Non-linear flow response and plane correlations

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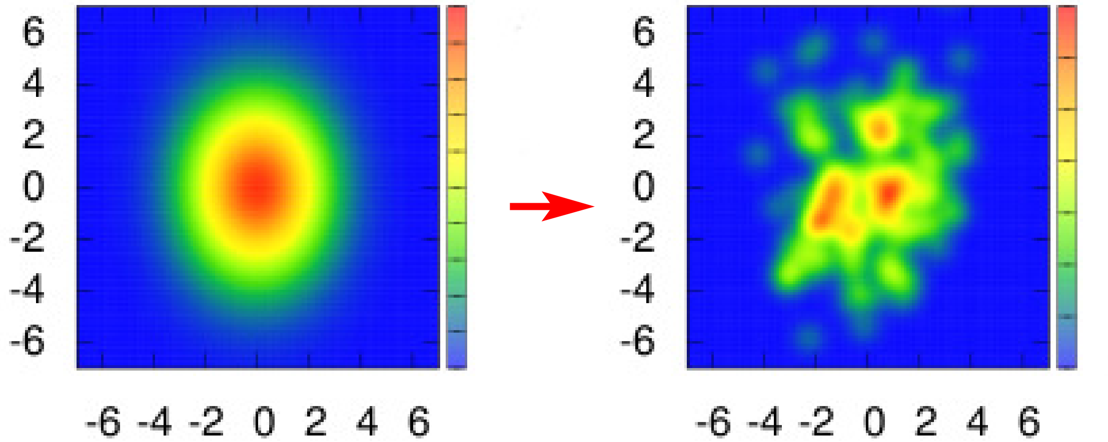
Collaborate with Derek Teaney

Outline of the talk

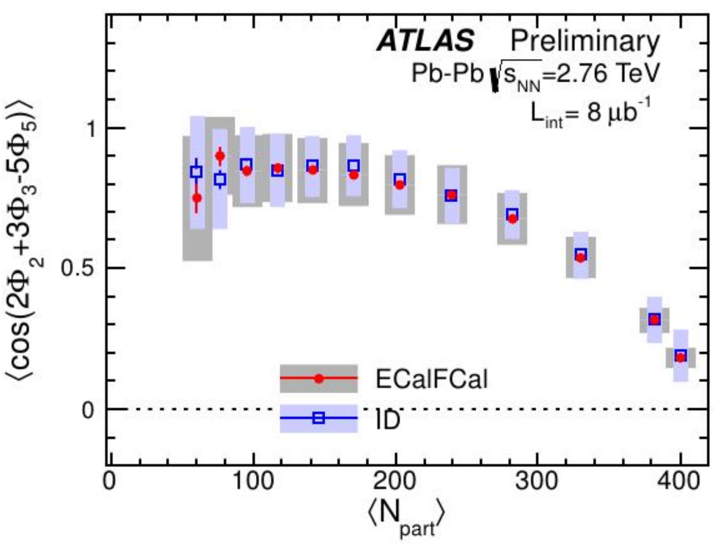
- Initial state with fluctuations: cumulants or moments ?
- Linear and non-linear flow response.
  1. Details of non-linear response formalism for hydrodynamics.
  2. Properties of non-linear flow response: dependence on p_T, η/s and centrality.
  3. Some discussions on the origins of non-linearity.
- Understand the observed plane correlations (ATLAS).

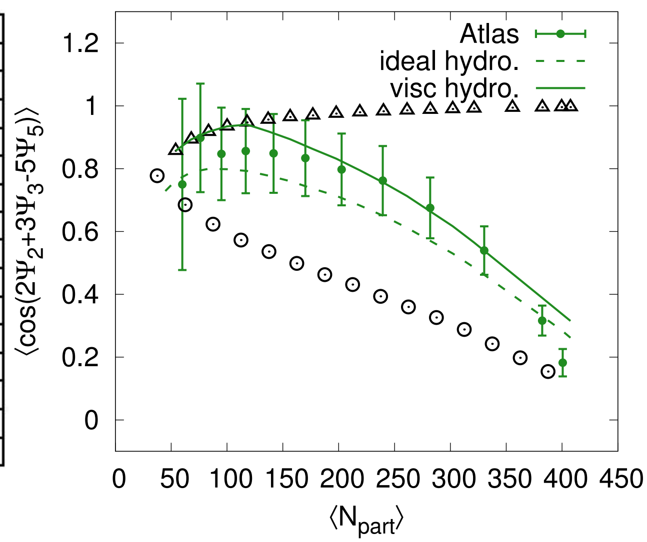
Fluctuated initial state and heavy ion collisions

Fluctuations in initial state – Fluctuated IC, event-by-event hydro. ....



