Recent Results from the RBC collaboration

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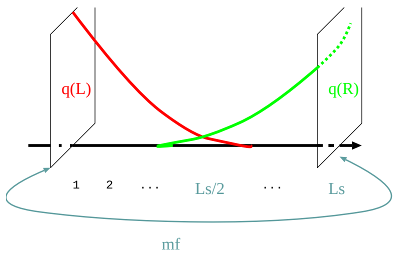
[RBC Collaboration]

Domain Wall Fermions



- Lattice fermions traditionally break either flavour or chiral symmetry.

- Domain Wall Fermions preserve flavour symmetry and have greatly reduced chiral symmetry breaking.

  - at the expense adding a extra, fifth, dimension.

- The nearest neighbour derivative in the 5th dimension distinguishes left- and right- handed fermions

$$-\gamma_\mu \frac{1}{2}\left(\nabla_\mu^{+} + \nabla_\mu^{-}\right) + \frac{1}{2}\nabla_\mu^{-}\nabla_\mu^{+} + M_5 \qquad \text{4d piece}$$

$$+P_L \partial_5^{+} - P_R \partial_5^{-} \qquad \text{5d piece}$$

Domain Wall Fermions

- Define 4d quark fields on the wall

$$q_x = P_L \Psi_{x,0} + P_R \Psi_{x,L_s-1}$$

- Couple the two walls with a mass term

$$m_f \bar{q} q$$

- For finite L_s chiral symmetry is broken, leading to an additive renormalisation of the mass

$$m_f \to m_f + m_{\text{res}}$$

- m_{res} measures the order of the suppression due to one "trip" through the bulk.

Advantages of Domain Wall Fermions

- Both flavour and chiral (almost) symmetry preserved at finite lattice spacing : continuum and chiral limits decoupled.
  - (hopefully) good scaling.
  - simple renormalisation : no mixing with wrong flavour operators and greatly reduced mixing with wrong chirality operators.
    - Need to know how small is "small" : Later we will discuss this for the extraction of B_K .
  - simple, continuum like, χpt fitting forms.
- Positive determinant for positive mass m_f (even at finite L_s) [Furman and Shamir, 94]
  - No conceptual problem with simulating odd numbers of flavours.

Cost of Domain Wall Fermions....

For DWF to be practical we need $L_s \approx 10$, to give a values of m_{res} that are "small enough" (a few MeV or below).

- In the quenched approximation Domain Wall Fermions we have seen that the size of the residual chiral symmetry is breaking highly dependent on how the gauge action is discretised:
  - Wilson $m_{res} \sim 3\text{MeV}$
  - Iwasaki (rg-improved) $m_{res} \sim .3\text{MeV}$
  - DBW2 (non-perturbative Iwasaki) $m_{res} \sim .03\text{MeV}$

  for $a^{-1} \approx 2\text{GeV}$, and $L_s = 16$.

- Previous dynamical work (Columbia) was at very large lattice spacing and needed an impractically large fifth dimension to show good chirality.

Details

- Here I will report on a preliminary study of $N_f = 2$, Dynamical Domain Wall Fermions
  - using improved gauge actions (DBW2)
  - on large lattices ($16^3 \times 32$)
  - at weak coupling ($a^{-1} \approx 2\text{GeV}$)
  - using a practical size of the fifth dimension ($L_2 = 12$).

- As well as improving the action we have worked hard to implement several improvements to the standard dynamical algorithms:
  1. Improved fermion force term
  2. Chronological inverter
  3. Multiple gauge-step leapfrog.

  that give a factor of ~ 3 speed-up.

Run Details

- We are using
  - DBW2 gauge action with $\beta = 0.80$ (Educated guess + hard work : aiming for $a^{-1} \approx$ 2GeV).
  - Bare dynamical masses of $m_f = 0.02$, 0.03 and 0.04 .
- Lattice are generated using the HMC algorithm. Each trajectory is of length 0.5 in HMC time split up into 50 leapfrog integration steps for mass of 0.02 and 0.03 , and 40 integration steps for 0.04 .
- Total number of trajectories collected so far:
  - $am_f = 0.02$ - 5361 trajectories (8 months on 200GF)
  - $am_f = 0.03$ - 6195 trajectories
  - $am_f = 0.04$ - 5605 trajectories

  The acceptance is $\approx 78\%$ for $m_f = 0.02, 0.03$ and 68% for $m_f = 0.04$.
- Unless otherwise stated all the following results are based on 94 configurations, with each configuration being seperated by 50 trajectories

PRELIMINARY results follow. Quoted errors are statistical only.

Residual mass

- Residual chiral symmetry breaking generate "extra" term in WTI.

$$\Delta_\mu A_\mu^a(x) = 2m_f J_5^a(x) + 2J_{5q}^a(x)$$

- We absorb this term into the residual mass , m_{res}

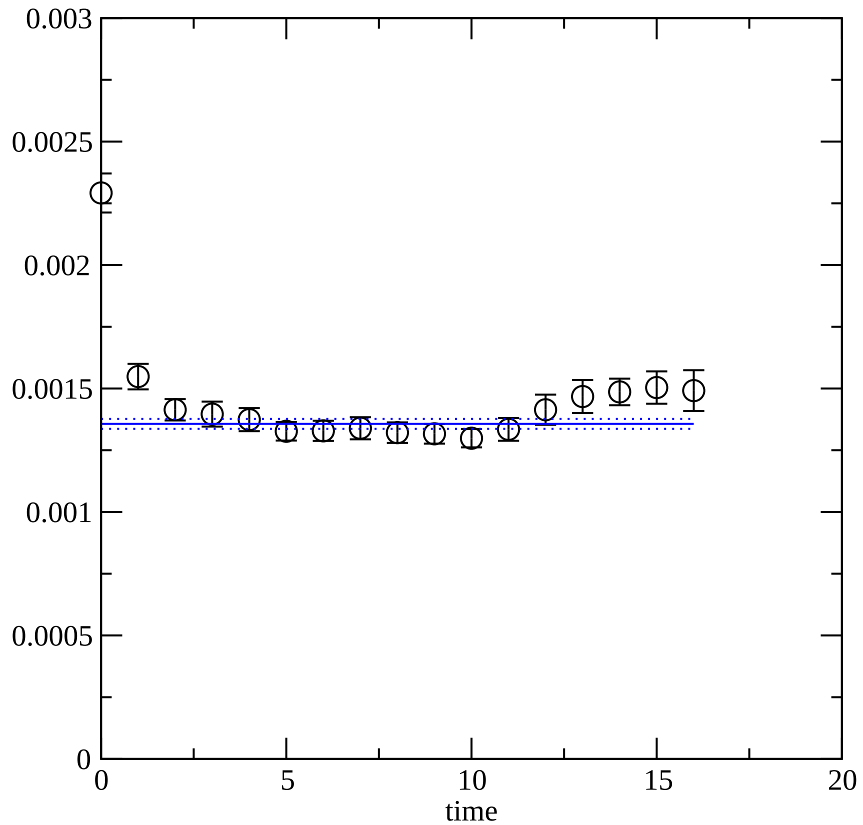
$$J_{5q}^a(x) \approx m_{\text{res}} J_5^a(x)$$

- Compare pion propagation along boundary to propagation to midpoint.

