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Defining the heavy quark potential in perturbation theory and on the lattice

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Lattice part in collaboration with T. Hatsuda & S. Sasaki

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THE UNIVERSITY OF TOKYO

Brookhaven Summer Program on

Quarkonium Production in Elementary and Heavy Ion Collision 2011

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Heavy Quarkonia: Motivation

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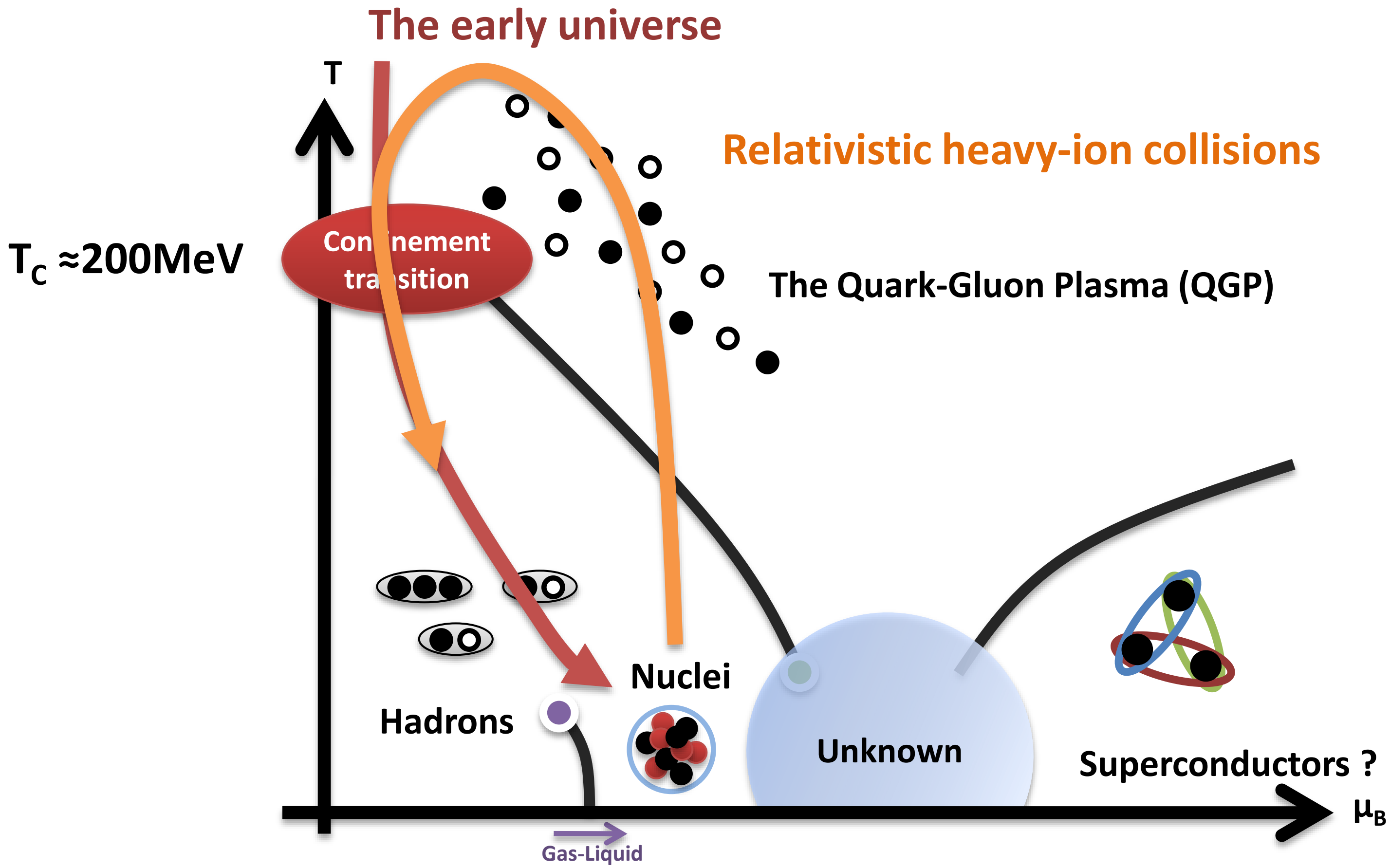

- Phase transition Quark-Gluon Plasma (**QGP**) $T>T_C$ vs. Confining phase $T<T_C$
- Recreate the QGP in the laboratory: **RHIC/LHC**
- **Heavy Quarkonium**: Clean probe for experiment and theory

The classic argument: Matsui & Satz

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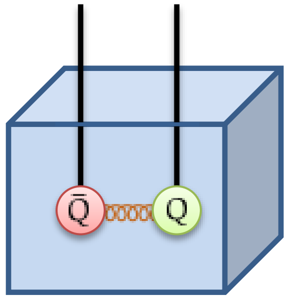
> If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect

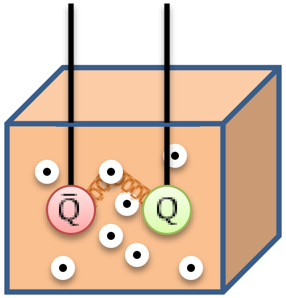
PLB 178 416 (1986)

$T<T_C$



QQbar à la Satz

$T>T_C$



- Model potentials: $V(R) = -\frac{\alpha}{R} + \sigma R$ confinement

  $V(R) = -\frac{\alpha e^{-m_D R}}{R}$ Debye screening

- Static color test charges: correlations from lattice QCD
  Polyakov loops

- Melting sets in already below $1.2T_C$

Goal for Theory ➡ What is the proper potential to use in a non-relativistic description?

How to derive a Schrödinger equation from first principles QCD?

Towards a T>0 Potential

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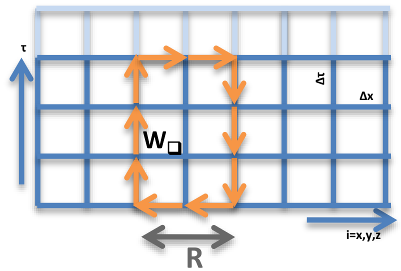
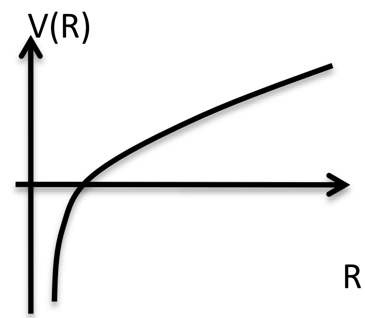
- Goal is to derive a Hamiltonian with: $H = \frac{p_1^2}{2m_Q} + \frac{p_2^2}{2m_Q} + V^{(0)}(R) + V^{(1)}(R)\frac{1}{m} + \ldots$

- At **T=0** systematic framework available: NRQCD, pNRQCD   Brambilla et al. 2005

Lattice QCD (LQCD): Monte Carlo

Derivation of $V^0(R)$:
Wilson Loop

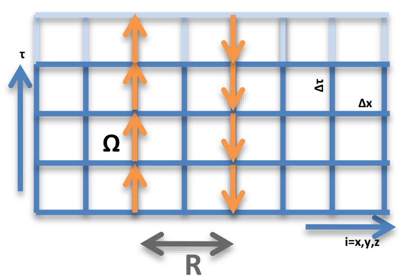


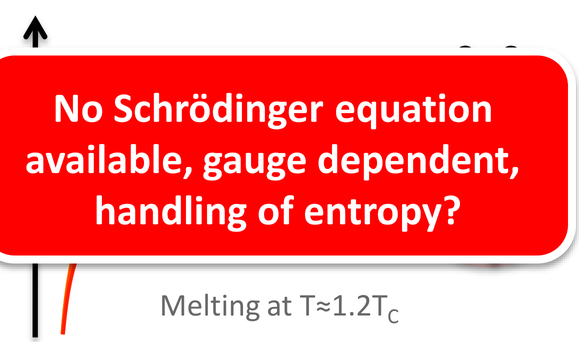


- Potential Models at T>0   Nadkarni, 1986

Lattice QCD: Monte Carlo

Ad-hoc choice:
Free Energies or
Internal Energies





No Schrödinger equation available, gauge dependent, handling of entropy?