More fruitful observation in experiment.

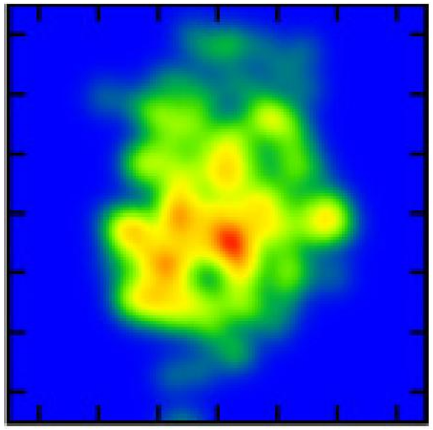




<u>Our notation</u>:

- $\Phi_n \rightarrow$ participant plane angle of the initial state in coordinate space.
- $\Psi_n \rightarrow$ reaction plane angle of the final state in momentum space.

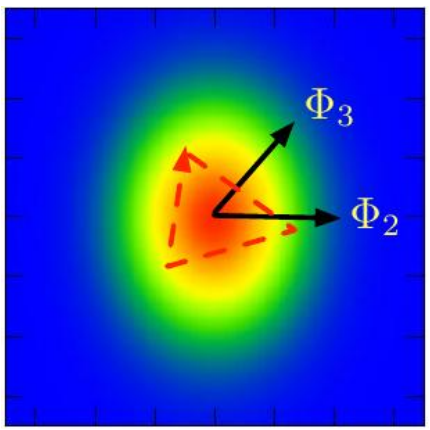
- If $\Psi_5(v_5)$ entirely comes from 5th order anisotropy in initial condition,



  - $\Psi_5 = \Phi_5$

    $\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) = \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)$

- If $\Psi_5(v_5)$ is determined by the interplay of $\Psi_2(v_2)$ and $\Psi_3(v_3)$,