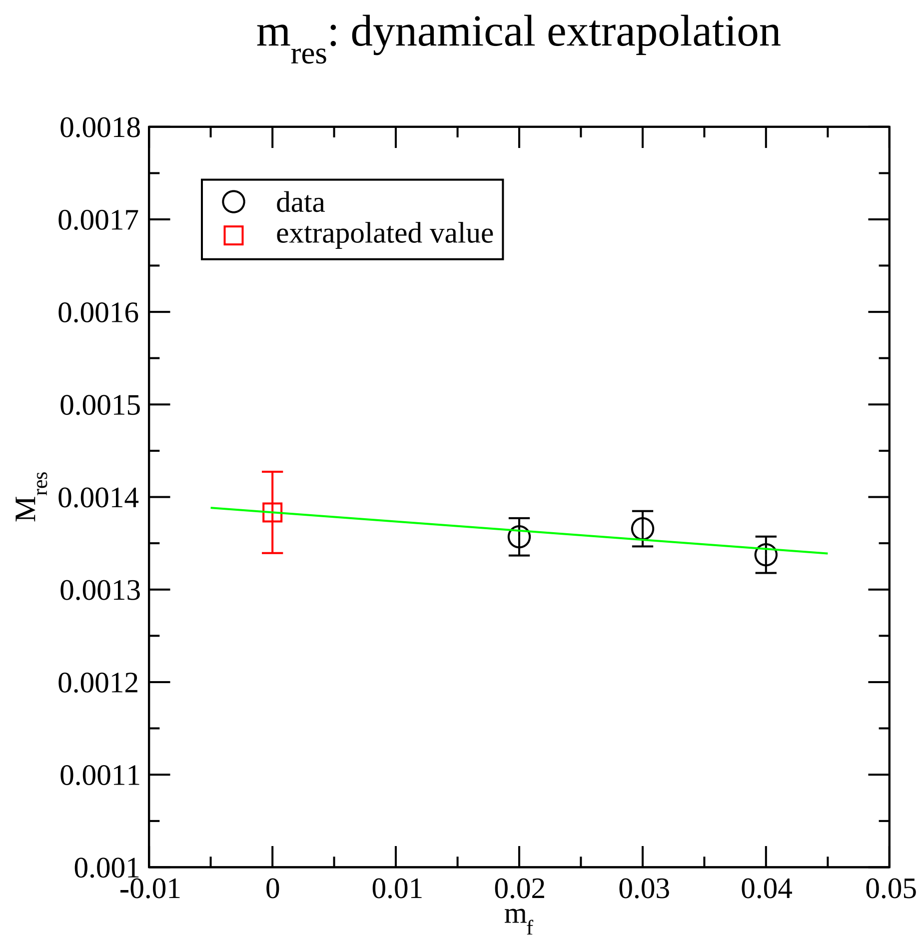
$$m_{\text{res}} = R(t) = \frac{\sum_x \langle J_{5q}(x,t) P(0,0)\rangle}{\sum_x \langle P(x,t) P(0,0)\rangle}$$

This should not be dependent on time seperation of correlators.

m_{res} extraction

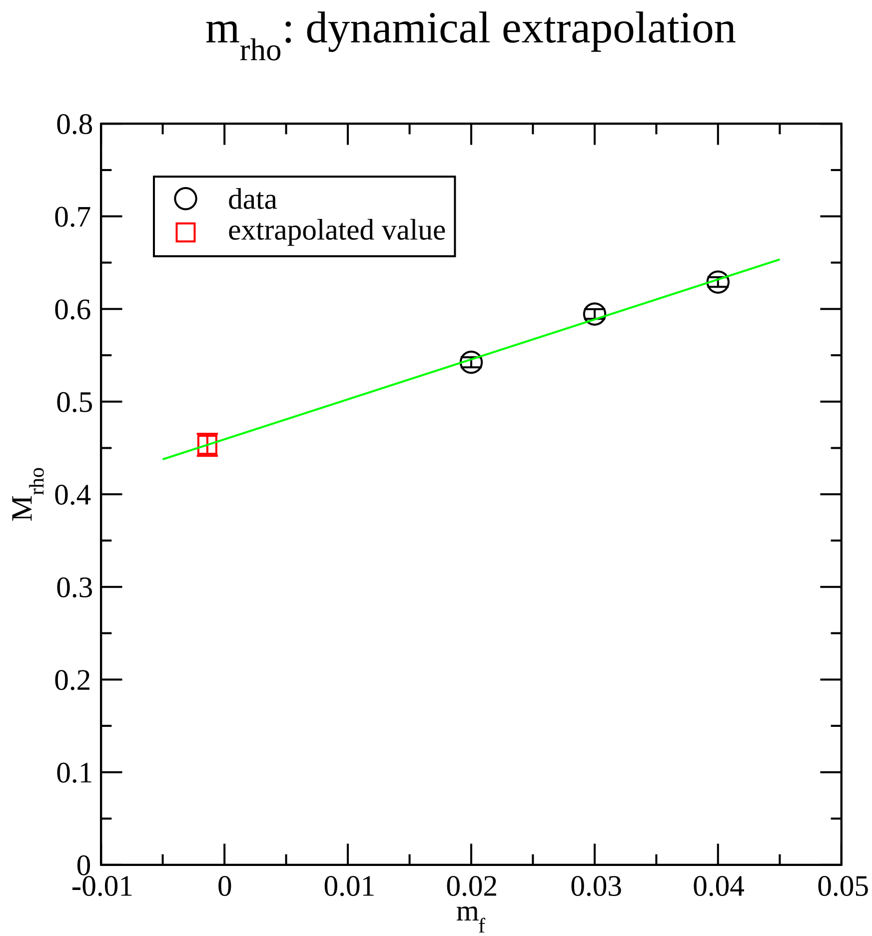


- m_{dyn} =0.02, dynamical point.
- Extract m_{res} by averaging between $t = 6$ and 16 .



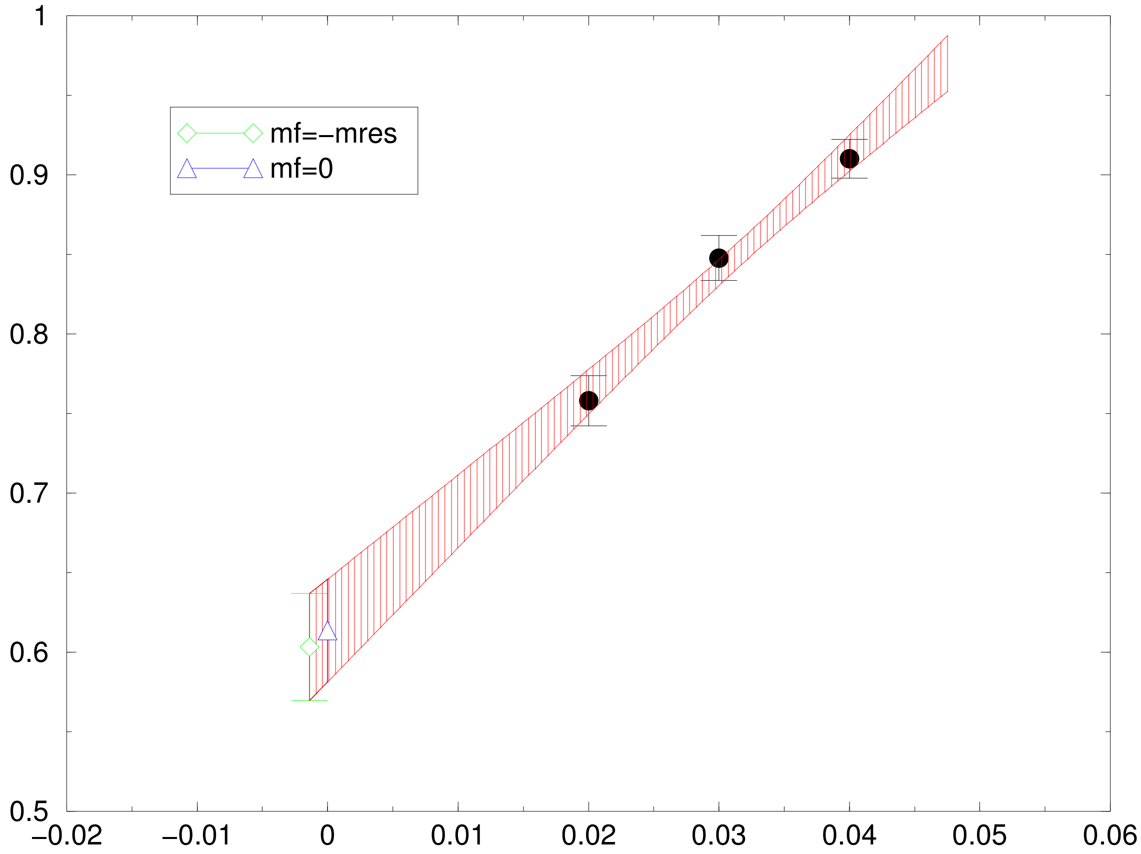
- Dynamical extrapolation gives $am_{\text{res}} = 0.001383(40)$

- $\approx 7\%$ of our smallest dynamical mass, but need to know scale before this can really be put into context.