Melting at $T \approx 1.2 T_C$

Defining a T>0 Potential

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- What is a non-relativistic potential?
  - Direct answer: The non-kinetic term in a Schrödinger type E.O.M.

$$i\partial_t D^>(R,t) = \left(\frac{p^2}{2m_q} + V(R)\right) D^>(R,t)$$

The first complex static potential

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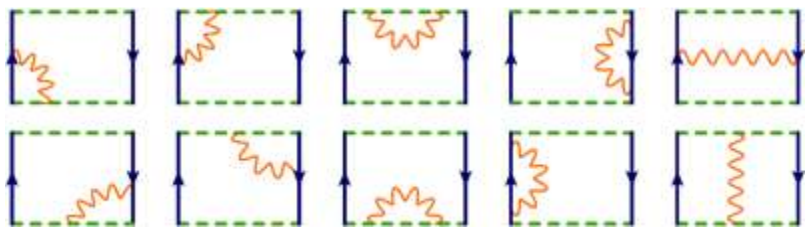
ABSTRACT: We derive a static potential for a heavy quark-antiquark pair propagating in Minkowski time at finite temperature, by defining a suitable gauge-invariant Green's function and computing it to first non-trivial order in Hard Thermal Loop resummed perturbation theory. The resulting Debye-screened potential could be used in models that attempt to describe the "melting" of heavy quarkonium at high temperatures. We show, in particular, that the potential develops an imaginary part, implying that thermal effects generate

The real-time thermal Wilson Loop

Laine et. al. JHEP03 (2007) 054; see also Beraudo et. al. NPA 806:312-338,2008

- Heavy quark propagation described by rectangular Wilson in the static limit

$$\langle \mathrm{Tr}[\exp[\oint A]]\rangle \equiv$$



HTL gluon propagator

Pisarski PRL 63 (1989) 1129

Braaten, Pisarski NPB 337 (1990) 569

- Wilson Loop in the infinite time limit: Potential emerges with **real and imaginary part**

$$\lim_{t\to\infty} V^0_{rT}(t,R) = -\frac{gC_F}{4\pi}\left[\boxed{m_D + \frac{e^{-m_D r}}{r}}\right] - \boxed{\frac{ig^2TC_F}{4\pi}\phi(m_D r)} \qquad \phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Debye screening: a cloud of quarks and gluons mitigates the interaction effects

Landau damping: collisions with the deconfined environment

Defining a T>0 Potential

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- What is a non-relativistic potential?
  - Direct answer: The non-kinetic term in a Schrödinger type E.O.M.

$$i\partial_t D^>(R,t) = \left(\frac{p^2}{2m_q} + V(R)\right) D^>(R,t)$$

  - Systematic answer: Renormalization coefficient in an effective theory
- How to obtain an effective field theory: Integrate out energy scales
  - Find a separation in scales and choose appropriate degrees of freedom

Hard scale: $E \gtrsim m_Q$ Physics of pair creation, gluons mediate interaction

$$\bar{Q}(x)\left(i\gamma^\mu D_\mu + m\right)Q(x)$$

Soft scale: $E \ll m_Q$ Physics of pauli spinors, gluons still appear explicitly

$$\bar{\chi}(x)\left(iD_0 + mc + C_0 + \frac{C_1}{2m}\sigma^i B_i + \frac{C_2}{2m}D_i^2\right)\chi(x)$$

Obtain C_i factors by comparing correaltion functions at a fixed energy (matching)

Even softer scale: $E \sim E_{bind}$ Physics of QQbar pairs, gluons do not appear explicitly

Effective Field Theory approach

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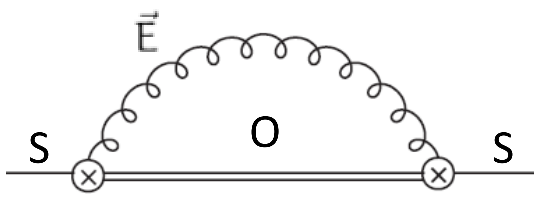
In a framework that makes close contact with modern effective field theories for nonrelativistic bound states at zero temperature, we study the real-time evolution of a static quark-antiquark pair in a medium of gluons and light quarks at finite temperature. For temperatures ranging from values larger to smaller than

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

- Effective theory that treats **q.m. singlet and octet pairs of heavy Q and Qbar** as d.o.f.

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{i=1}^{n_f}\bar{q}_i i\gamma^\mu D_\mu q_i + \int d^3r \mathrm{Tr}\left\{S^\dagger\left[i\partial_0 + V_S(r)\right]S + O^\dagger\left[iD_0 - V_O(r)\right]O\right\}$$
$$+ V_A \mathrm{Tr}\left\{O^\dagger\vec{r}\cdot g\vec{E}S + S^\dagger\vec{r}\cdot g\vec{E}O\right\} + \frac{V_B}{2}\mathrm{Tr}\left\{O^\dagger\vec{r}\cdot g\vec{E}O + O^\dagger O\vec{r}\cdot g\vec{E}\right\}$$

- Obtain potentials by comparing greens functions in the EFT and perturbative QCD
- If $T \gg E_B$ and $m_D \gg E_b$ the following diagram gives the leading contribution



Interaction with the color electric field temporarily turns singlet into octet

between the two static sources, and calculate their energy and thermal decay width. Two mechanisms contribute to the thermal decay width: the imaginary part of the gluon self-energy induced by the Landau damping phenomenon, and the quark-antiquark color-singlet to color-octet thermal breakup.

Potential from lattice spectral functions

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- Goal: **Non-perturbative** determination of the potential at **any temperature** by using **first principles lattice QCD**
  - Use only the following separation of scales:

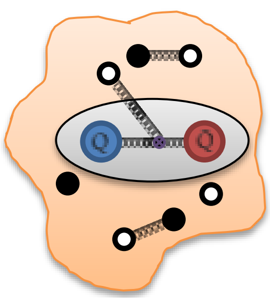
$$\frac{\Lambda_{QCD}}{m_Q c^2} \ll 1, \quad \frac{T}{m_Q c^2} \ll 1, \quad \frac{\mathbf{p}}{m_Q c} \ll 1$$

  - Select appropriate degrees of freedom:

Relativistic thermal field theory

Derivation of the heavy quark potential $\longrightarrow$

Quantum mechanics



Dirac fields

$\bar{Q}(x), Q(x)$

$\bar{q}(x), q(x), A^\mu(x)$

Pauli fields

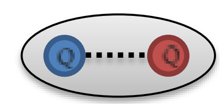
$\bar{\chi}(x), \chi(x)$

$\bar{q}(x), q(x), A^\mu(x)$

Position & Momentum

$\mathbf{z}(t), \mathbf{p}(t)$

$V^{(0)}(R) \leftarrow \rho_\square(\omega, R)$



Lattice QCD: Monte Carlo

Maximum Entropy Method

  - Derive a Schrödinger equation with a **non-perturbative**, spin-independent potential

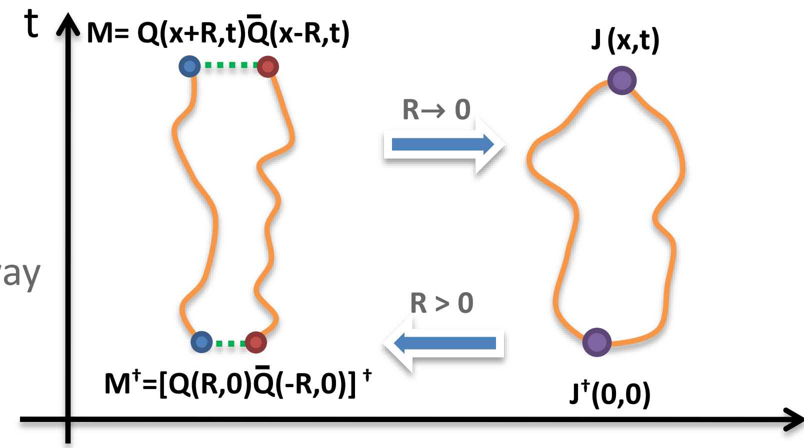
Derivation of V(R)

- Starting point: $\bar{Q}Q$ in the language of field theory (Minkowski – time)
  - Meson current $J(x) = \bar{Q}(x)\Gamma Q(x)$
  - Test charges: Introduce an **external separation R**

$$M = \bar{Q}(x)\Gamma \boxed{W(\mathbf{x},\mathbf{y})} Q(y)$$

  - Describe the time evolution in a **gauge invariant** way

$$D^{>}(\mathbf{R},t) = \langle M(\mathbf{R},t)M^{\dagger}(\mathbf{R},0)\rangle$$



  - Another choice: do not insert W(**x**,**y**) and fix a gauge
  - Only naturally gauge invariant quantity: current - current correlator

$$\Big[-i\partial_t + H(\mathbf{R},\mathbf{p})\Big]D^{>}(\mathbf{R},t) = 0 \quad \Rightarrow \quad \lim_{R\to 0} D^{>}(\mathbf{R},t) = \Big\langle J(t)J^{\dagger}(0)\Big\rangle$$

  - Use separation of scales to simplify the expression for $D^{>}(\mathbf{R},t)$

$$D^{>}(\mathbf{R},t) = \Big\langle \mathcal{T}\Big[\int \mathcal{D}[\bar{Q},Q]\,\Gamma\bar{\Gamma}\,WW^{\dagger}\;Q(y')\bar{Q}(y)Q(x)\bar{Q}(x')\;e^{iS_{QQ}[Q,\bar{Q},A]}\Big]\Big\rangle_{q,\bar{q},A}$$