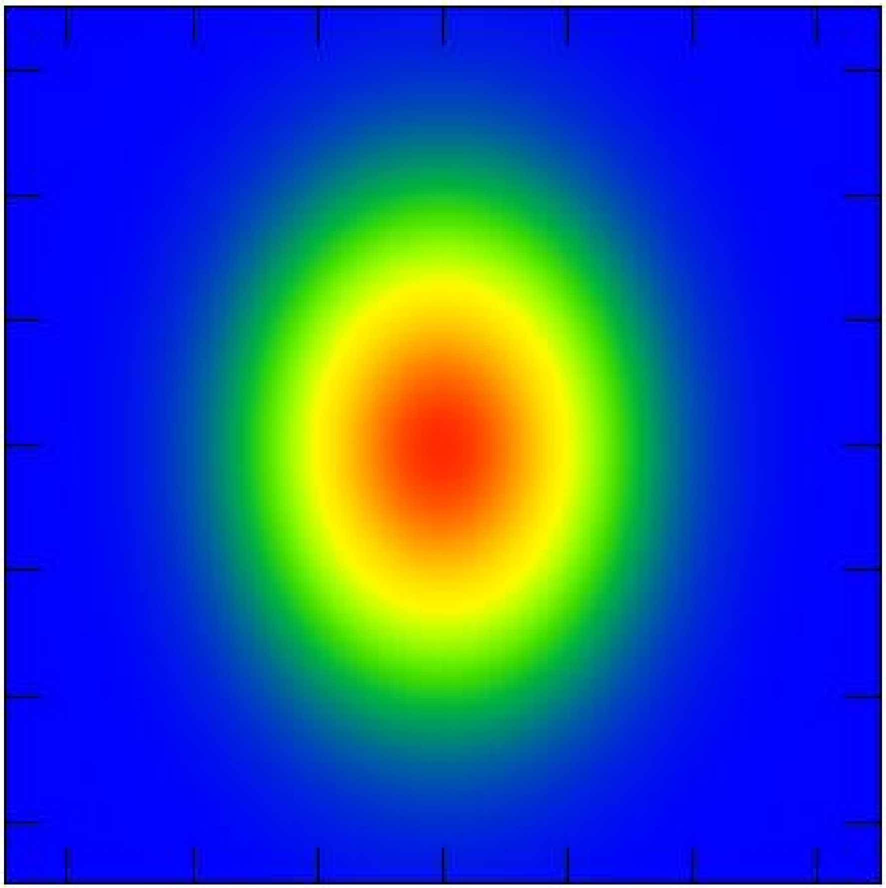


  - $\Psi_5 = 2\Psi_2 + 3\Psi_3$

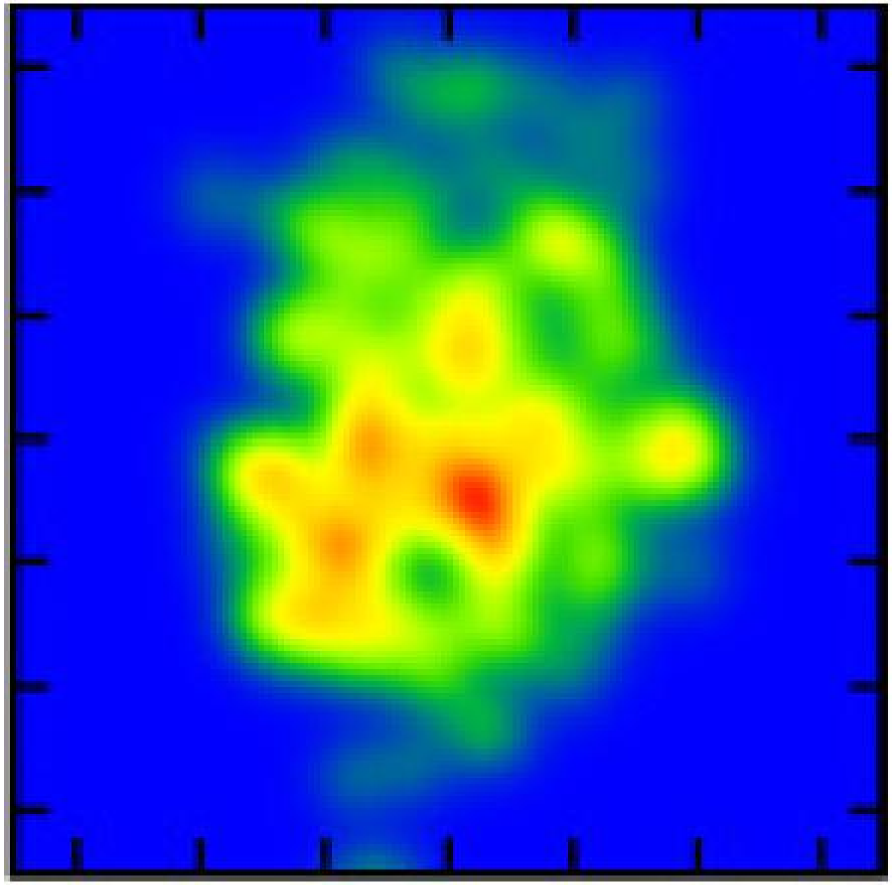
    $$\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) = 1$$

- Exp. is between these two scenarios, determined by the relative contributions.
- The senond scenario gives rise to the idea of non-linear flow response.

In heavy ion collisions: without/with fluctuations



$\approx$ Gaussian



$\approx$ Gaussian + fluctuations?

Cumulant expansion

What is cumulant expansion?

- Cumulant expansion: long wavelength expansion

$$
\begin{aligned}
e^{W(\mathbf{k})} &= \int d^2x e^{-i\mathbf{k}\cdot\mathbf{x}} \rho(\mathbf{x}) \\
\rho(\mathbf{x}) &= \int d^2k e^{i\mathbf{k}\cdot\mathbf{x}} e^{W(\mathbf{k})} = \int d^2k e^{i\mathbf{k}\cdot\mathbf{x}} \exp\left[\sum W_{n,m} k^m \cos(n\phi_k)\right] \\
&= \text{Gaussian} + \underbrace{\text{1st cumulant}}_{\propto W_{1,3}} + \underbrace{\text{3rd cumulant}}_{\propto W_{3,3}} + \underbrace{\text{4th cumulant}}_{\propto W_{4,4}} + \ldots
\end{aligned}
$$

Note:

- A simple rule to obtain $W_{n,m}$: with complex form $\bar{r} = x + iy$,

$$
W_{n,n} \sim \langle \bar{r}^n \rangle - \text{subtractions as in Wick's theorem}
$$

e.g.,

$$
\begin{aligned}
W_{4,4} \sim & \langle \bar{r}^4 \rangle - 3\langle \bar{r}^2 \rangle^2 \\
\sim & \langle r^4 \cos 4\phi_r \rangle - 3\langle r^2 \cos 2\phi_r \rangle^2
\end{aligned}
$$

- convention: n,m are the angular and radial indices.

Definitions in cumulants (not moments!)