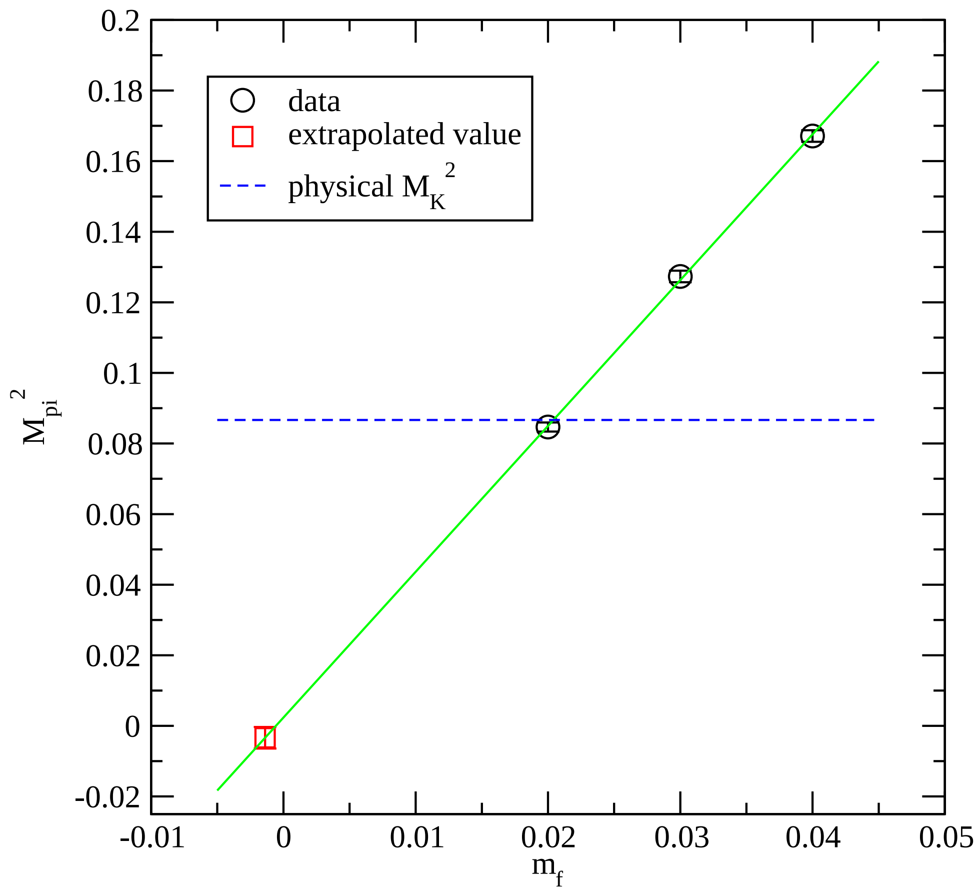


- Dynamical extrapolation gives: $a^{-1} = 1.698(44)\text{GeV}$.
- Residual mass is therefore $\approx$ 2MeV .