Integrating out $E=m_Q c^2$

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- Replace the degrees of freedom for the heavy fermions Q -> Q = (χ,ξ)

$$S_{QQ}[A] = \bar{Q}(x)\Big(i\gamma^\mu D_\mu(x;A) - mc\Big)Q(x)$$

  - **Foldy-Tani-Wouthuysen** transformation: an expansion in the inverse rest energy $1/m_Q c^2$

$$S^{FTW}_{QQ}[A] = \bar{Q}(x)\left[\boxed{\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} D_0 - mc + \frac{1}{2mc}D_i^2 + \frac{g}{2mc^2}\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} B^i}\right]Q(x)$$

  - FTW Transformation leaves the path integral unchanged at this order
  - No coupling between upper χ and lower ξ components of Q. **No creation/annihilation** possible

- Integrate out (χ,ξ) explicitly: Replace by quantum mechanical **Green`s functions S =** $\boxed{}^{-1}$

$$D^{>}_{FTW}(\mathbf{R},t) = \left\langle \mathcal{T}\left[\int \mathcal{D}[\bar{Q},Q]\, \Gamma\bar{\Gamma}\, WW^\dagger\, Q(y')\bar{Q}(y)Q(x)\bar{Q}(x')\, e^{iS^{FTW}_{QQ}[Q,\bar{Q},A]}\right]\right\rangle_{q,\bar{q},A}$$

$$\Downarrow$$

2x2 sub matrix

$$D^{>}_{q.m.} = \left\langle \mathcal{T}\Big[W(\mathbf{x},\mathbf{y})\, G\, S(\mathbf{y},\mathbf{y}')\, \bar{G}\, W^\dagger(\mathbf{x}',\mathbf{y}')\, S^\dagger(\mathbf{x},\mathbf{x}')\Big]\right\rangle_{q,\bar{q},A}$$

Temperature dependence

q.m. heavy quark **Green`s functions** (2x2)

Q.M. Path Integrals

- Determine the heavy quark Green`s function S **beyond the static limit**: Barchielli et. al. 1988

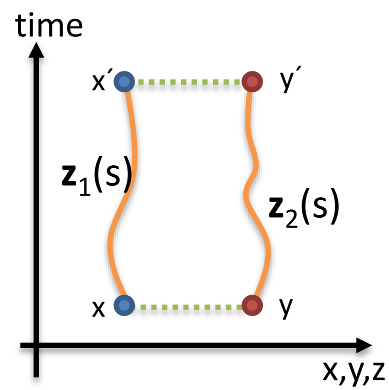
$$i\partial_t S(x,x') = \left[\boxed{\frac{1}{2m}\Big(-i\nabla - \frac{g}{c}\mathbf{A}(x)\Big)^2 + gA^0(x) - \frac{g}{mc}\sigma_i B^i(x)}\right] S(x,x') \qquad \& \qquad S(x,x')|_{t=t'} = \delta^3(\mathbf{x}-\mathbf{x}')$$

$$S(x,x') = \int_x^{x'} \mathcal{D}[\mathbf{z},\mathbf{p}]\mathcal{T}\exp\left[i\int_t^{t'} dt\Big(\mathbf{p}(t)\dot{\mathbf{z}}(t) - \boxed{H(z(t),p(t))}\right]$$

- The full forward propagator $D^>$ is the product of two S factors:

$$D^>_{q.m.} = \exp[-2imc^2 t]\int \mathcal{D}[\mathbf{z}_1,\mathbf{p}_1]\int \mathcal{D}[\mathbf{z}_2,\mathbf{p}_2]\exp\left[i\int_t^{t'} ds \sum_i \Big(\mathbf{p}_i(s)\dot{\mathbf{z}}_i(s) - \frac{\mathbf{p}_i^2(s)}{2m}\Big)\right]\times$$
$$\left\langle \frac{1}{N}\mathrm{Tr}\left[\mathcal{P}_{\mathcal{C}}\boxed{\exp\left[\frac{ig}{c}\oint_{\mathcal{C}} dx^\mu A_\mu(x)\right]}\right]\right\rangle$$

This is not just the rectangular Wilson loop: fluctuating paths



- To read off the **Hamiltonian for the two-body system** we need to rewrite:

$$\langle \mathrm{Tr}[\exp\left[\oint A\right]]\rangle \equiv \exp\left[i\int_t^{t'} ds\, U(\mathbf{z}_1(s),\mathbf{z}_2(s),\mathbf{p}_1(s),\mathbf{p}_2(s),s)\right]$$

The Proper Static Potential

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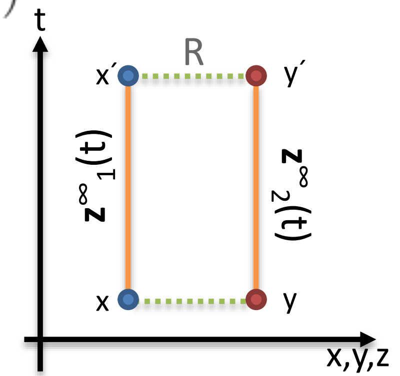
- Systematic expansion of the potential in p/mc: Barchielli (1988) uses v/c instead

$$i\log\left[\left\langle W(z(t),t)\right\rangle\right] = \int_t^{t'} ds\, U(z(s),p(s),s) = \int_t^{t'} ds\left(\boxed{u(z,s)|_{p=0}} + w_n^i(z,s)|_{p=0}\frac{p_n^i(s)}{mc} + \dots\right)$$



  - For p→0: $$i\log\left\langle \frac{1}{N}\mathrm{Tr}\left[\mathcal{P}_C\exp\left[\frac{ig}{c}\oint_C dx^\mu A_\mu(x)\right]\right]\right\rangle\Bigg|_{p_{1,2}=0} = \int_t^{t'} ds\, u(R,s)$$

Real-time thermal **rectangular** Wilson loop

  - Take the time derivative to obtain the potential u(R,t)

  - Use **spectral function** of the Wilson Loop: $W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho_\square(R,\omega)$

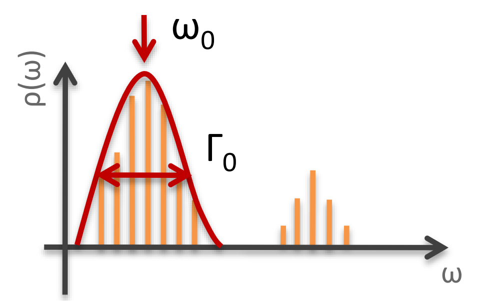
Well defined peaks

$$u(R,t) = \frac{1}{W_\square(R,t)}\int d\omega e^{-i\omega t}\,\omega\,\rho_\square(R,\omega)$$

  - **Intuitive**: Two analytically solvable cases (Breit-Wigner & Gaussian)

$$u_{BW}(R) = \omega_0(R) + \boxed{i\Gamma_0(R)}$$ constant damping

$$u_G(R,t) = \omega_0(R) + \boxed{i\Gamma_0^2(R)\,t}$$ Resembles a **diffusion** process



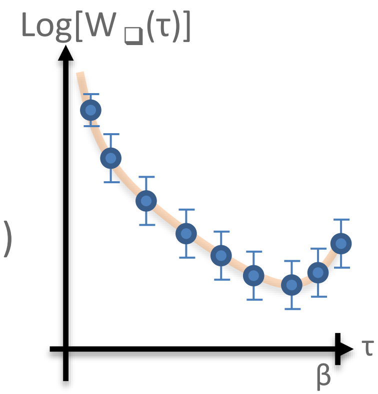
Spectral functions & LQCD

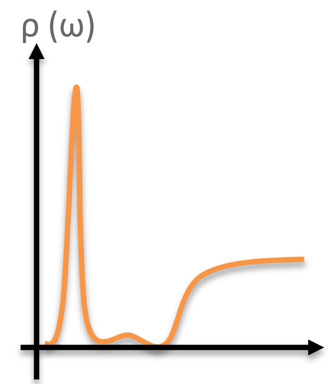
- We have connected the **spectral function** ρ of the rectangular **real-time** Wilson loop $W_\square$ to **V(R)**
  - Cannot measure ρ or $W_\square(R,t)$ directly in Lattice QCD: **Analytic continuation**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho_\square(R,\omega) \xleftarrow{t=-i\tau} \boxed{W_\square(R,\tau)} = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\boxed{\rho_\square(R,\omega)}$$

O(10) + noise **O(1000)**

  - Cannot use simple χ^2 fitting: ill defined
  - Since ρ is spectral function: positive definite