Definition of anisotropies and participant plane angle,

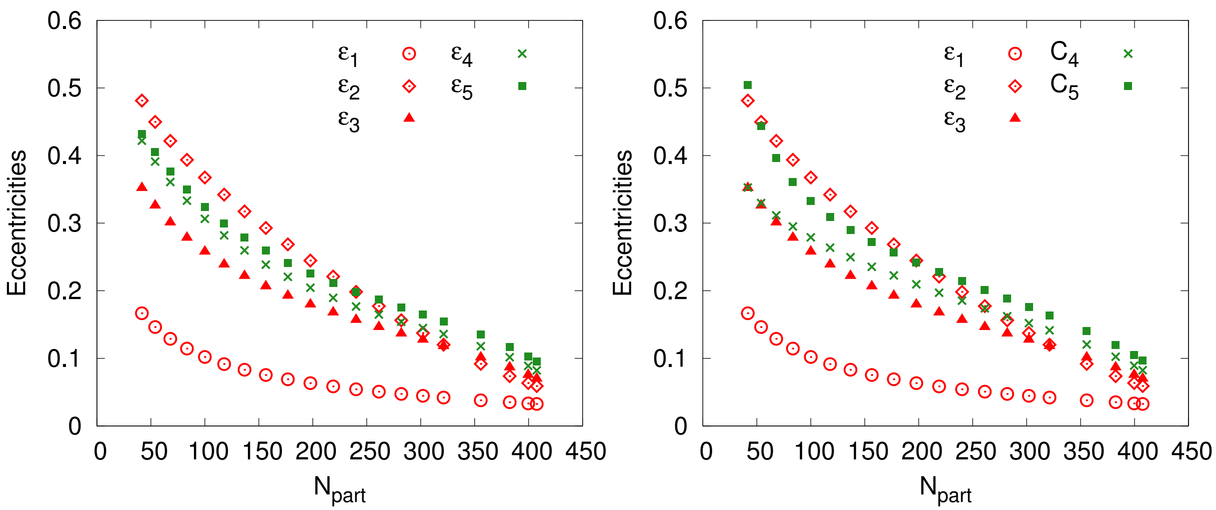
$$\varepsilon_n = \frac{\sqrt{(W^c_{n,n})^2 + (W^s_{n,n})^2}}{\langle r^n \rangle}, \qquad n\Phi_n = \mathrm{atan2}(W^s_{n,n}, W^c_{n,n}) + \pi$$

- $\langle r^n \rangle$ weight is naturally introduced.
- There exists $\varepsilon_{1,3} \propto W_{1,3}$, responsible to rapidity even v_1.[1]
- Note that for $n \geq 4$, we use $\mathcal{C}_n$ instead of ε_n, e.g.,

$$\mathcal{C}_4 \propto W_{4,4} \propto \langle \bar{r}^4 \rangle - 3\langle \bar{r}^2 \rangle^2$$

$$\varepsilon_4 \propto \langle \bar{r}^4 \rangle$$

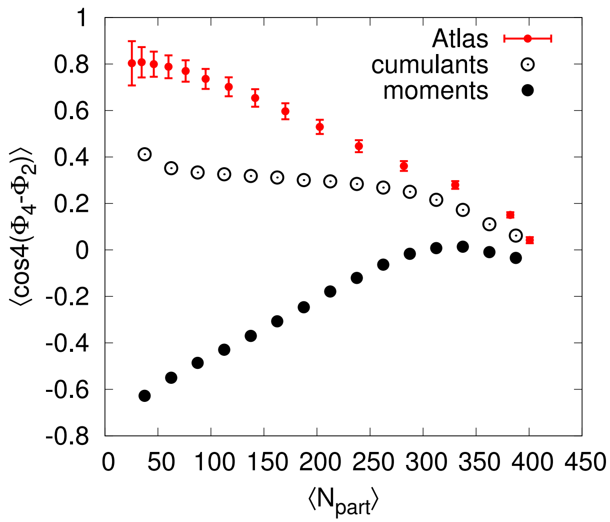
Cumulant def. $\neq$ Moment def. if $n \geq 4$.

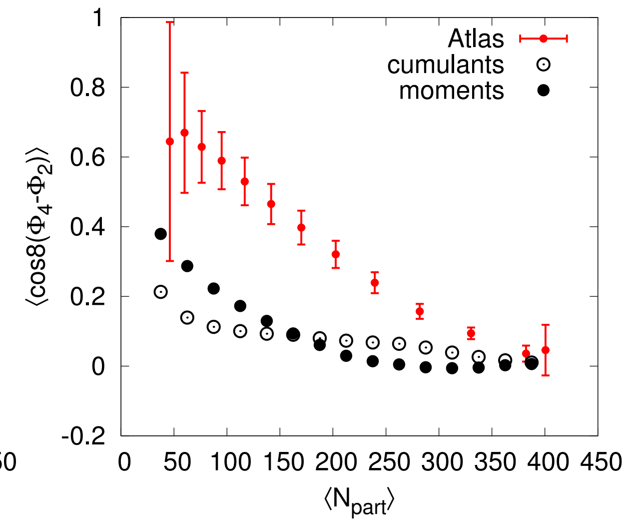


[1] See for example, ATLAS measurement in arxiv:1203.3087.