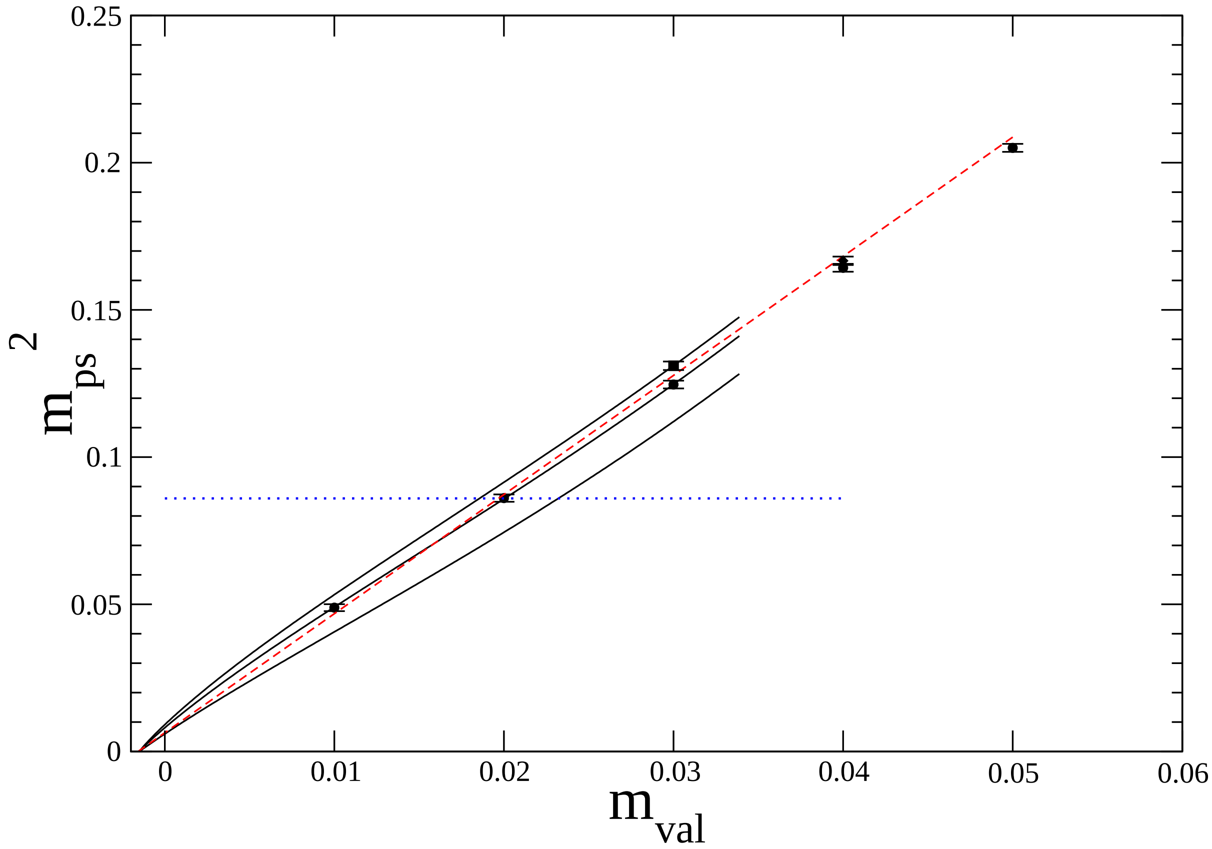
a M_N vs mf a



m_{pi}^2: dynamical extrapolation

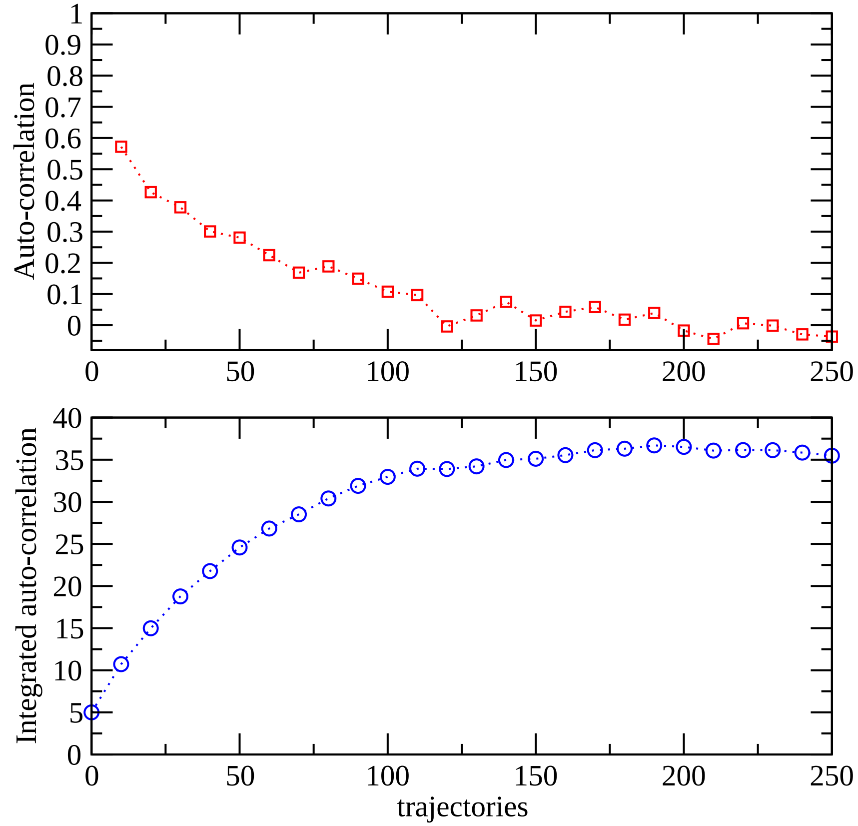


- Lightest dynamical mass $\sim$ half the strange quark mass.
- M_π^2 extrapolates to 0 at $m_f \approx -m_{res}$ (within stat. errors)



- (Preliminary) NLO in chiral perturbation theory fit.

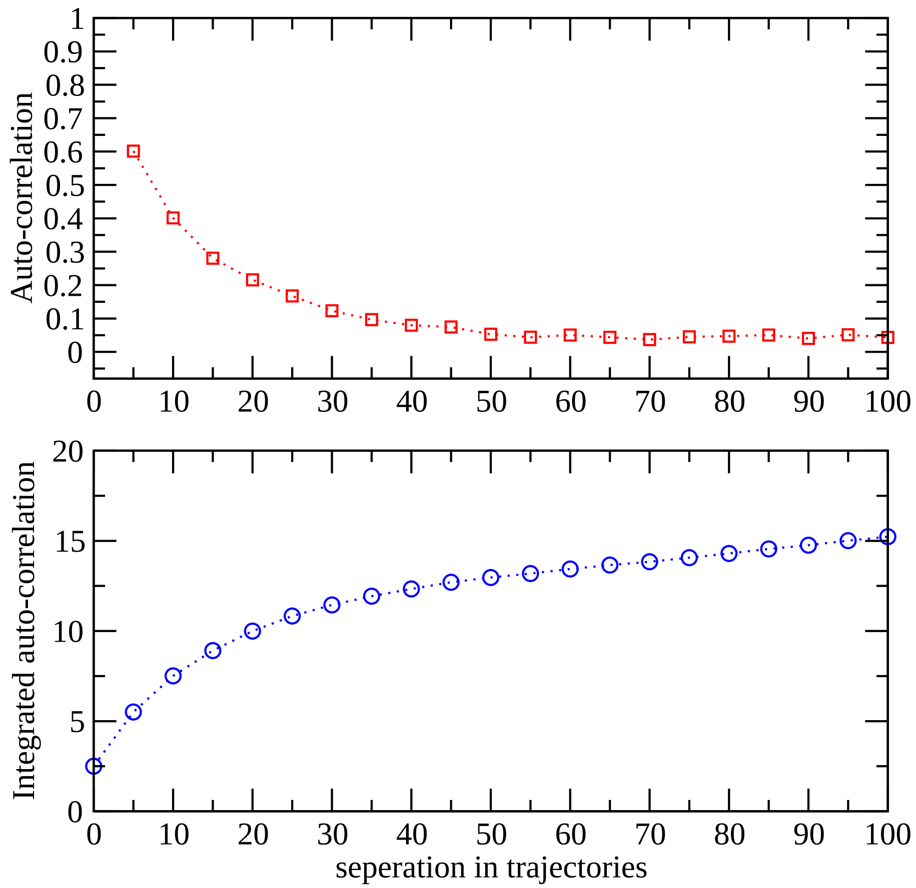
Auto-correlation time; Axial correlator



- We have run one set of spectrum data, measuring every 10 trajectories to try and resolve the auto-correlation length.

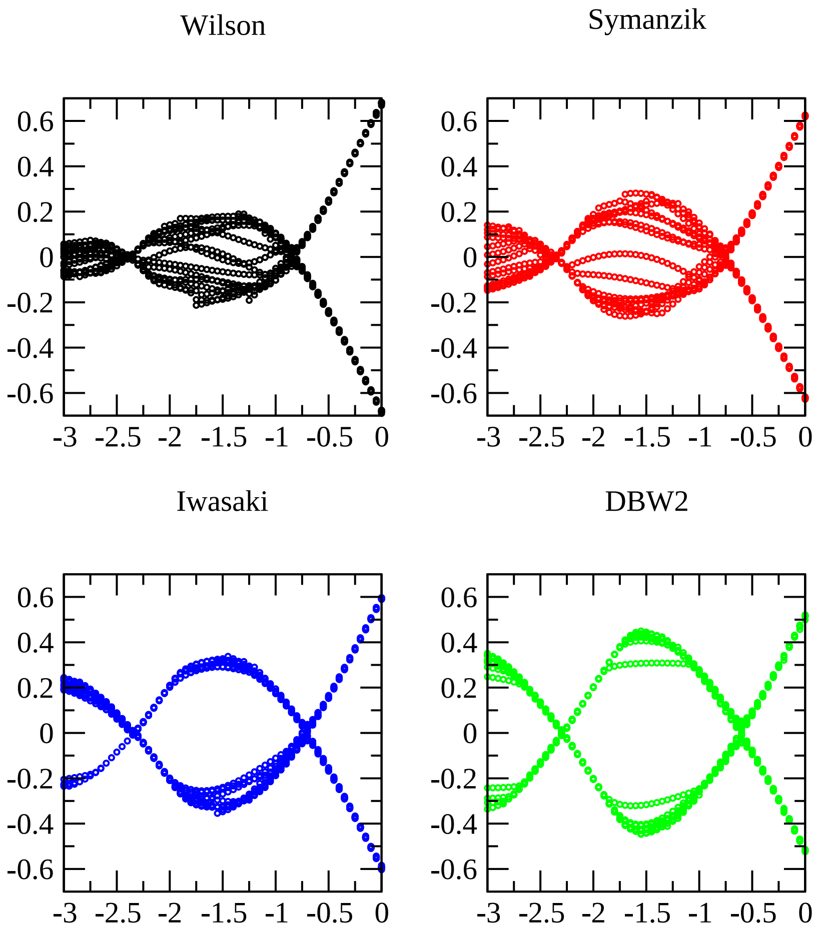
- The above shows the result for the Axial-Axial , box-point correlator at timeslice 12.

Auto-correlation time; plaquette



- Plaquette has same order of magnitude auto-correlation time, but slightly smaller.

Quenched Spectral Flows



A transfer matrix in the 5^{th} dimension can be defined for DWF .

