On Phase Transitions In Strongly Coupled Quantum Field Theories: QCD vs. QED_3

J. A. Bonnet together with C. S. Fischer
- JLU Gießen -

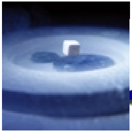
December 20, 2012

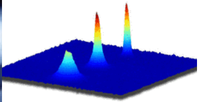
Outline

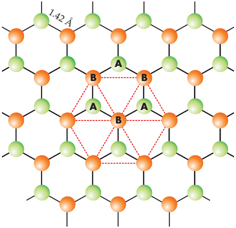
Phase transitions in strongly coupled QFTs

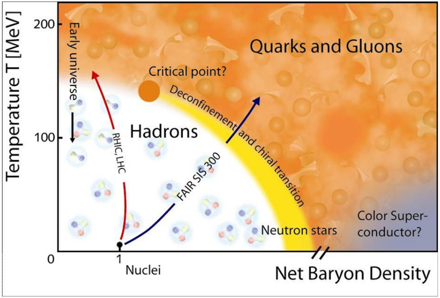
Why?

$\Rightarrow$ numerous strongly coupled systems in focus of interest









http://www.gsi.de/forschung/fair_experiments/CBM/1intro.html
http://www.pro-physik.de/phy/physik/dossier.html?qid=1200979
http://www.iap.tu-darmstadt.de/apqneu/research/ultracold-quantum-gases
A.Geim, A.H.MacDonald, Phys. Today 60, 35 (2007)

QED_3 vs. QCD...

...share several features in common

- strongly coupled
- asymptotically free
- show dynamical chiral symmetry breaking
- (pseudo-)conformal window $\Rightarrow$ e.g. Sannino Acta Phys. Polon.B 40(2009)
- confining (though QED_3 only trivially due to dimensionality)

...but QED_3 remains a lot "simpler" due to

- its Abelian nature which means
- no gauge boson self-interaction (no ghosts)
- its super-renormalizability

QED_3

$\rightarrow$ study application as a low-energy effective theory for HTSs

HIGH T_C SUPERCONDUCTING MATERIALS

Discovery of high-temperature superconductivity in 1986.

J. G. Bednorz and K. A. Müller, Zeitschrift f. Physik B Condensed Matter **64***, no. 2, 189 (1986).*

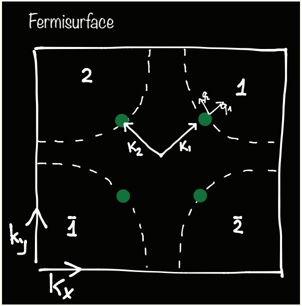
- critical temperature > 77 K
- ceramical compounds
- need 'critical doping'
- non-superconducting phase is insulating anti-ferromagnetic

No proven theory to describe the phenomenon, yet.
$\rightarrow$ Which features would have to be described?

EXPERIMENTAL FEATURES OF HTS

Experiments with high temperature superconductors (HTSs) show:
H. Ding, M. R. Norman and J. C. Campuzano, Phys. Rev. B **54**, *R 9678 (1996)*

- energy gap function with "d-wave-symmetry"
  H. Ding, M. R. Norman and J. C. Campuzano, Phys.Rev. B **54**, *R 9678 (1996).*
- nodal quasiparticles
- linear energy dispersion relation at the nodes of the energy gap function



I. F. Herbut, Phys. Rev. B **66**, *094504 (2002).*

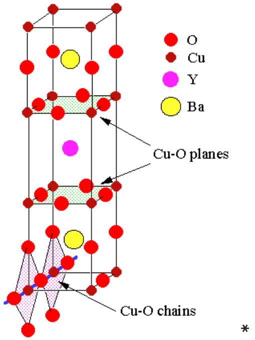
$\Rightarrow$ We get a hint to *QED*.