Why we take cumulant definition?

Initial (Φ_2, Φ_4) correlations:





E.g., consider a pure Gaussian distribution($\varepsilon_2 \neq 0$): $\rho(x, y) =$ Gaussian

- $\mathcal{C}_4 = 0$, but $\varepsilon_4 \sim \varepsilon_2^2 \neq 0$. & Correlation: Φ_4(moment def.) $- \Phi_2 = \pi/4$

Non-linearity is subtracted in cumulants definition:

1. No interference between different eccentricities based on definition.
2. Avoid double counting of non-linearity in initial state.

Cumulants definition is necessary for the study of non-linear flow generation.

Non-linear flow response in hydrodynamics

Linear flow response assumption

Spectrum of observed particles is decomposed into harmonics,

$$E\frac{d^3N}{dp^3} = \frac{N}{2\pi}\left[1 + \sum_n v_n e^{in(\phi_p - \Psi_n)} + c.c.\right]$$

1. When ε_n's are small enough, we assume flow linear response to ε_n.
2. Linear response assumes, in the spectrum,

$$\underbrace{v_n e^{-in\Psi_n}}_{\text{final state}} = \underbrace{\frac{w_n}{\varepsilon_n}}_{\text{linear resp.}} \times \underbrace{\varepsilon_n e^{-in\Phi_n}}_{\text{initial state}}$$

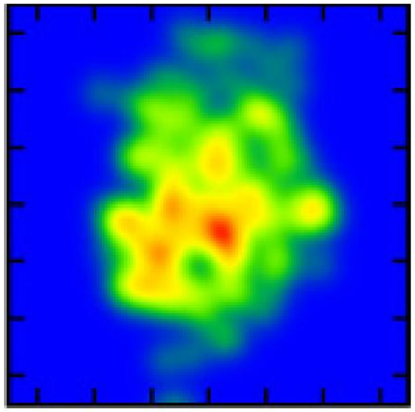
   with i. $v_n \propto w_n \propto \varepsilon_n$ and ii. $\Phi_n = \Psi_n$.
3. Linear response coefficient w_n/ϵ_n is independent of (ϵ_n, Φ_n).
4. Linear response coefficient is obtained by,

$$\text{Symmetric Gaussian} \quad + \quad \underbrace{(\varepsilon_n, \Phi_n)}_{\text{perturbation}} \quad \underbrace{\Longrightarrow}_{\text{hydro.}} \quad \frac{w_n}{\varepsilon_n}$$

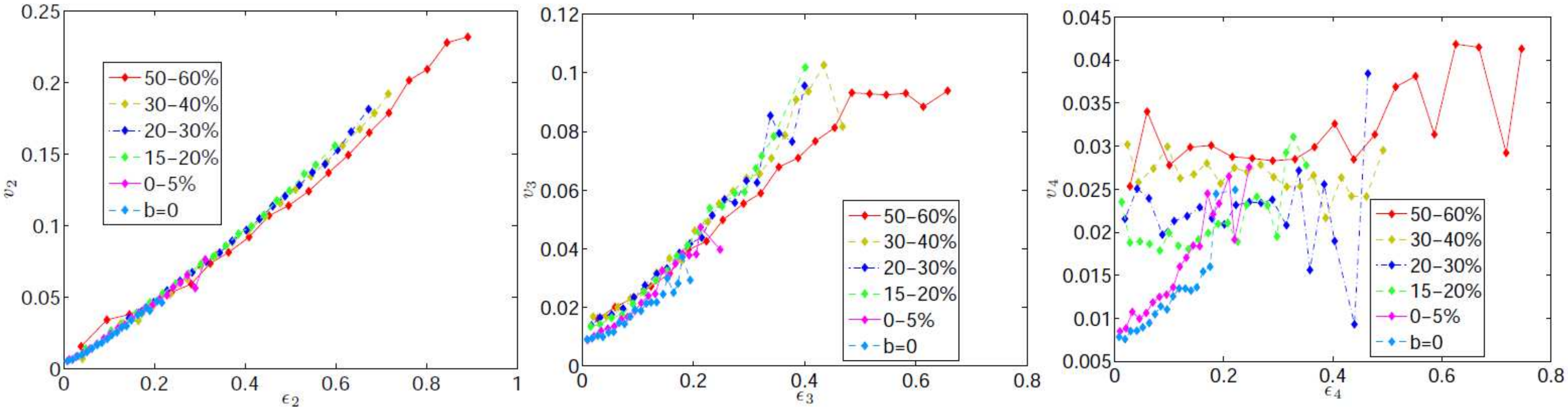
5. Observables of final state: $v_n\{2\} = \frac{w_n}{\varepsilon_n}\langle\langle\varepsilon_n^2\rangle\rangle^{1/2}$