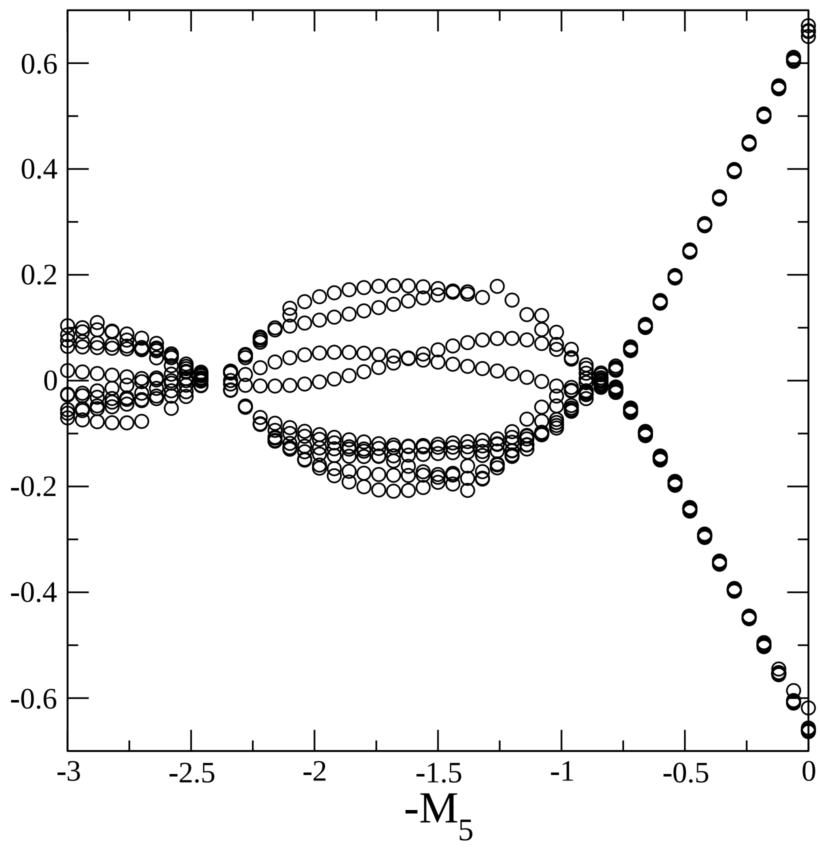
$$T = \frac{1 - H_T}{1 + H_T}$$

$$H_T \approx \gamma_5 D_W$$

Zero eigenvalues of $\gamma_5 D_W$ propagate 5^{th} dimension.

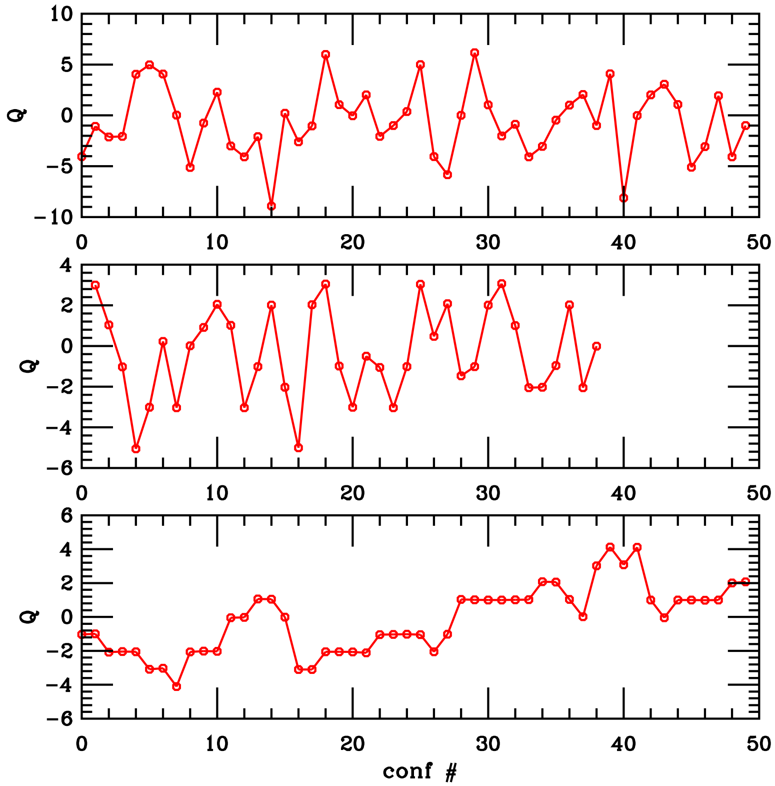
1. x-axis is $-M_5$
2. y-axis is eigenvalue

Example m_{dyn}=0.03 spectral flow



- Spectral flow shows the existance of a gap.

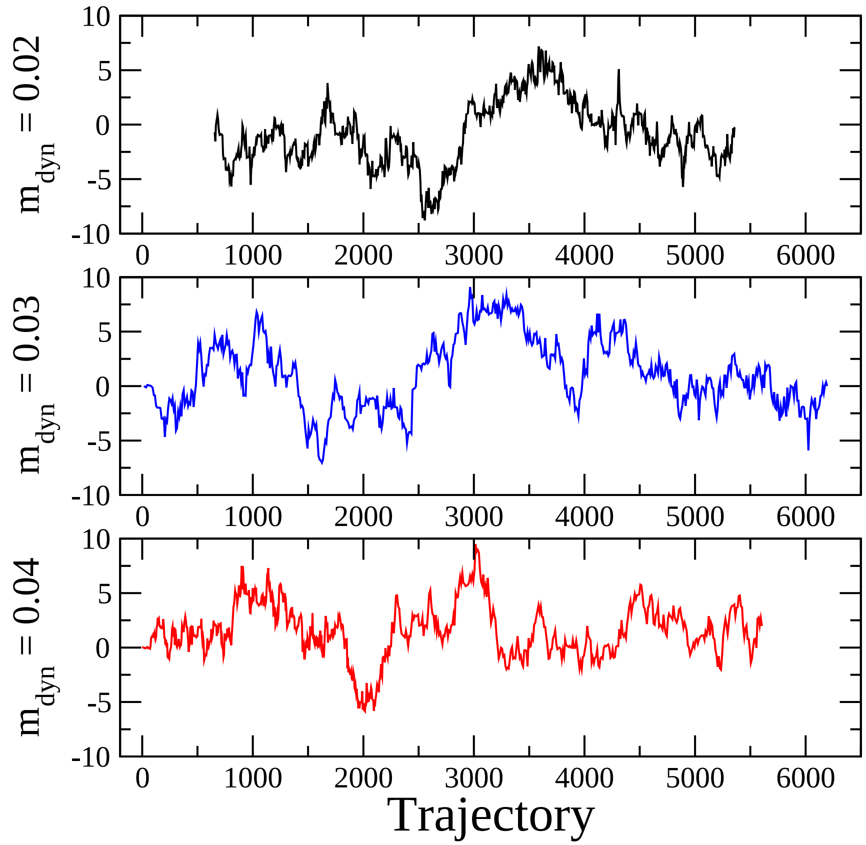
Topological Charge Tunneling (Quenched)



- From the top:
  1. Symanzik
  2. Iwasaki
  3. DBW2

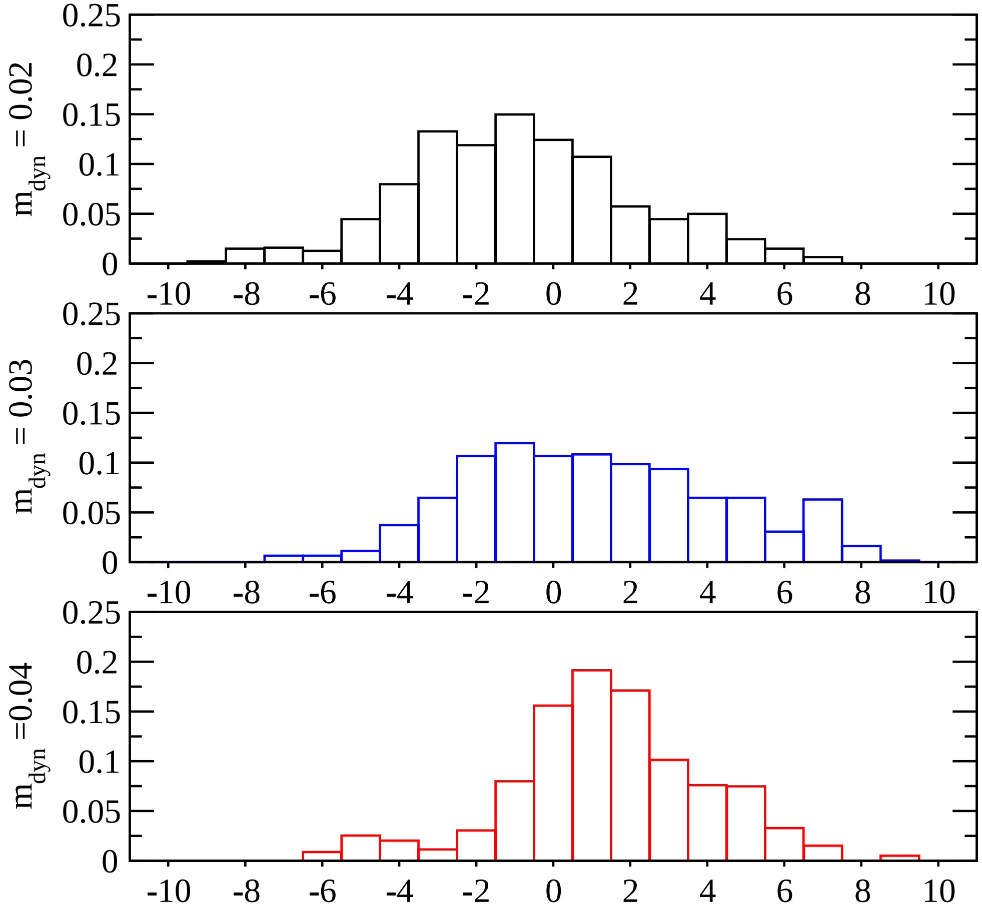
- 1000 heatbath sweeps between configurations, all around $a^{-1} = 2\text{GeV}$.