The Features

Further experimental input:

- superconducting CuO_2-planes
- inherent anisotropy in form of two different speeds of light
  $\rightarrow (c_s, \ v_f\ , v_\Delta)$

  (e.g.M. Chiao et al., arXiv: cond-mat/9910367; J. Mesot et al., Phys. Rev. Lett. **83***,840 (1999).)*
- theoretically recover v_f, v_Δ from Fermi- and gapenergy



$\Rightarrow$ Superconducting dynamics in (2+1)-dimensions

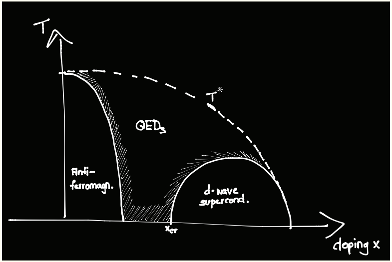
*http://www.ch.ic.ac.uk./rzepa/mim/century/html/ybco_text.htm

A POSSIBLE SOLUTION: QED$_3$

How to describe the transition to the insulating phase?

mapping

pseudogap phase
$\overset{*}{\leftrightarrow}$
chiral symmetric phase



*M. Franz, Z. Tešanović and O. Vafek, PRB **66**,054535 (2002).*

Reformulate task:

→study order parameter of the transition
→find critical quantities

** M. Franz and Z. Tešanović , Phys. Rev. Lett. **87**, 257003 (2001) & Phys. Rev. Lett. **84**, 3 (2000);O. Vafek, A. Melikyan, M. Franz and Z. Tešanović, Phys. Rev. B **63**, 134509; M. Franz, Z. Tešanović and O. Vafek, Phys. Rev. B **66**,054535 (2002); I. F. Herbut, Phys. Rev. B **66**, 094504 (2002).*

How To Include Anisotropy?

Define a metric-like quantity...

$$(g^{\mu\nu}) = \begin{pmatrix} 1 & & \\ & (v_F)^2 & \\ & & (v_\Delta)^2 \end{pmatrix}$$

..and modify the Lagrangian:

$$\mathcal{L}^{aniso} = \sum_{j=1,2} \bar{\Psi}_j \left\{ \sum_{\mu=0}^{2} \gamma_\nu \sqrt{g^j}_{\nu\mu} \left(\partial_\mu + \mathrm{i} a_\mu\right) \right\} \Psi_j$$

The Dyson-Schwinger Equations..

- having the Lagrangian $\mathcal{L}$ of a given theory,
- derive the Green's functions' equation of motion $\leftrightarrow$ functional derivatives of the full generating functional

$$
\begin{aligned}
0 &= \int \mathcal{D}\phi \frac{\delta}{\delta J} \mathcal{Z}[J] \\
\mathcal{Z}[J] &= \exp\left(\int d^4x \mathcal{L} - \int \phi \cdot J \right)
\end{aligned}
$$

- obtain an infinite tower of coupled integral equations for the propagators,...

- in practice: need truncation scheme

Why Dyson-Schwinger?

Lattice Gauge Theory

- ☺ Ab-initio
- ☹ Sign problem (Only for small μ)

Effective field theories

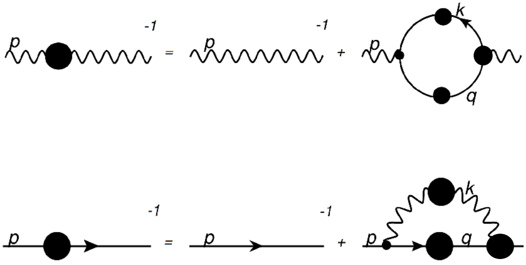
- ☺ No sign problem
- ☹ Effective degrees of freedom

FUNctional Methods, e.g. Dyson-Schwinger

- ☺ No sign problem
- ☺ Original degrees of freedom
- ☹ Truncation scheme needed

$\rightarrow$ **write DSEs down for our specific case**

The Dyson-Schwinger Equations



- Landau gauge
- study within finite volume by introduction of boundary conditions
- have to choose appropriate truncation

$$S_{F,i}{}^{-1}(p) = S_{0i}{}^{-1}(p) + Z_1\, e^2\, T \sum_{n_t} \int \frac{d^2 q}{(2\pi)^2} \Big(\sqrt{g}_{i,\mu\alpha} \gamma^\alpha S_{F,i}(q) \sqrt{g}_{i,\nu\beta} \Gamma_i^\beta(q,p)\, D_{\mu\nu}(k) \Big),$$

$\rightarrow$ with the tools at hand, return to the problem...