Why we need non-linear flow response?

1. In a real collision event, all ε_n's simultaneously exist.



2. E-BY-E hydro. shows for $n \geq 4$ linear response assumption fails. [2]



3. We need to understand the flow generation in event-by-event hydro.

[2] (Heinz and Qiu, see also F. Gardim et al.)

Non-linear response assumption

Similar to linear response, assume non-linear response to product of ε_n's

1. Taking into account the combined deformation $\varepsilon_n\varepsilon_m$, especially $\underline{\varepsilon_2\varepsilon_n}$
2. Take n=5 as an example,

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}} + \underbrace{\frac{w_{5(23)}}{\varepsilon_2\varepsilon_3}}_{\text{non-linear resp.}} \times \underbrace{\varepsilon_2\varepsilon_3 e^{-i(3\Phi_3+2\Phi_2)}}_{\text{initial state}} + \dots$$

note 1. $w_{a(bc)} \propto \varepsilon_b\varepsilon_c$, and we expect $\frac{w_{a(bc)}}{\varepsilon_b\varepsilon_c} \sim \underline{\frac{w_b}{\varepsilon_b} \times \frac{w_c}{\varepsilon_c}}$.

note 2. We have cut at the order proportion to $\varepsilon_2\varepsilon_3$.

3. Non-linear response coefficient should be independent of (ε_n, Φ_n).
4. Non-linear response coefficient obtained from single-shot hydro.:

$$\text{Symmetric Gaussian} \quad + \quad \underbrace{(\varepsilon_2, \Phi_2), (\varepsilon_n, \Phi_n)}_{\text{perturbation}} \quad \underbrace{\Longrightarrow}_{\text{hydro.}} \quad \text{NL resp. coefficient}$$

5. Observables of final state, e.g., $v_5\{2\}$: v_5

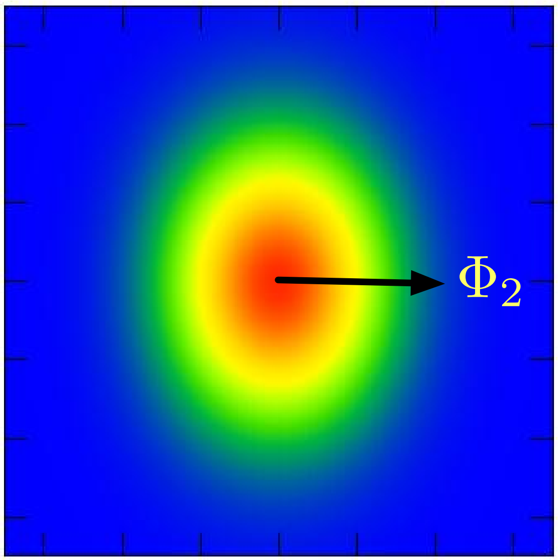
$$v_5\{2\} = \langle\langle |w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(3\Phi_3+2\Phi_2)}|^2\rangle\rangle^{1/2}$$

$$v_5 \sim (\text{linear}) + (\text{non-linear}) + (\text{interference} \propto \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)).$$

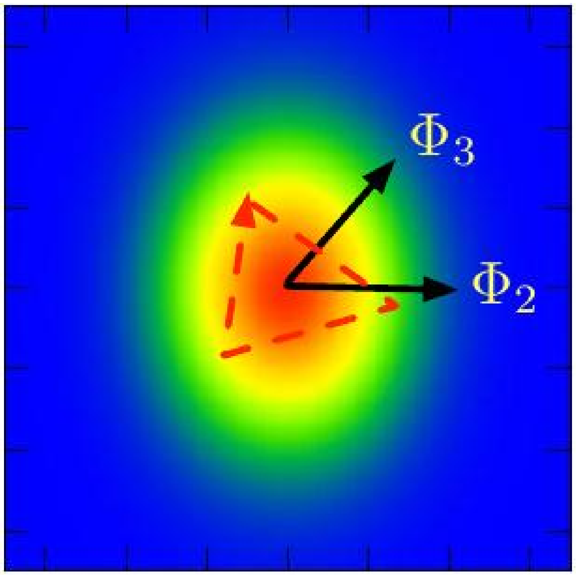
Non-linear response coefficients from single-shot hydro. simulations