- Bayes Theorem can help (**Maximum Entropy Method**)

$$P[\rho|Dh] = \frac{\boxed{P[D|\rho h]}\,\boxed{P[\rho|h]}}{P[D|h]}$$

$$\boxed{\propto \mathrm{Exp}\Big[-\frac{1}{2}\sum_{ij}\big(D(\tau_i)-D_\rho(\tau_i)\big)C_{ij}^{-1}\big(D(\tau_j)-D_\rho(\tau_j)\big)\Big]} \qquad \boxed{\propto \mathrm{Exp}\Big[\alpha\int_{-\infty}^{\infty}\Big\{\rho(\omega)-h(\omega)-\rho(\omega)\mathrm{Log}\Big(\frac{\rho(\omega)}{h(\omega)}\Big)\Big\}d\omega\Big]}$$

Likelihood: the usual χ^2 fitting term

Prior probability: Shannon-Janes entropy

$$\Rightarrow \frac{\delta}{\delta\rho}P[\rho|Dh] \stackrel{!}{=} 0$$

Exploring the potential

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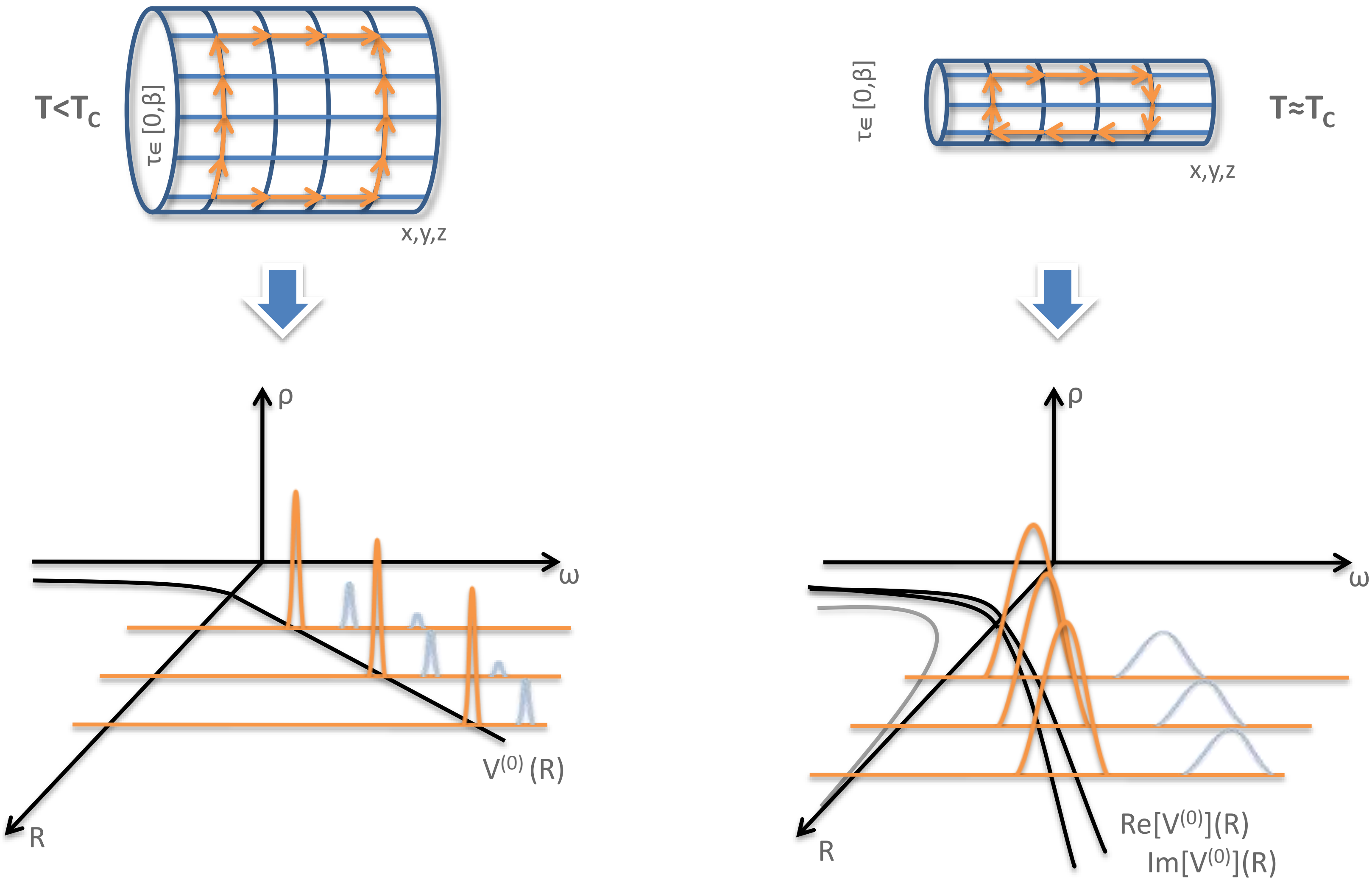
- Using **Lattice QCD** and the **MEM**, we can obtain the spectral function at any temperature



Numerical Results: $T=0.78T_C$

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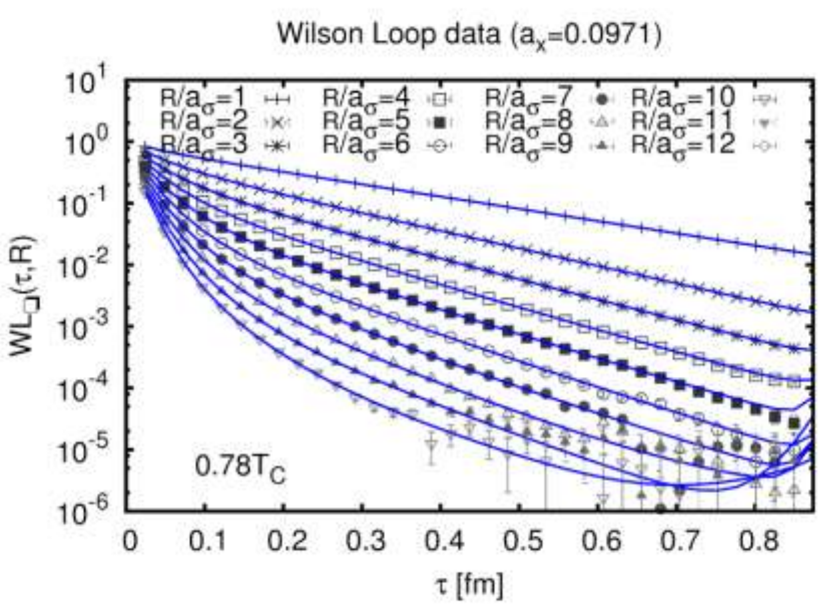
Quenched QCD Simulations

- Anisotropic Wilson Plaquette Action
- $NX=20 \quad NT=36 \quad \beta=6.1 \quad \xi_b=3.2108$
- Box Size: 2fm Lattice Spacing: 0.1fm
- HB:OR 1:4 with 200 sweeps/readout