THE TASK

study dynamical generation of mass via $B(p)$

$\rightarrow$Know: depends on number of fermion flavours
$\rightarrow$Effects of finite temperature?
$\rightarrow$Look at B_{max} depending on T, v_f, v_Δ, N_f
$\rightarrow$Know from $N_{f,c}$ if the 'physical' system is in the chirally broken or symmetric phase

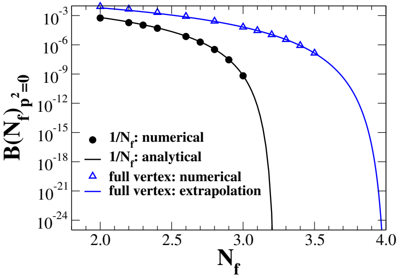
Probe the anisotropic finite temperature plane for N_f^{crit}**.**

$\rightarrow$ But first: What was the starting point?

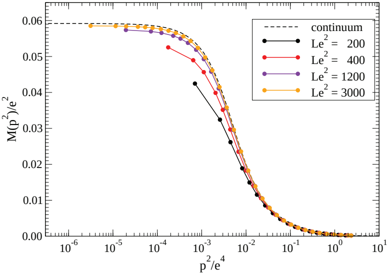
Isotropic Continuum Results To Build On ..

... within the Dyson–Schwinger framework

- continuum studies of isotropic equations at zero temperature
  Fischer, Alkofer, Maris PRD 70(2004)



- finite volume studies of isotropic equations at zero temperature
  Goecke,Fischer,Williams PRB 79(2009)



$\Rightarrow$ **extended to finite anisotropies**

WHAT WAS DONE...

...within other approaches

- lattice calculations for certain values of anisotropy

  Hands, Thomas PRB 72(2005), PRB 75(2007)

- small anisotropy calculations within roughly approximated DSE framework + RG studies

  Franz,Tesanovic,Vafek PRB 66 (2002), PRB 65 (2002); Franz, Tesanovic PRL 87(2001); Herbut PRB 66(2002),PRL 88(2002);Lee, Herbut PRB 66, 094512 (2002)

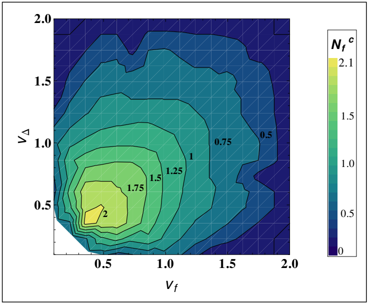
- calculations within gauge boson exchange model

  Concha, Stanev, Tesanovic PRB 79(2009)

$\Rightarrow$ **checked, better understood**
$\Rightarrow$ **generalization to finite temperatures AND finite anisotropies**

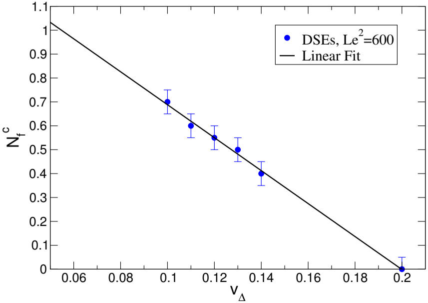
The Phase Diagram

T=0



JAB, Fischer, Williams: Phys. Rev. B 84 (2011)

"physical points"



JAB, Fischer, Williams: Phys. Rev. B 84 (2011)

$\Rightarrow$ Decreasing N_f^c with increasing anisotropy v_f and v_Δ.
$\Rightarrow$ How does temperature affect these findings ?

Critical Scaling

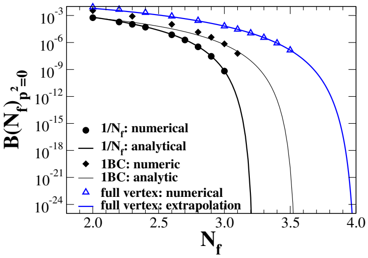
Reminder: Features of QED_3

- strongly coupled
- asymptotically free (in an RG sense)
- chiral phase transition to pseudoconformal phase

T= 0 : Miransky scaling

$$k_{SB} \sim \exp\left(\frac{-2\pi N_f}{\sqrt{N_f(N_f - N_f^c)}}\right)$$

Gusynin, Miransky,Shpagin PRD 58 (1998); Miransky,Yamawaki, Mod. Phys. Lett. A4 (1989), PRD 55, 5051(1997)



Fischer, Alkofer, Maris PRD 70, 073007 (2004)

"Beyond Miransky Scaling"