Non-linear response coefficients from single-shot hydro.:

$w_{4(22)}$



$w_{1(23)}$ & $w_{5(23)}$

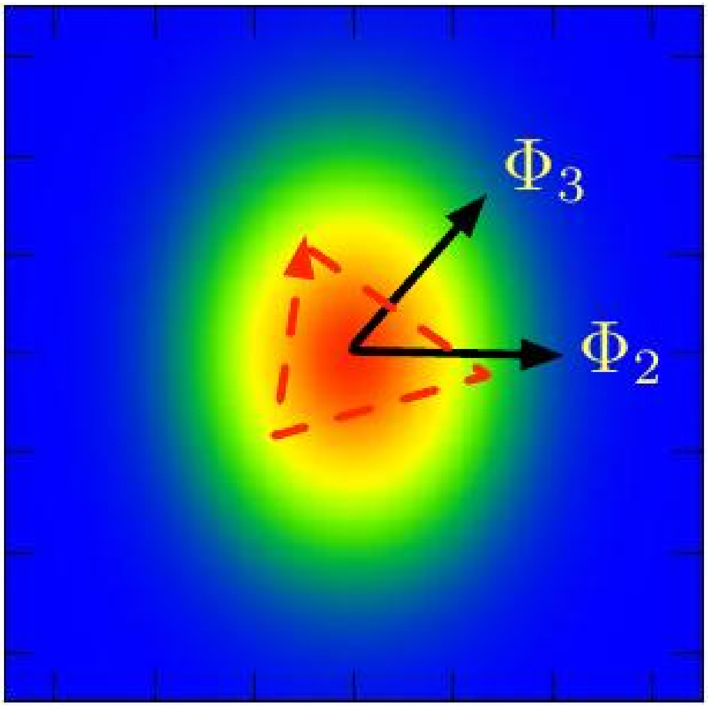


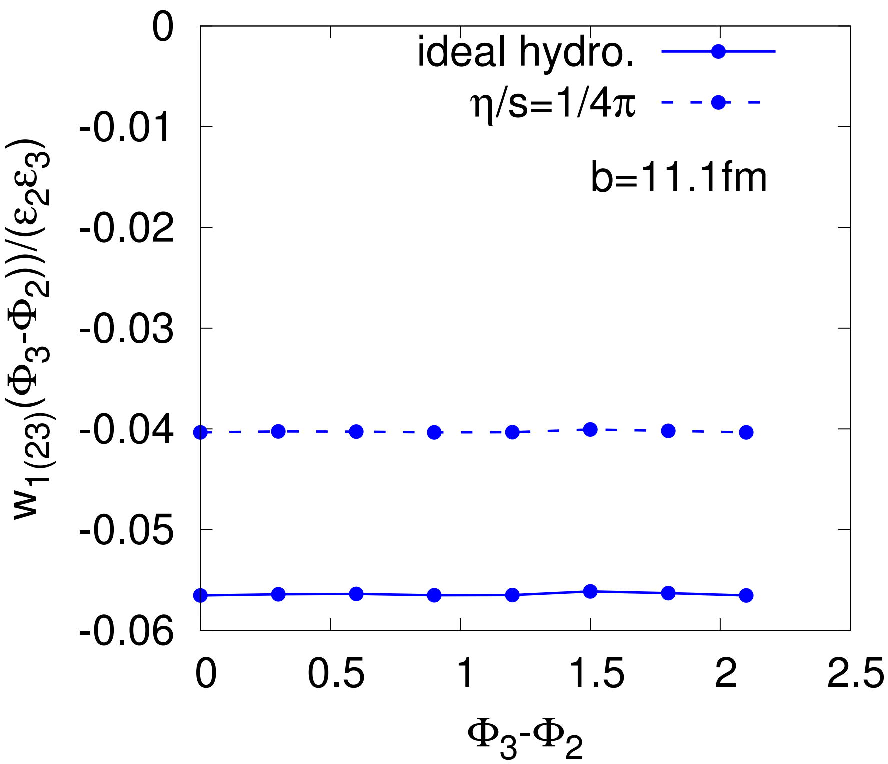
- Initial state: take $(\varepsilon_2 = 0.05, \mathcal{C}_4 = 0) \Longrightarrow \frac{w_{4(22)}}{\varepsilon_2 \varepsilon_2}$.
- Initial state: take $(\varepsilon_2 = \varepsilon_3 = 0.05, \mathcal{C}_5 = 0) \Longrightarrow \frac{w_{1(23)}}{\varepsilon_2 \varepsilon_3}, \frac{w_{5(23)}}{\varepsilon_2 \varepsilon_3}$.

