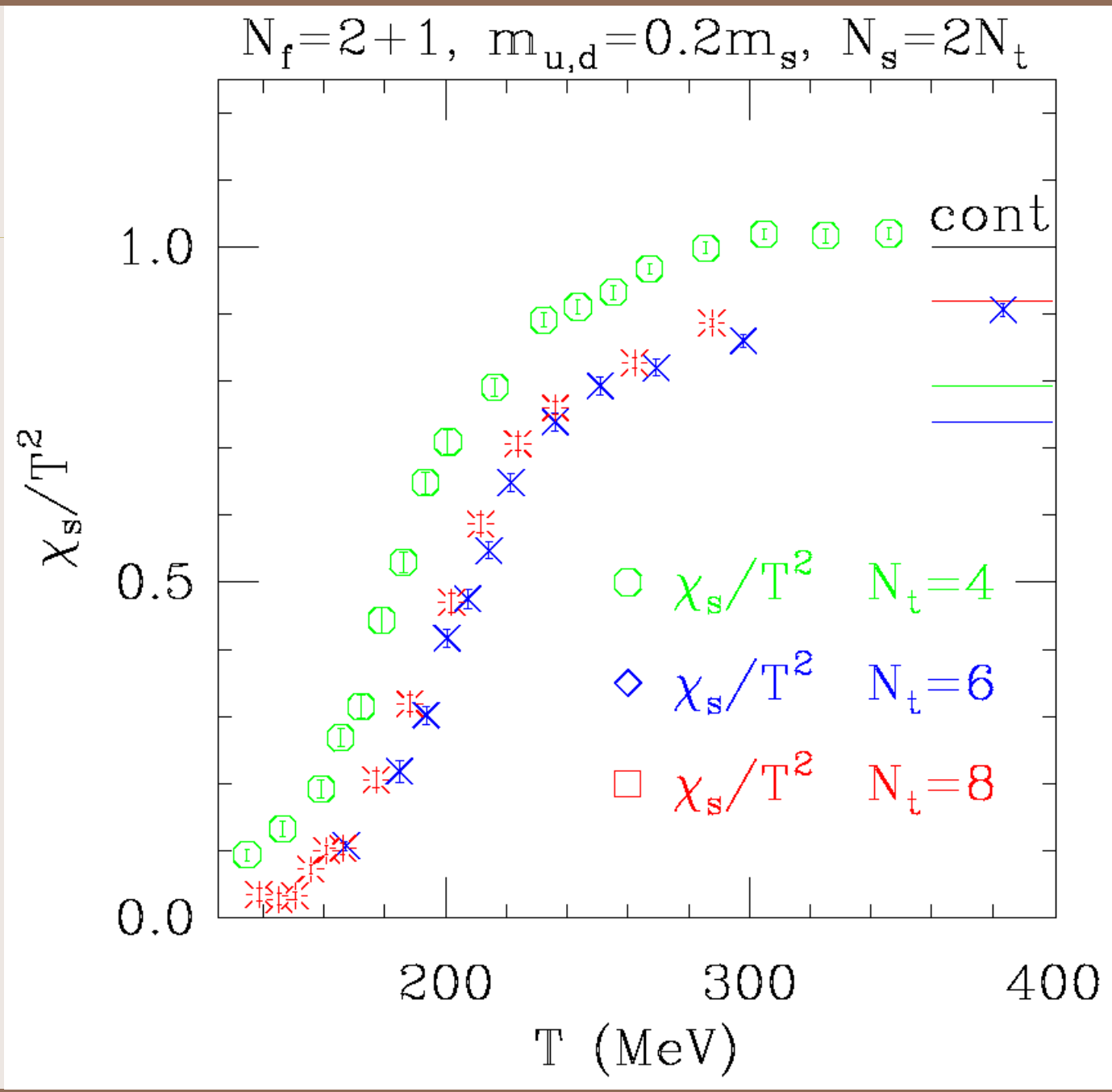

$N_f=3,\ 16^3x8$
0.2
0.1
0.0
$a^3<\bar{\psi}\psi>$
m=m_s
m=0.6m_s
m=0.4m_s
m=0.2m_s
m=0.0
150
200
250
T (MeV)


$N_f=3,\ 12^3x6$
χ_{tot}
T (MeV)
3
2
1
150
200
250
300
$m=m_s$
$m=0.6m_s$
$m=0.4m_s$
$m=0.2m_s$


$N_f=2+1,\ 16^3x8$
0.15
0.10
0.05
0.00
$\langle\bar{\psi}\psi\rangle$
$m_{u,d}=0.6m_s$
$m_{u,d}=0.4m_s$
$m_{u,d}=0.2m_s$
$m_{u,d}=0.1m_s$
$m_{u,d}=0.0$
$m_{u,d}=0.0$
100
150
200
250
300
T (MeV)


N_f=2+1, 12^3x6
2.5
2.0
1.5
1.0
0.5
χ_{tot}
200
300
400
T (MeV)
$m_{u,d}$=0.4m_s
$m_{u,d}$=0.2m_s


$N_f=2+1,\ m_{u,d}=0.2m_s,\ N_s=2N_t$
cont
χ_{NS}/T^2 $N_t=4$
χ_{NS}/T^2 $N_t=6$
χ_{NS}/T^2 $N_t=8$
χ_{NS}/T^2
0
1
2
200
300
400
T (MeV)


$N_f=2+1,\ m_{u,d}=0.2m_s,\ N_s=2N_t$
cont
χ_s/T^2 $N_t=4$
χ_s/T^2 $N_t=6$
χ_s/T^2 $N_t=8$
χ_s/T^2
1.0
0.5
0.0
200
300
400
T (MeV)

Future

- D, B decays (leptonic, semileptonic)
- η' correlator (tests of $\det^{1/4}$)
- Quark plasma Equation of State
- Hybrid exotics and decays