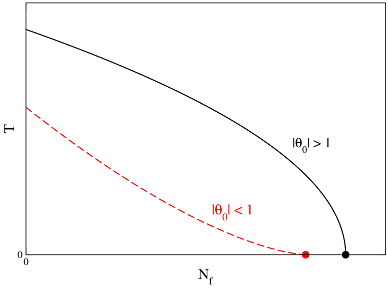
...universal power-law corrections to Miransky scaling

$$k_{SB} \quad \sim \quad \theta(N_f^C - N_f)|N_f - N_f^C|^{-\frac{1}{\Theta_0}} \exp\left(\frac{\pi}{2\epsilon\sqrt{\alpha_1|N_f^C - N_f|}}\right)$$

Braun, Fischer, Gies, PRD 84, 034045 (2011)

- including running coupling effects
- critical exponent determines scenario

connection between scenarios?

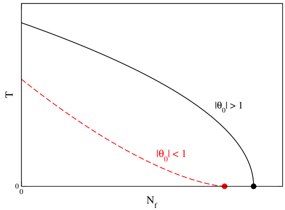


Critical Scaling

generic scaling

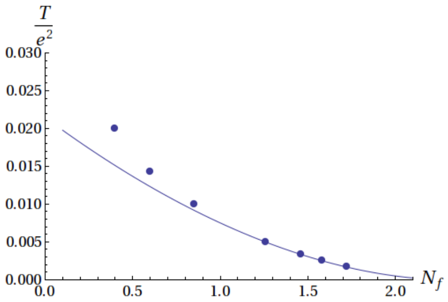
to leading order:

$$T_{cr} \sim k_0 |N^c_{f,0} - N_f|^{-\frac{1}{\Theta_0}}$$



v_f=0.8

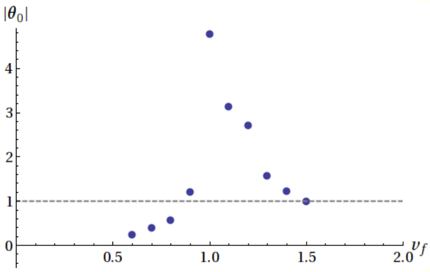
..as an example



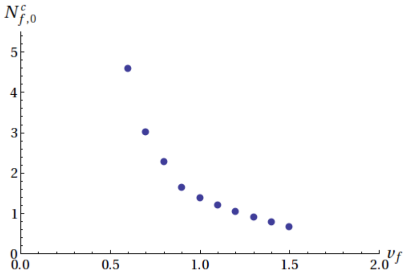
- critical Exponent smaller than 1
- small power-law corrections

The Critical Exponent

Extraction of critical exponent and $N_{f,0}^c$ from phase diagram



JAB, Fischer, Phys. Lett. B 718(2012)

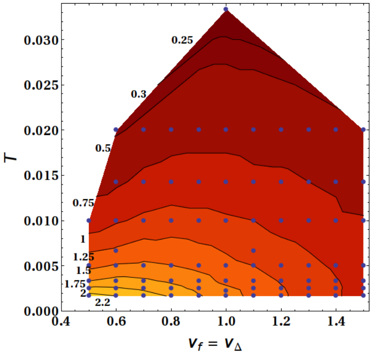


JAB, Fischer, Phys. Lett. B 718(2012)

$\Rightarrow$ Anisotropy provides a parameter to drive theory from regime $|\Theta_0| < 1$ to $|\Theta_0| > 1$.

Phasediagram at finite temperature

Finally generalize for all anisotropy values



JAB, Fischer, Phys. Lett. B 718(2012)

- color-coded N_f
- lines to guide the eye
- grid of calculated points
- "off-diagonal" anisotropies possible, here omitted for convenience
- magnitude of N_f smaller due to finite volume

N_f^c decreases with increasing T at fixed anisotropy v_f.

Pre-Conclusion

Summary

- phase diagram for N_f^c in dependence of anisotropy and temperature
- critical exponent in dependence on anisotropy
- anisotropy provides a parameter to modify the generic scaling behavior of QED$_3$

What is left to do...

- extrapolation to infinite volume
- solve photon equation explicitly
- apply knowledge to graphene, anisotropic QCD...