Topological Charge Tunneling (Dynamical)



- Topological charge calculated using the A.P.E. smearing approximation to cycling [DeGrand et al] and a classically $O(a^4)$ improved definition of the topological charge operator.

Topological Charge Distribution

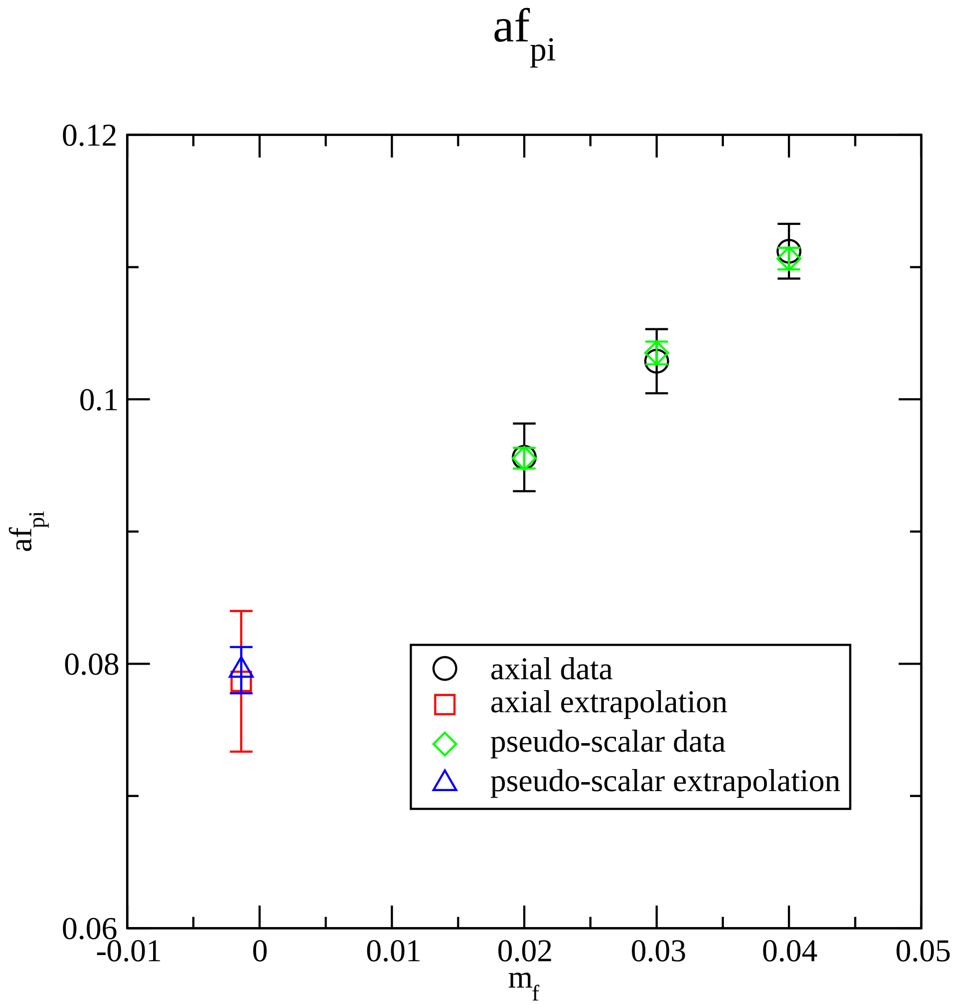


- Distribution of toplogical charge for the different ensembles.

Pseudo-scalar decay constant

Technique

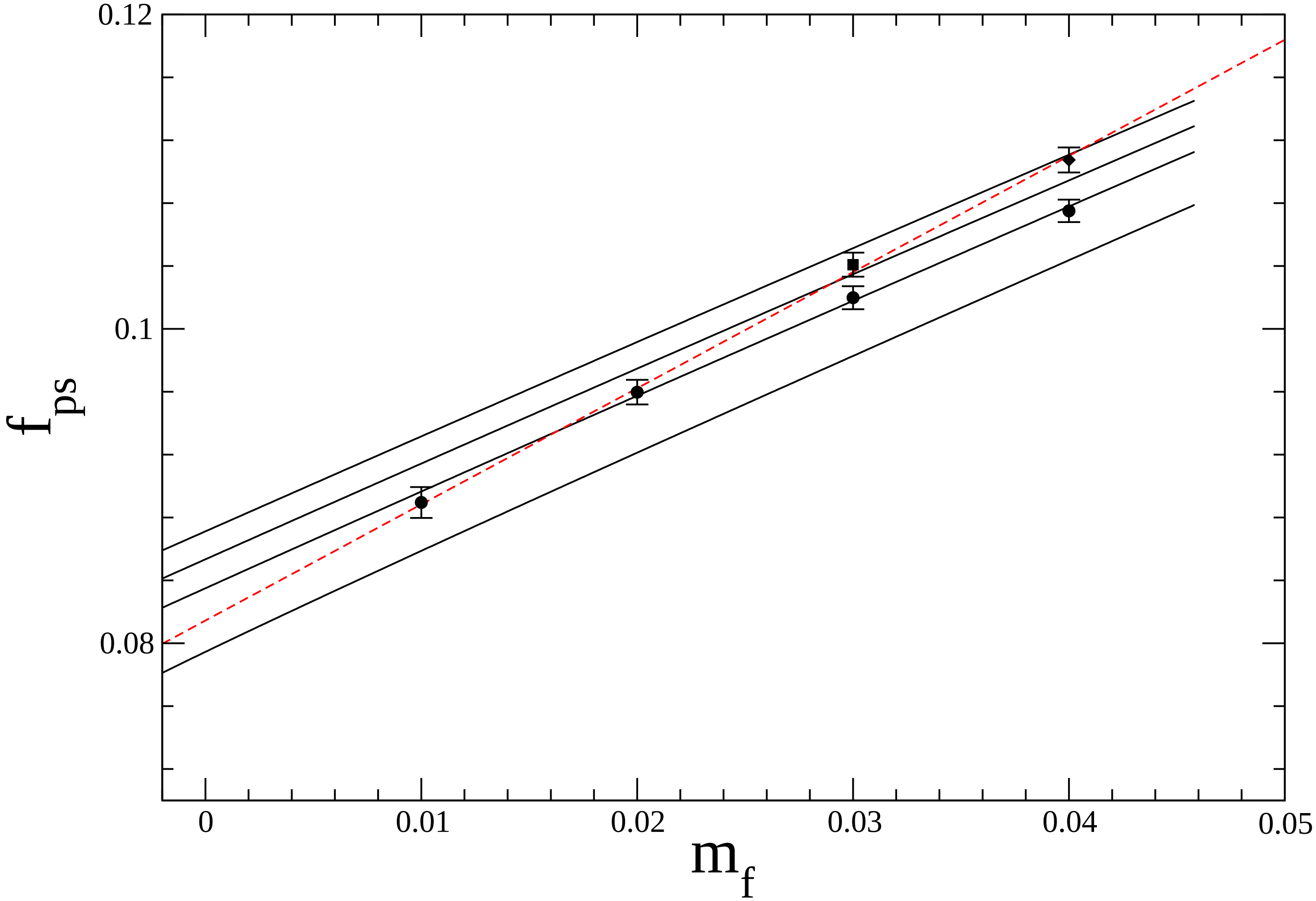
- Can get f_π two different ways:
  1. From the Axial-Axial correlator
$$\frac{f_\pi^2}{Z_A^2}\frac{m_\pi}{2}e^{-m_\pi t} = \langle \int d^3x \; A_0^a(x,t)A_0^a(0,0)\rangle$$
  2. or from the Pseudo-Scalar correlator using the WTI to tell us that at low energies
$$\Delta_\mu A_\mu^a(x) \equiv 2\left(m_f + m_{\text{res}}\right) J_5^a(x)$$
     or
$$-\frac{f_\pi^2}{(m_f + m_{\text{res}})^2}\frac{m_\pi^3}{8}e^{-m_\pi t} = \langle \int d^3x \; J_5^a(x,t)J_5^a(0,0)\rangle$$