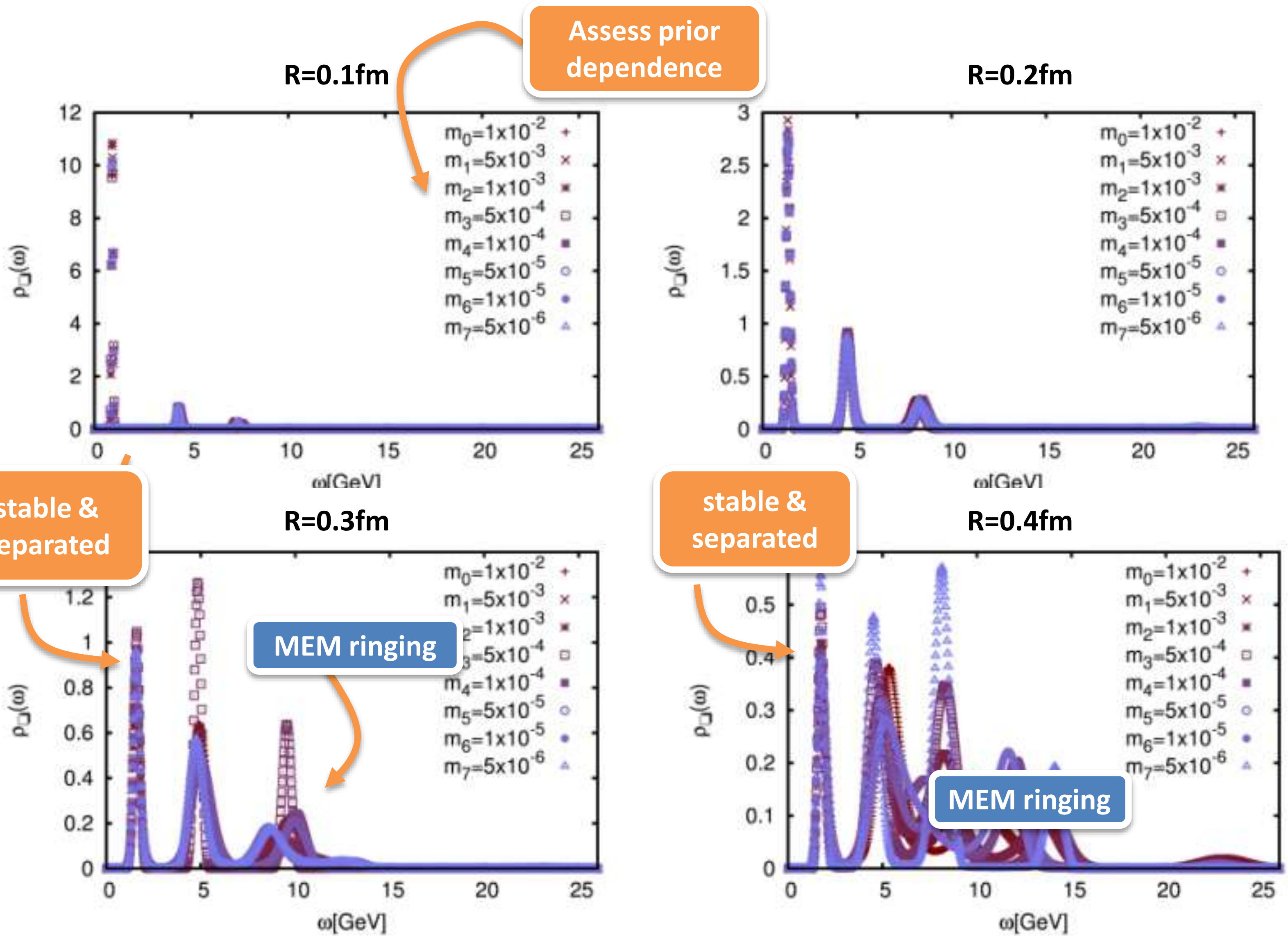
Maximum Entropy Method

- Singular Value Decomposition
- $N_\omega=1500$
- Prior: m_0/ω, varied over 4 orders
- 384bit precision



Note that the Wilson Loop is non-symmetric since heavy quarks are not thermalized

Numerical Results: $T=0.78T_C$



Numerical Results: $T=0.78T_C$

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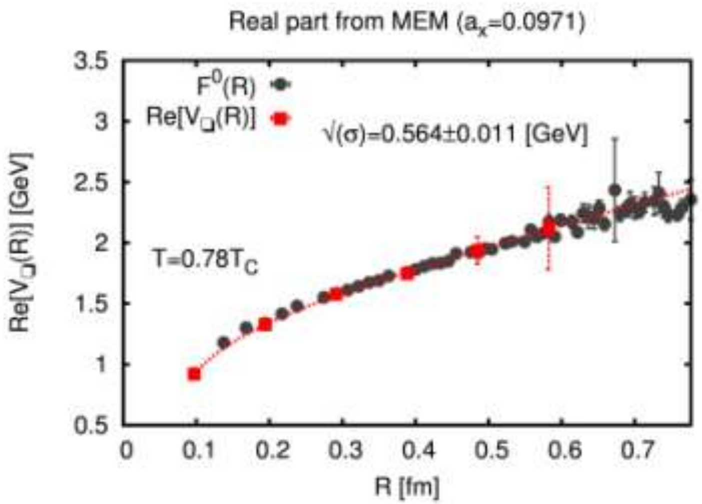

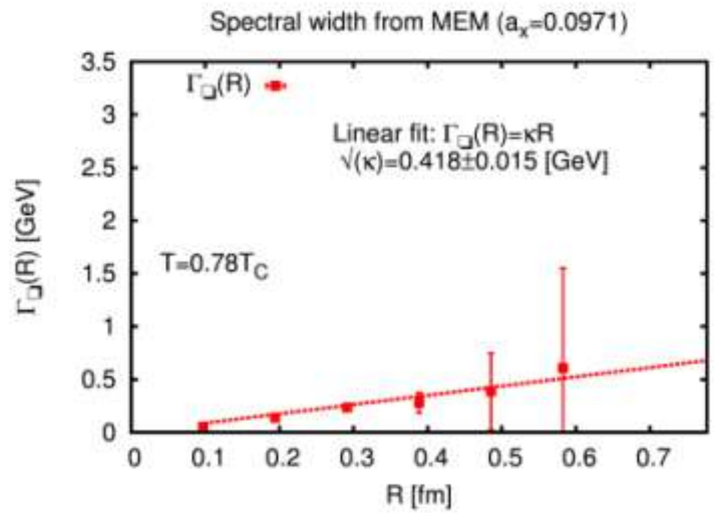

- The **real part coincides** with the color singlet free energies in Coulomb gauge

- Spectral width **consistent with zero** due to large error bars
(Note: MEM induces artificial width)

Numerical Results: $T=1.17T_C$

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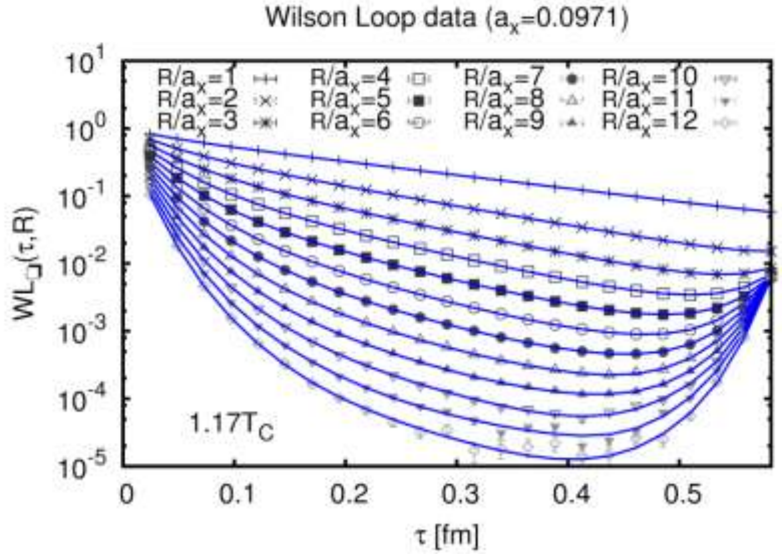

Upward trend becomes visible

Numerical Results: T=1.17T$_C$

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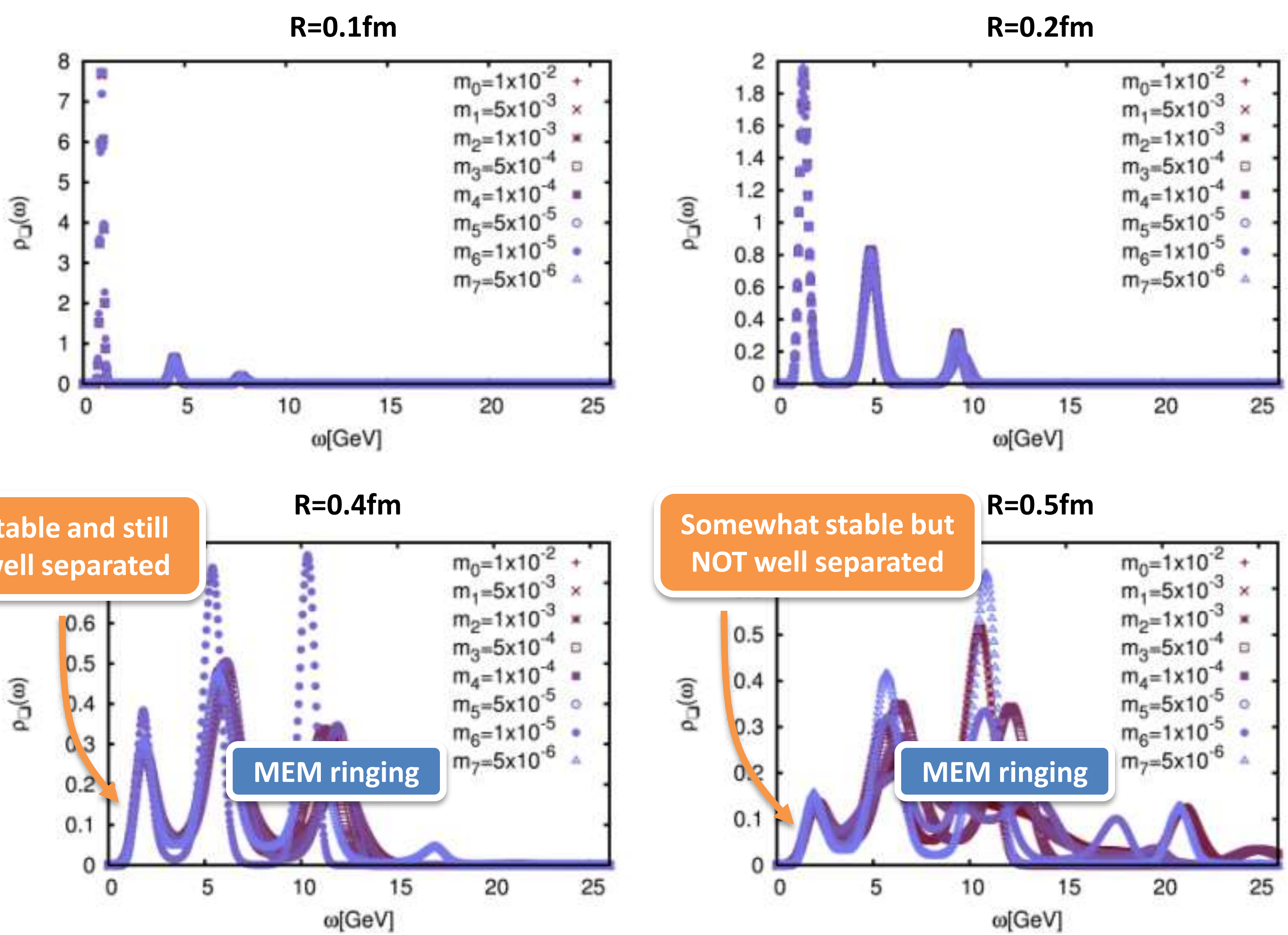