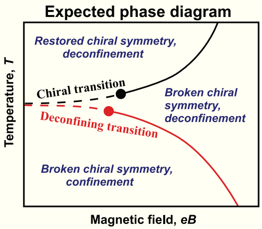
Why are magnetic fields interesting?

- Heavy Ion Collisions
- Cosmological electroweak phase transition
- Neutron Stars
- Condensed Matter Systems
- (Color-) Superconductors

(first three items:) $\sim 10^{14-16}$ Tesla

Open Issues

- Modification of the (QCD) vacuum structure ?!
- Chiral Magnetic Effect ?
- Magnetic catalysis ?



Mizher, Chernodub, Fraga: PRD 82,105016 (2010)

A First Step: Magnetic Catalysis

Enhancement of chiral symmetry breaking due to an external magnetic field

Gusynin, Miransky, Shovkovy: PRL 73,26 (1994)

Issues...

- realization of magnetic field ?
- mechanism driving enhancement ?
- effect of approximations/truncations ?
- origin of discrepancies between lattice and effective model results ?

Aim ...

$\Rightarrow$ obtain complementary information by non-perturbative finite volume study in a Dyson–Schwinger framework

Magnetic Background

Implementation

- Abelian field in z-direction

$$A_\mu = (0, \mathcal{B}\, z, 0, 0)^{\mathsf{T}}$$

$$\Rightarrow \mathcal{F}_{\mu\nu,ab} = \mathcal{F}_{\mu\nu,ab} + f_{\mu\nu} \cdot \mathbb{1}_{ab}$$

- Principle of minimal coupling

$$\mathrm{D}_\mu = \partial_\mu - ieA_\mu \qquad\qquad \mathcal{D}_\mu = \mathrm{D}_\mu + \mathrm{i}g\, t^a\, \mathcal{A}_{\mu,a}$$

- Leads to the underlying Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\not{\mathcal{D}} - m)\psi + \frac{1}{4}\mathcal{F}_{\mu\nu}\,\mathcal{F}^{\mu\nu}$$

Setting the equations

- work in the Dyson–Schwinger approach
- propagator from Green's function identity:

$$(\mathrm{i}\not{\Pi} - m)\, S(x, x') = \delta(x - x')$$

  with $\Pi_\mu = \partial_\mu - eA_\mu$
- 'standard' approach: expand in plane wave functions
  find diagonal propagator in momentum space
- challenge: $[\Pi_\mu, p_\nu] \neq 0$
  $\rightarrow$'standard' not applicable

Following Ritus' method

...to obtain the (inverse) propagator in momentum space

V.I. Ritus: Annals of Phys. 69,555 (1972)

Ritus Method

The Idea

- Observation I:
  S can only depend on scalar structures built from γ^μ contracted with $\Pi_\mu, F_{\mu\nu}, ...$
- Observation II:
  $[(\not{\Pi})^2, S(x, x')] = 0$

The Procedure

- Use *eigen*functions of $\not{\Pi}^2$ to diagonalize propagator
- End up with 'modified' propagator
  diagonal in momentum space
  depending on special subset of momenta

$$
\begin{aligned}
(\mathrm{i}\not{\Pi} - \mathrm{m})\, S(x, x') &= \int \mathrm{d}p\, \mathbb{E}_p\, (\bar{\not{p}} - \mathrm{m}) S(p)\, \bar{\mathbb{E}}_p \\
&\stackrel{!}{=} \delta(x - x')
\end{aligned}
$$

The Propagator

- Diagonalization procedure shows

$$S(\bar{p}) = (\bar{\not{p}} - \mathrm{m})^{-1}$$

- With momenta given by

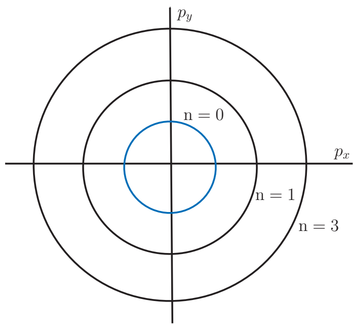
$$\bar{p} = (p_0, 0, \sqrt{k}, p_z)^{\mathrm{T}}$$

- And $\sqrt{k}$ encoding the particles' Landau Levels

$$\sqrt{k} = \sqrt{|e\mathcal{B}|(2\mathrm{n}+1) + \sigma e\mathrm{B}\, sgn(e\mathcal{B})}$$