- Simple linear fit to pseudo-scalar data gives

$$f_\pi \approx f = 135(5)\ \text{MeV}$$

(M_ρ to set the scale)



- (PRELIMINARY) NLO in chiral perturbation theory fit

PRELIMINARY NLO results (statistical errors only)

$$
\begin{aligned}
f_\pi \approx f &= \text{131(5) MeV} \\
f_K &= \text{156(5) MeV} \\
f_K/f_\pi &= 1.188(18) \\
f_\pi/M_\rho &= 0.170(6)
\end{aligned}
$$

The kaon B parameter B_K (CP violation in the SM)

- The $\Delta S = 2$ operators needed to calculate B_K is in the class of operators

$$O_\Gamma = \overline{s}\Gamma_i^A d \overline{s}\Gamma_i^A d$$

- Where in the continuum the operator relevent for B_k has the spinor structure

$$VV + AA \equiv \gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$$

- With B_K :

$$B_K = \frac{\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3}\langle \overline{K}^0 | A_\mu | 0 \rangle \langle 0 | A_\mu | K^0 \rangle}$$

- (potential) Problem; in the absence of exact chiral symmetry O_{VV+AA} can mix with four other operators. It is important to note that these operators are suppressed by a factor of $O(am_{\text{res}}^2)$.

- The lattice matrix element will consist of the matrix element we want plus a contribution from the wrong chirality operators:

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{latt}} = Z_{11} \langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{ren}} + \sum_{i \geq 2} z_{1i} \langle \overline{K}^0 | O_i | K^0 \rangle_{\text{ren}}$$

- First order chiral perturbation theory predicts that

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle \propto M_k^2$$

and, unfortunately, that

$$\langle \overline{K}^0 | O_{\text{THE REST}} | K^0 \rangle \propto 1$$

so... as the chiral limit is approached the wrong chirality operators will dominate .

- We always work at relatively large values of the pseudo-scalar mass ($\sim M_K$), but still it is important to understand the expected size of the z_i coefficients.

- We can characterise the form of the chiral symmetry breaking in DWF by adding a spurion field , Ω , to the action such that

$$\Omega \to U_R \Omega U_L^\dagger$$

under an extended $SU(3)_L \otimes SU(3)_R$ transformation which is a symmetry of the action [Blum et. al , 2002]

  – just like a standard mass term ("mass flips chirality")

$$\begin{aligned} \Omega q_L &\to U_R(\Omega q_L) \\ \Omega^\dagger q_R &\to U_L(\Omega^\dagger q_R) \end{aligned}$$

- with each factor of Ω being $O(m_{res})$ when evaluated

- To analyse the chiral properties of the operators, it is convenient to explicitely write the operators in left- and right-handed components :

$$
\begin{aligned}
O_{VV+AA} &\propto \overline{s}_L\sigma_\mu d_L\overline{s}_L\sigma_\mu d_L + \overline{s}_R\overline{\sigma}_\mu d_R\overline{s}_R\overline{\sigma}_\mu d_R \\
O_{VV-AA} &\propto \overline{s}_L\sigma_\mu d_L\overline{s}_R\overline{\sigma}_\mu d_R \\
O_{SS-PP} &\propto \overline{s}_L d_R\overline{s}_R d_L \\
O_{SS+PP} &\propto \overline{s}_L d_R\overline{s}_L d_R + \overline{s}_R d_L\overline{s}_R d_L \\
O_{TT} &\propto \overline{s}_R\overline{A}^{\mu\nu} d_L\overline{s}_R\overline{A}^{\mu\nu} d_L + \overline{s}_L A^{\mu\nu} d_R\overline{s}_L A^{\mu\nu} d_R
\end{aligned}
$$

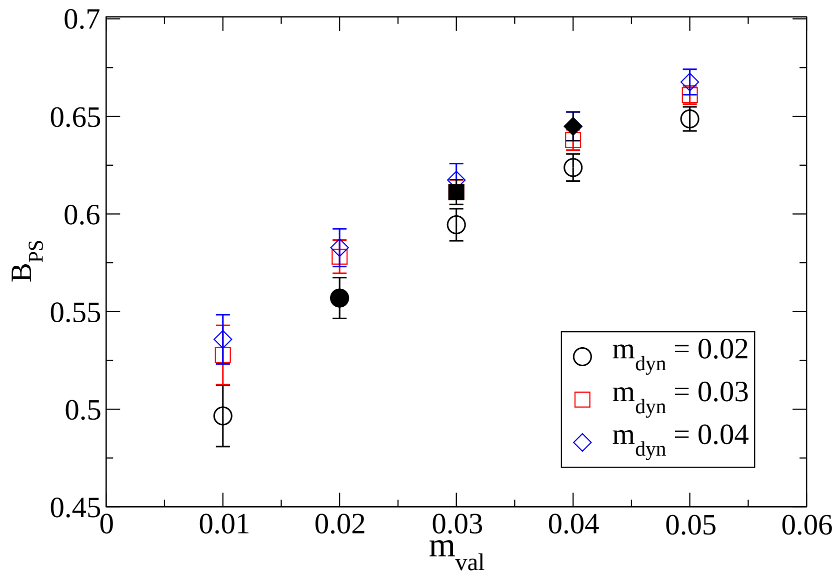
- Looking for potential mixing; at each order new order im m_{res} we can flip one left(right)-handed quark into a right(left)-handed one:

$$
\begin{array}{ll}
O(1) & \overline{s}_L d_L\overline{s}_L d_L + \overline{s}_R d_R\overline{s}_R d_R \\
 & \Downarrow \\
O(m_{res}) & \overline{s}_R d_L\overline{s}_L d_L,\ \overline{s}_L d_R\overline{s}_L d_L,\ \overline{s}_L d_L\overline{s}_R d_L \cdots \\
 & \overline{s}_L d_R\overline{s}_R d_R,\ \overline{s}_R d_L\overline{s}_R d_R,\ \overline{s}_R d_R\overline{s}_L d_R \cdots \\
 & \Downarrow \\
O(m_{res}^2) & \overline{s}_R d_R\overline{s}_L d_L,\ \overline{s}_R d_L\overline{s}_R d_L,\ \overline{s}_R d_L\overline{s}_L d_R \cdots
\end{array}
$$

- The basic scale for chiral symmetry breaking mixing is $O(m_{res}^2)$; 10^{-6} in the current case.