Numerical Results: T=1.17T$_C$

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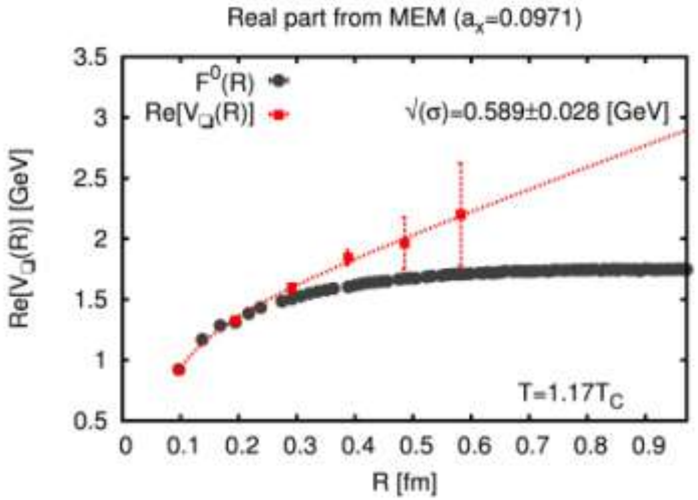

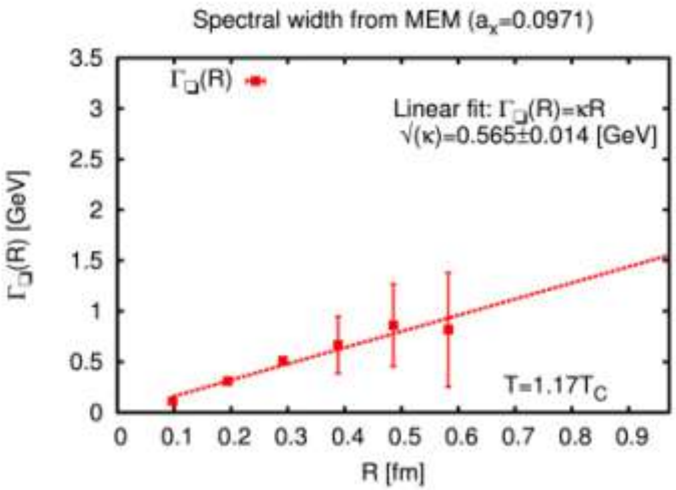

- Real part is slightly stronger than color singlet free energies but error bars are quite large.
- Spectral width is finite and larger than below T_c

Comparison at different T

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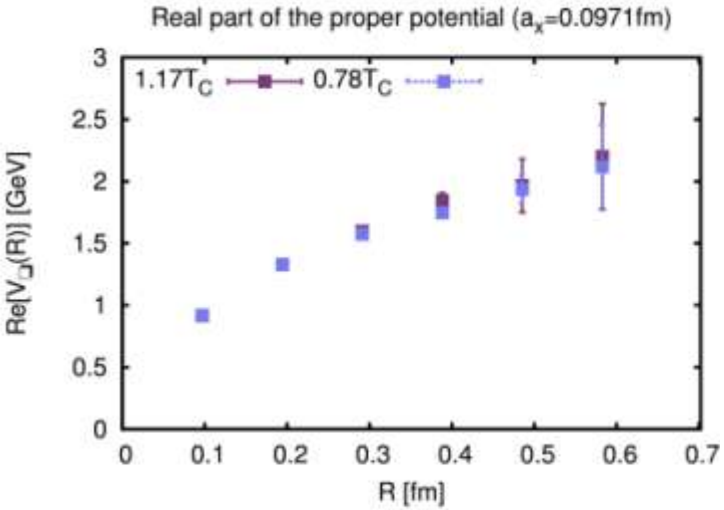
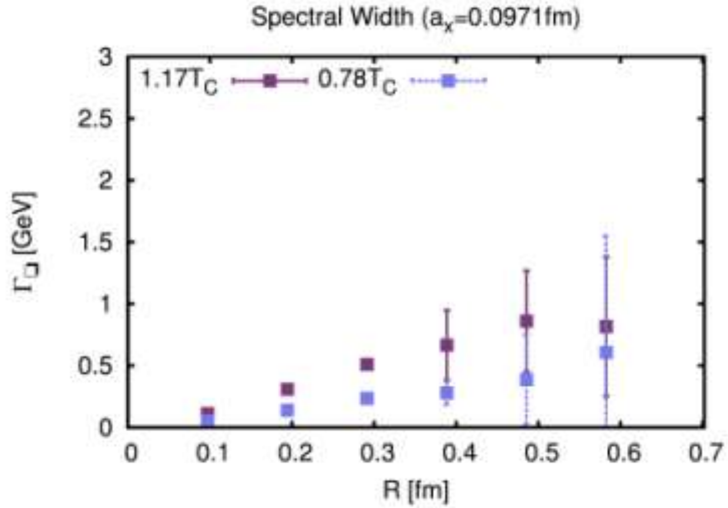
- The simulations around T_C show:

From the Wilson Loop





- **Real part** up to and around T_C **insensitive** to thermal fluctuations
- **Imaginary part increases** with temperature

Numerical Results T=2.33T_C

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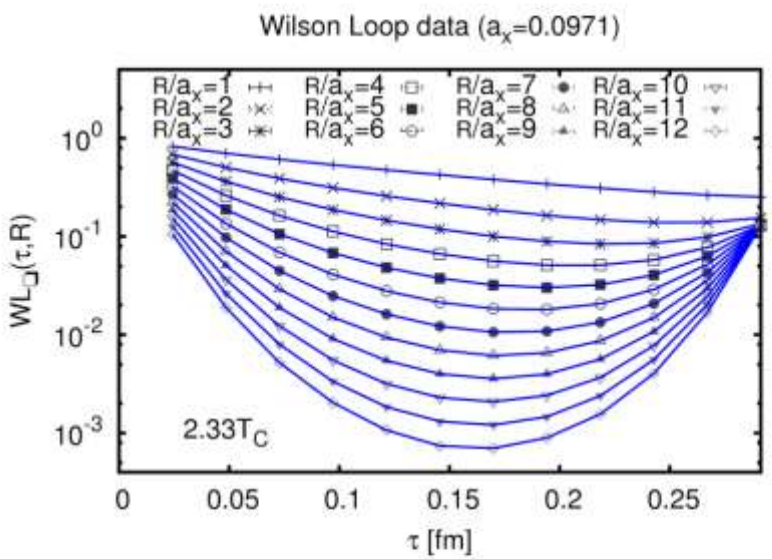
Quenched QCD Simulations

- Anisotropic Wilson Plaquette Action
- NX=20 NT=12 β=6.1 ξ_b=3.2108
- Box Size: 2fm Lattice Spacing: 0.1fm
- HB:OR 1:4 with 200 sweeps/readout

Maximum Entropy Method

- Singular Value Decomposition
- N_ω=1500
- Prior: m_0/ω, varied over 4 orders
- 384bit precision