- States per unit area:
  $\frac{|e\mathcal{B}|}{2\pi}$ for n = 0
  $\frac{|e\mathcal{B}|}{\pi}$ for n $\geqslant 1$

The Landau Levels

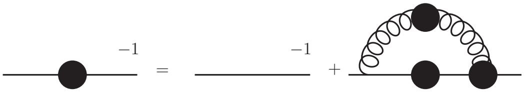


- Lowest Landau level approximation (LLLA) : n = 0 (spin polarized state)
- Dimensional reduction $n = 0 \Rightarrow \sqrt{k} = 0$

$$\bar{p} = (p_0, 0, 0, p_z)^T$$

- NO application of the Mermin–Wagner theorem $\rightarrow$ gluons are 4-dimensional
- Drawback/Limit of LLLA: $\mathcal{B} \rightarrow 0$
- Beyond LLLA: include $n = 1, 2, ...$

THE DYSON-SCHWINGER EQUATIONS



with the dressed propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\,(A_0(\bar{p})\gamma^0\bar{p}_0 + A_2(\bar{p})\gamma^2\bar{p}_2 + A_3(\bar{p})\gamma^3\bar{p}_3)$$

- $A_1(p)$ and $A_2(p)$ are not accessible in LLL approximation
- Gluonic input from lattice calculations

  $\rightarrow$ Fischer, Maas, Pawlowski: Annals Phys.324 (2009)

- Landau gauge
- Modified bare vertex approximation
- Solution in a finite volume $\rightarrow$ (1+1) torus

Magnetic Fields In A Finite Volume

Magnetic Flux

$$\int dx_\mu A_\mu = \mathcal{B} \cdot \mathrm{F}$$

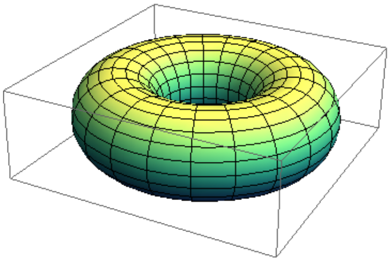
$$\int dx_\mu A_\mu = \mathcal{B} \cdot (\mathrm{F} - L_x L_y)$$

Charged Particles

$$\exp(\mathrm{i} q \mathcal{B} \mathrm{F}) \stackrel{!}{=} \exp(\mathrm{i} q \mathcal{B}(\mathrm{F} - L_x L_y))$$

$$\Rightarrow q\mathcal{B} = \frac{2\pi}{L_x L_y} b$$

with b= 0,1,2...



Magnetic Catalysis

Full Quark Propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\, A_\mu(\bar{p})\gamma^\mu \bar{p}_\mu$$

Quantized B-Field

$$|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$$

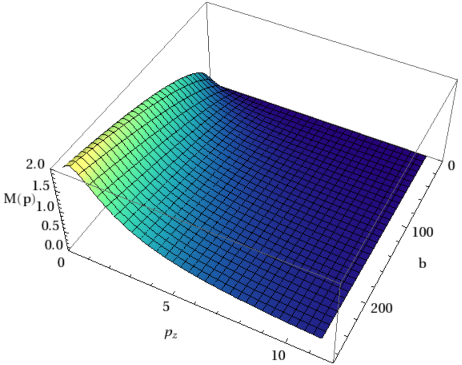
$$b \in [0, L_x \cdot L_y]$$

Typical Tori

- Box Length: 6 fm
- Mom. points: 8×33
- $\mathcal{B}_{max} \sim 1.8\ \mathrm{GeV}^2$

Dynamically Generated Mass

$$M(p) = \frac{B(p)}{A(p)} \qquad [M(p)] = GeV$$



Result

- Lowest Landau Level approximation ⇒
  Enhanced mass generation with increasing magnetic field

Preliminary

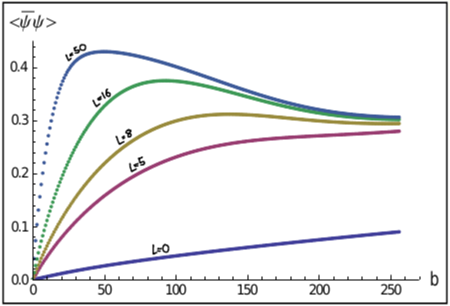
Chiral Condensate

$$\langle\bar{\psi}\psi\rangle_b \sim b \sum_{n_t,n_z} \frac{B_b(p)}{B_b(p)^2+(A_{0b}(p)\,p_0)^2+(A_{2b}(p)\,p_2)^2+(A_{3b}(p)\,p_3)^2}$$