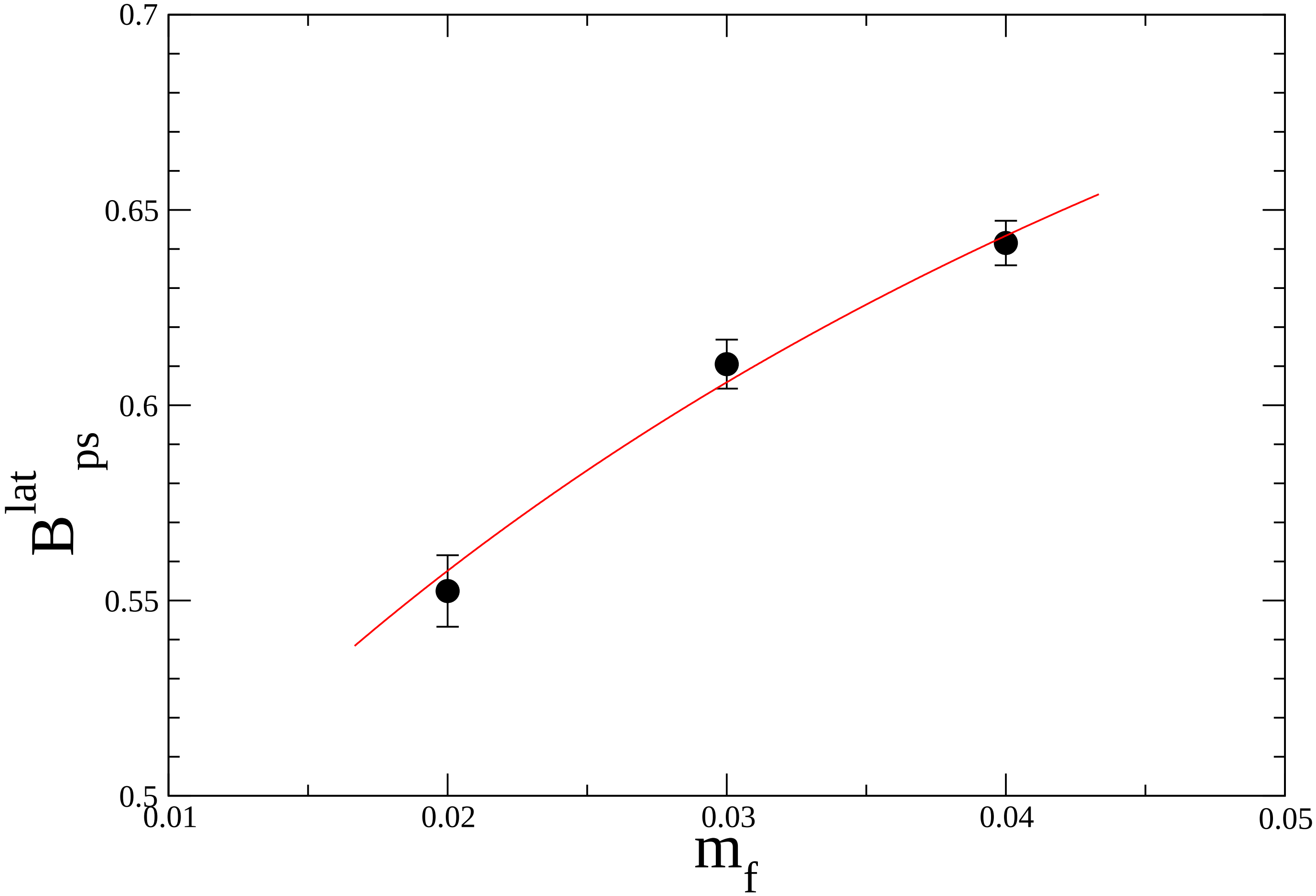
Preliminary dynamical B_K

[see the Lattice 2003 proceedings of Taku Izubuchi]



- Bare B_{ps} for all dynamical masses.

- We have calculated the renormalisation factor using both the NPR method of the Rome-Southampton group and perturbation theory. We get $"Z''_{B_K} = 0.93(2)$ and $0.92(2)$ respectively.



NLO in chiral perturbation theory fit to $m_{sea} = m_{val}$ points

Preliminary dynamical B_K.....

- Looking at just the fully dynamical points and extrapolating to the physical kaon point gives $B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.503(20)$ (naive m_s).

- We can also, for each dynamical mass, extrapolate to $m_{val} = m_s/2$, and this is shown in the table below

m_{dyn}	B_K^{lat}	$B_K^{\overline{\text{MS}}}(2\text{GeV})$	$\hat{B}_K$
0.02	0.537(11)	0.499(22)	0.692(34)
0.03	0.557(9)	0.518(20)	0.719(32)
0.04	0.568(10)	0.529(21)	0.733(34)
∞ RBC		0.536(6)	
∞ CPPACS		0.564(14)	

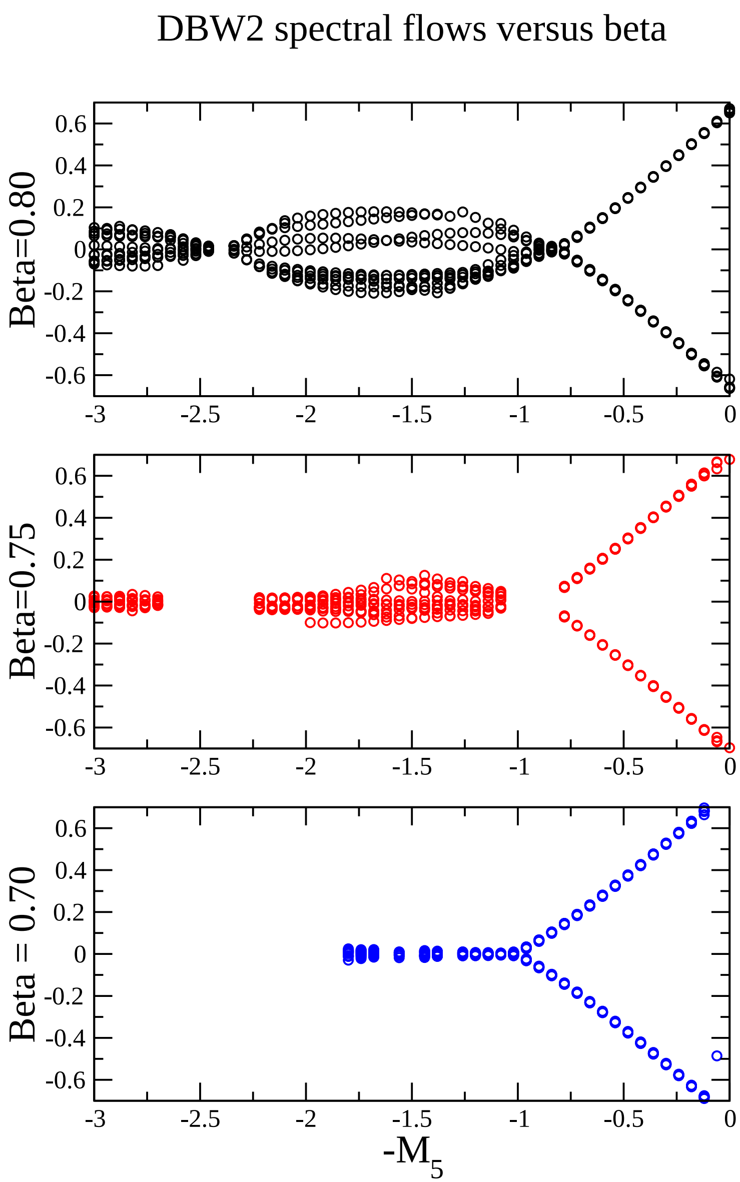
- These numbers, of course, are subject to systematic errors:

  1. Finite volume.
  2. Auto-correlations (real error bar might be bigger).
  3. Plateau
  4. ** value of $m_s/2$ ** from NLO fit to m_{ps}^2, $m_{sea} \to 0$ gives $m_s/2 \approx 0.023$ instead of 0.018: $B_K \uparrow$.

Summary

- Wrap up $n_f = 2$ study
- Have begun exploratory $n_f = 3$ study $(m_f \sim m_s)$ to investigate residual chiral symmetry breaking.
- QCDOC: 2+1 flavor DWF simulation

Spectral flow and the gauge coupling



- As we move to smaller beta the “gap” in the spectral flow quickly dies.
- $n_f = 3$: smaller β for the same lattice spacing