Numerical Results T=2.33T$_C$

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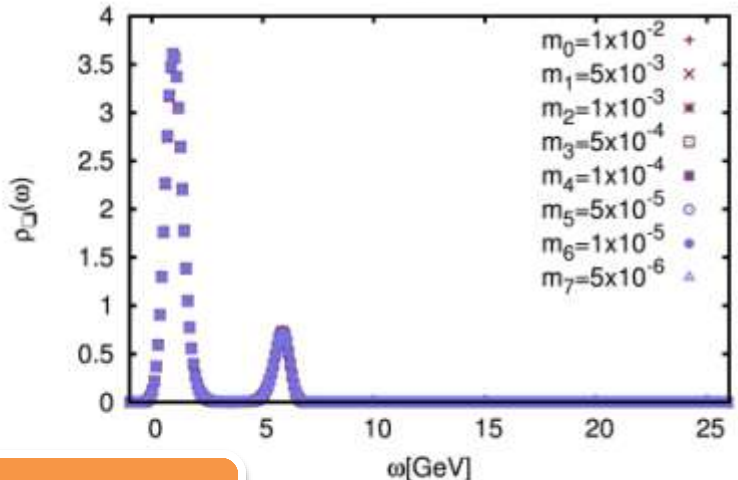
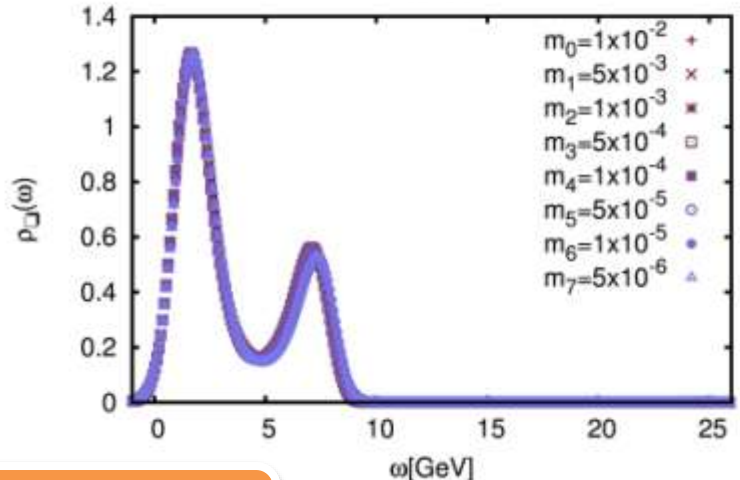
R=0.1fm

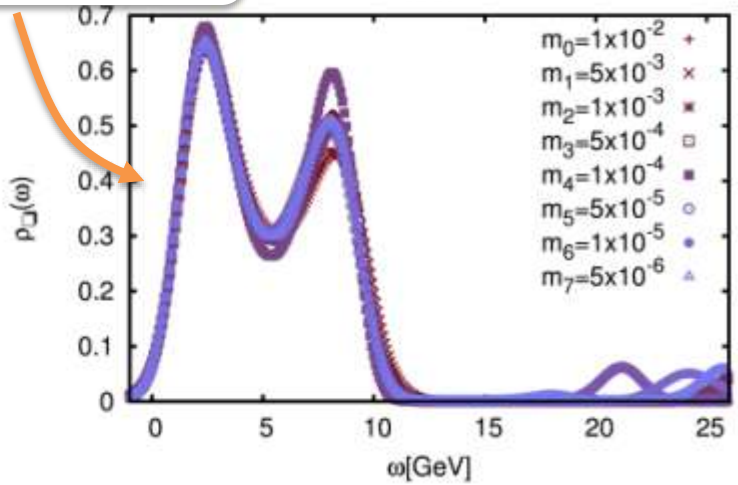


R=0.2fm



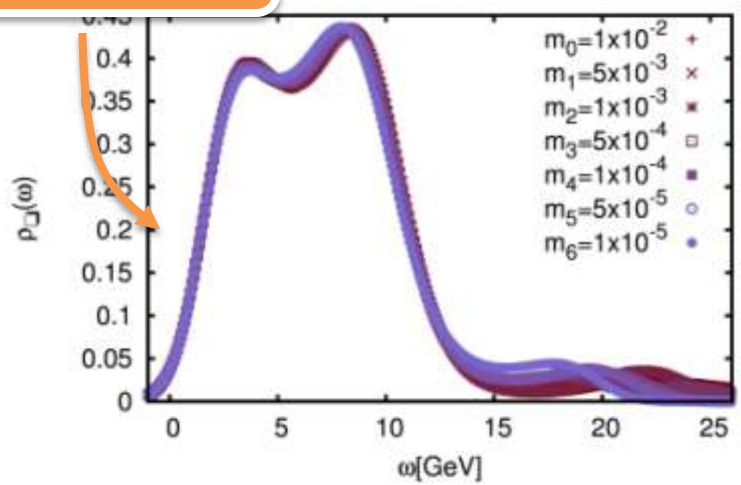
stable but not well separated

R=0.3fm



stable but NOT separated

R=0.4fm



Comparison at different T

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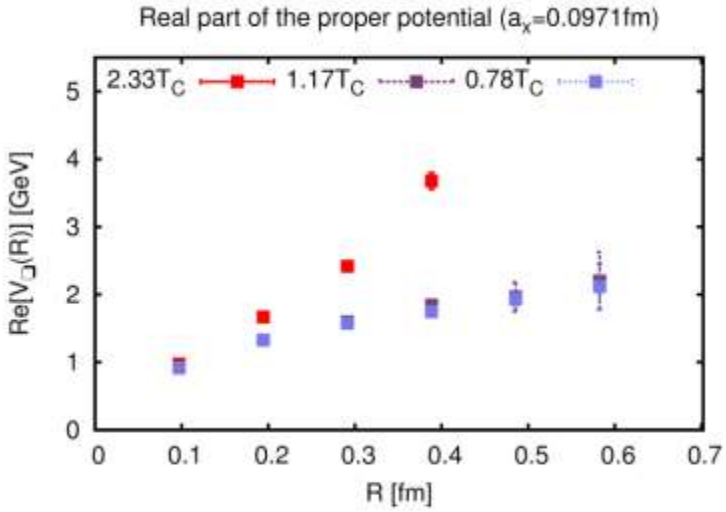
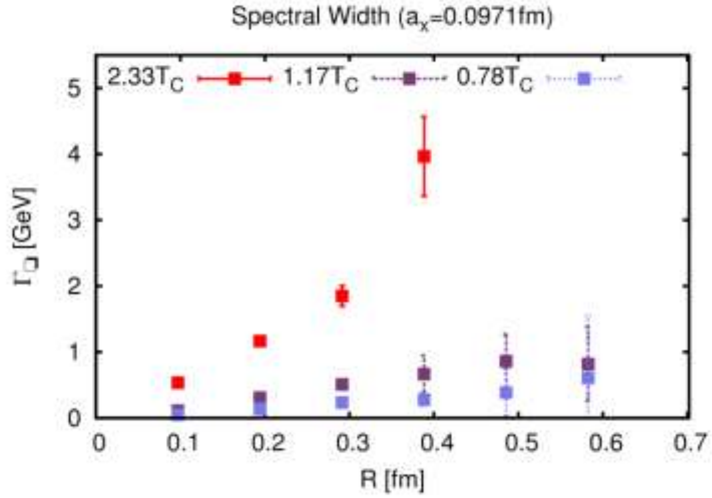
- The simulations at $T=2.33T_C$ show:





  - **Real part** shows steep rise if we trust the position of the lowest peak
  - **Imaginary part increases** with almost equal slope and magnitude

Check the numerical results $T=2.33T_C$

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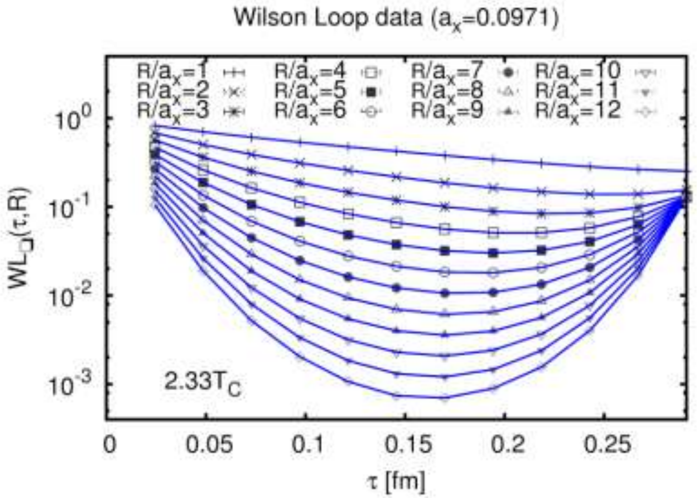
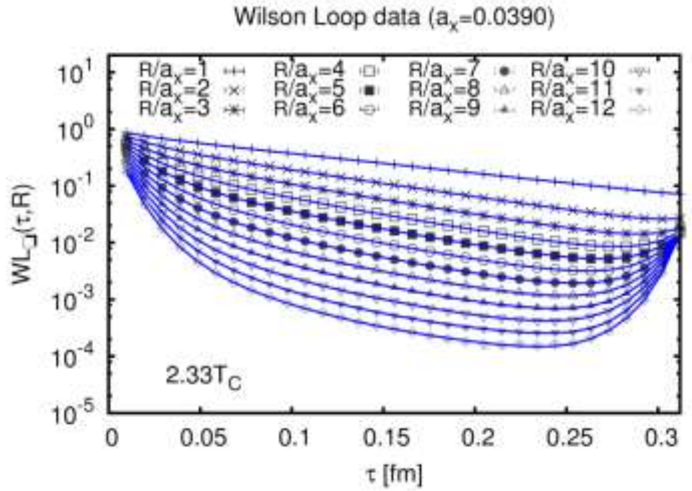
Quenched QCD Simulations