Max. B-field

- $|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$
- $\mathcal{B}_{max} \sim 1.8 \text{ GeV}^2$

- Limit $\mathcal{B} \to 0$ is not reliable in LLLA

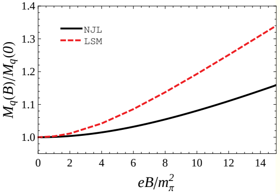


Result

- Including more Landau Level $\Rightarrow$ Non-monotonic chiral condensate with increasing magnetic field

Different Approaches...

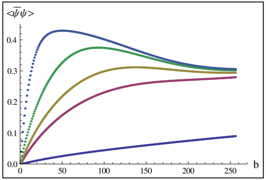
Effective Model Approaches



Ferrari, Garcia, Pinto: arXiv 1207.3714v2

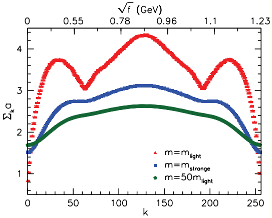
- Mostly LLLA
- Infinite volume
- Increasing chiral symmetry breaking with B

Dyson–Schwinger Approach



- Beyond LLLA
- Finite volume
- Saturation effects in chiral condensate

Lattice Gauge Theory



Bruckmann, Endrödi: PRD 84 (2011)

- Beyond LLLA
- Finite volume
- Saturation effects in chiral condensate

The Dual Condensate

Bruckmann and Endrödi,PRD 84, (2011)

- Fourier transform of the chiral condensate $\langle\bar{\psi}\psi\rangle_b$
- Dual variables: magnetic field quanta and the (by the loop) enclosed area

$$\widetilde{\Sigma}(s) = \frac{1}{S_{\mu\nu}} \sum_{b=0}^{b_{max}} e^{-2i\pi bs/N_{\mu\nu}} \langle\bar{\psi}\psi\rangle_b$$

- 'Dressed Wilson loops', since for $m \to \infty$: Contact to Wilson loops

$$\widetilde{\Sigma}(s)_{m\to\infty} \sim \exp(-\sigma A)$$

- Observable for confinement, connected with chiral symmetry breaking

$\rightarrow$ **probe for (spatial) string tension** σ

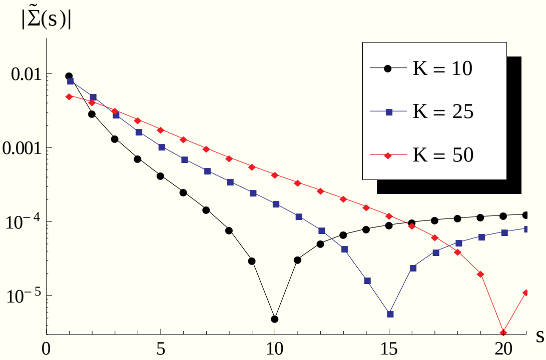
Preliminary

Dual Condensate/Dressed Wilson loop

$$\tilde{\Sigma}(s) = \frac{1}{S_{\mu\nu}} \sum_{b=0}^{b_{max}} e^{-2i\pi bs/N_{\mu\nu}} \langle\bar{\psi}\psi\rangle_b$$

- beyond LLLA
- finite FT

K	$\sqrt{\sigma}$ (GeV)
10	0.39
25	0.32
50	0.27



→ **Decay with expected area-law**

Summary

- Concept of constant external magnetic fields
- Effects on particles' propagators and momenta
- Discussion of (Lowest) Landau Level approximation
- Chiral condensate and dressed Wilson loop
- Still many 'technical' open questions...

What is left to do ...

- Volume & Charge dependence studies

  $\longrightarrow$ D'Elia, Negro, PRD 83 (2011)
- Finite temperatures

  $\longrightarrow$ Bali et al. JHEP 1202 (2012), Bali et al. arXiv:1206.4205, Fukushima, Pawlowski arXiv:1203.4330
- Finite chemical potential

Thank you for your attention!

Questions??