- Anisotropic Wilson Plaquette Action
- NX=20 NT=12 $\beta=6.1$ $\xi_b=3.2108$
- Box Size: 2fm Lattice Spacing: 0.1fm
- NX=20 NT=32 $\beta=7$ $\xi_b=3.5$
- Box Size: 0.8fm Lattice Spacing: 0.04fm
- HB:OR 1:4 with 200 sweeps/readout

Maximum Entropy Method

- Singular Value Decomposition
- $N_\omega=1500$
- Prior: m_0/ω, varied over 4 orders
- 384bit precision





Back

Numerical Results T=2.33T_C

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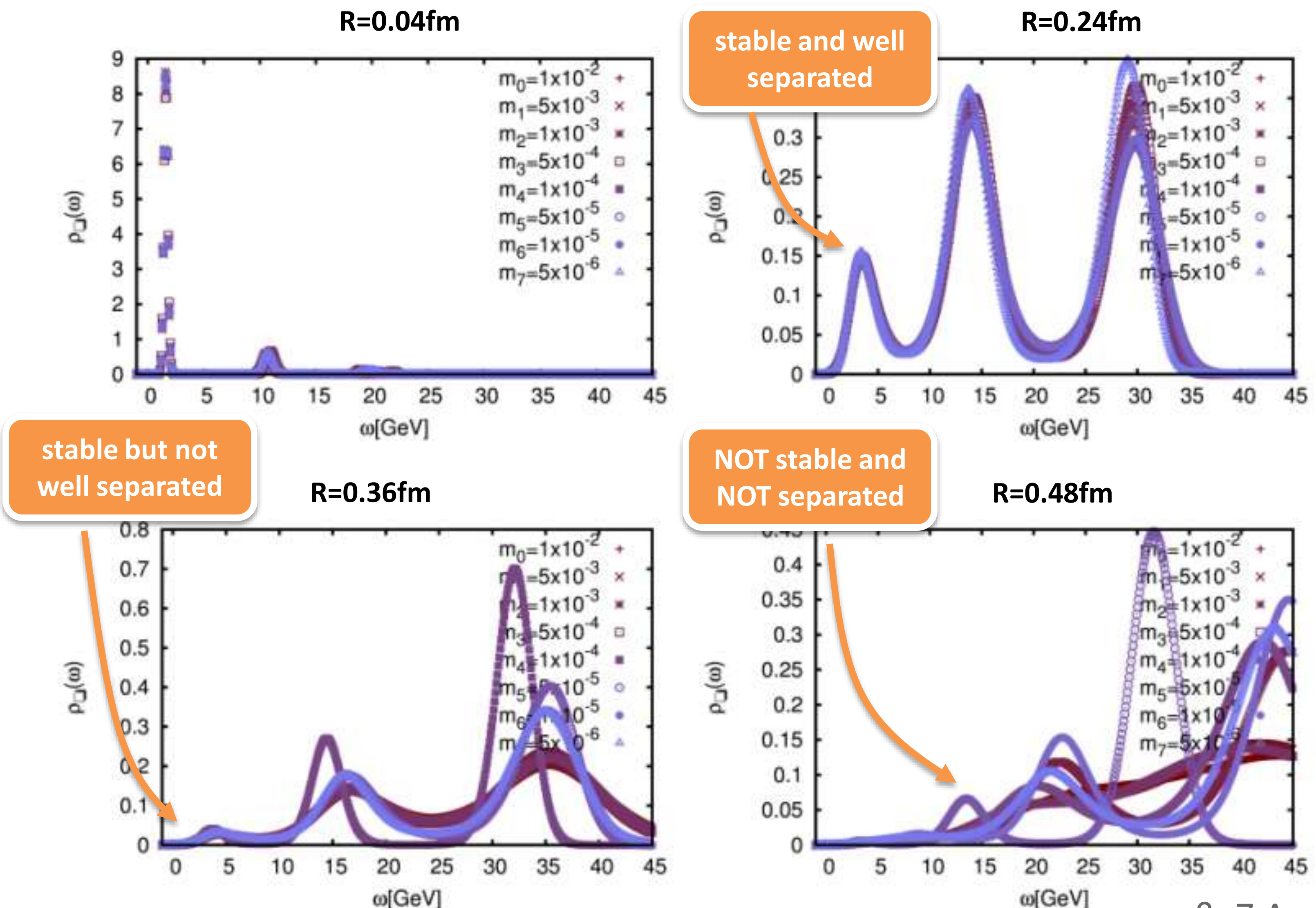

β=7 Δx=0.04fm

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Comparison at different lattice spacing

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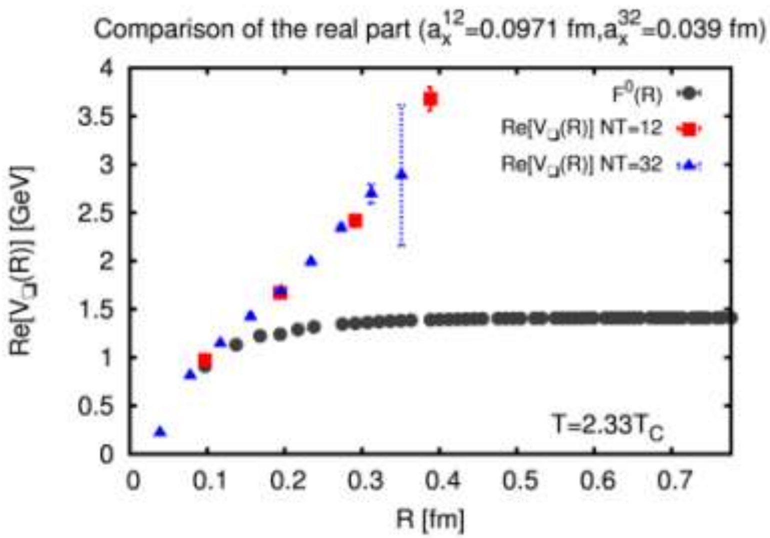
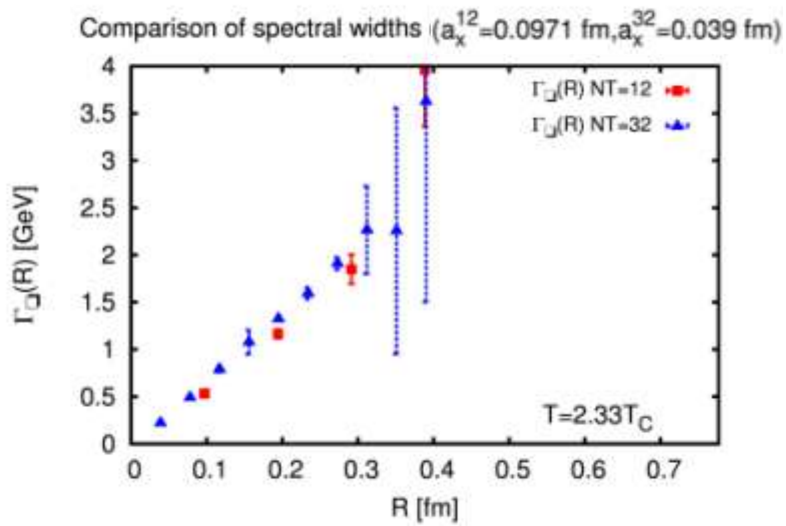
- The simulations at $T=2.33T_C$ show:





  - Steep rise of both real and imaginary part **does not dependent** on the **lattice spacing**

Summary & Conclusion

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- Treatment based on effective theories allows a **derivation** of the heavy quark potential

- **Perturbative** evaluation of the potential:

  Landau damping and **singlet to octet breakup** can induce an imaginary part

  There is more to the physics of J/Ψ melting than Debye screening

- **Non-perturbative** derivation of an effective in-medium **Schrödinger equation**

$$u(R,t) = \frac{1}{W_\square(R,t)} \int d\omega e^{-i\omega t}\ \omega\ \rho_\square(R,\omega)$$

  Possibility to **check the applicability** of the potential picture

  Complex Potential is obtained from the spectral function of the real-time Wilson loop

- Numerical Results and Discussion

  At $T<T_C$ **real part coincides with color singlet free energies**

  Around T_C the real-part appears to be **insensitive to thermal fluctuations**

  Above T_C both **real and imaginary part** become of the **same (large) magnitude**

Future Studies

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- Future work

➡ Check **consistency** with perturbation theory **at very high T**

Extract the heavy quark potential at **larger separation distances** R

Include **dynamical fermions** into the medium

➡ Solve the **time dependent Schrödinger equation** to assess the **temporal** and **spatial** structure during the evolution of J/ψ in the QGP

The End