Non-linear response coefficients do NOT depend on ε_n's, as long as ε_n's are small.

Non-linear response dependence on angle

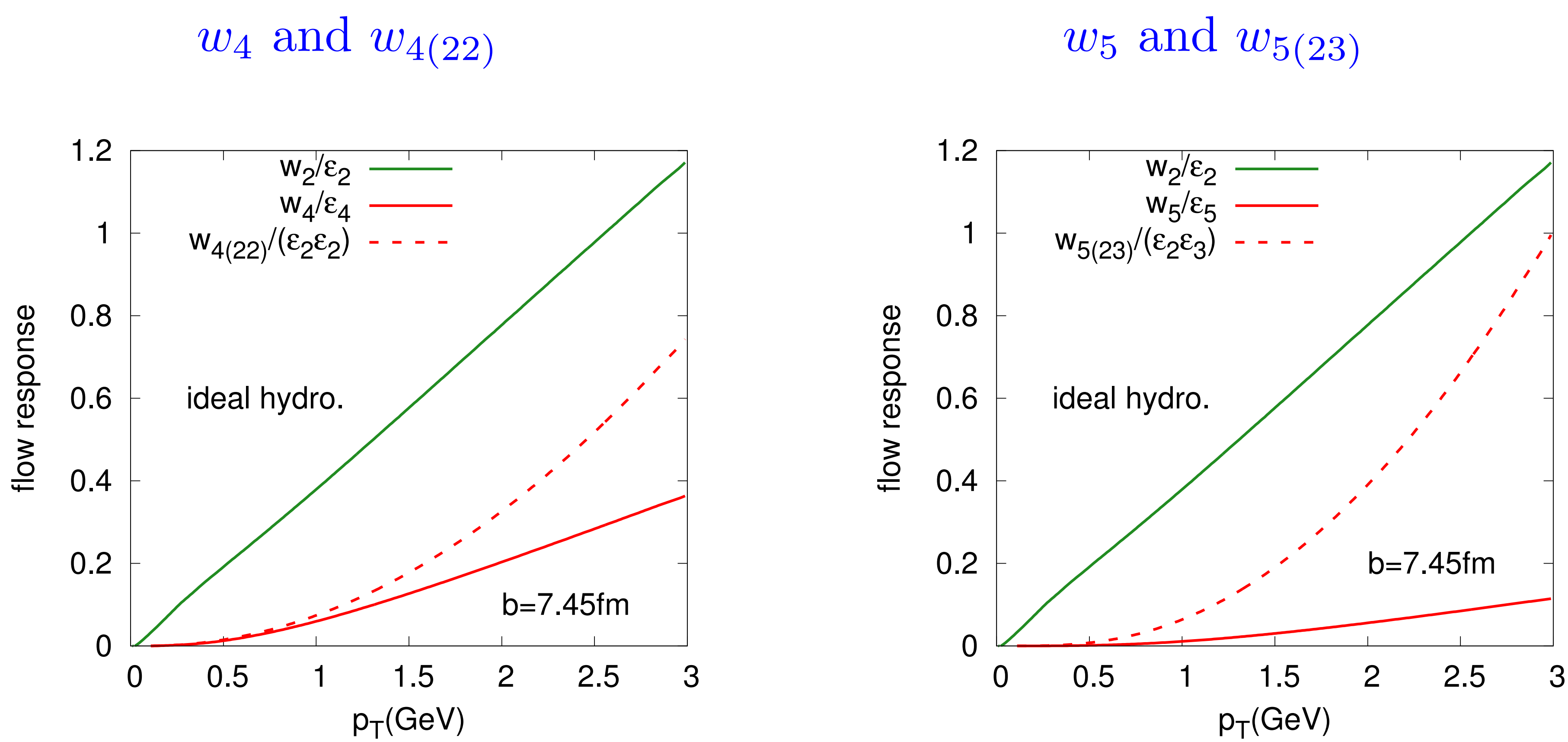
- For $w_{4(22)}$ the angle dependence is trivial.
- $\Phi_2 = \Phi_R$ fixed, while Φ_3 rotates $\rightarrow w_{1(23)}$, ($w_{5(23)}$ similar!).





Non-linear response coefficients are independent of relative angle.

Non-linear response dependence on p_T

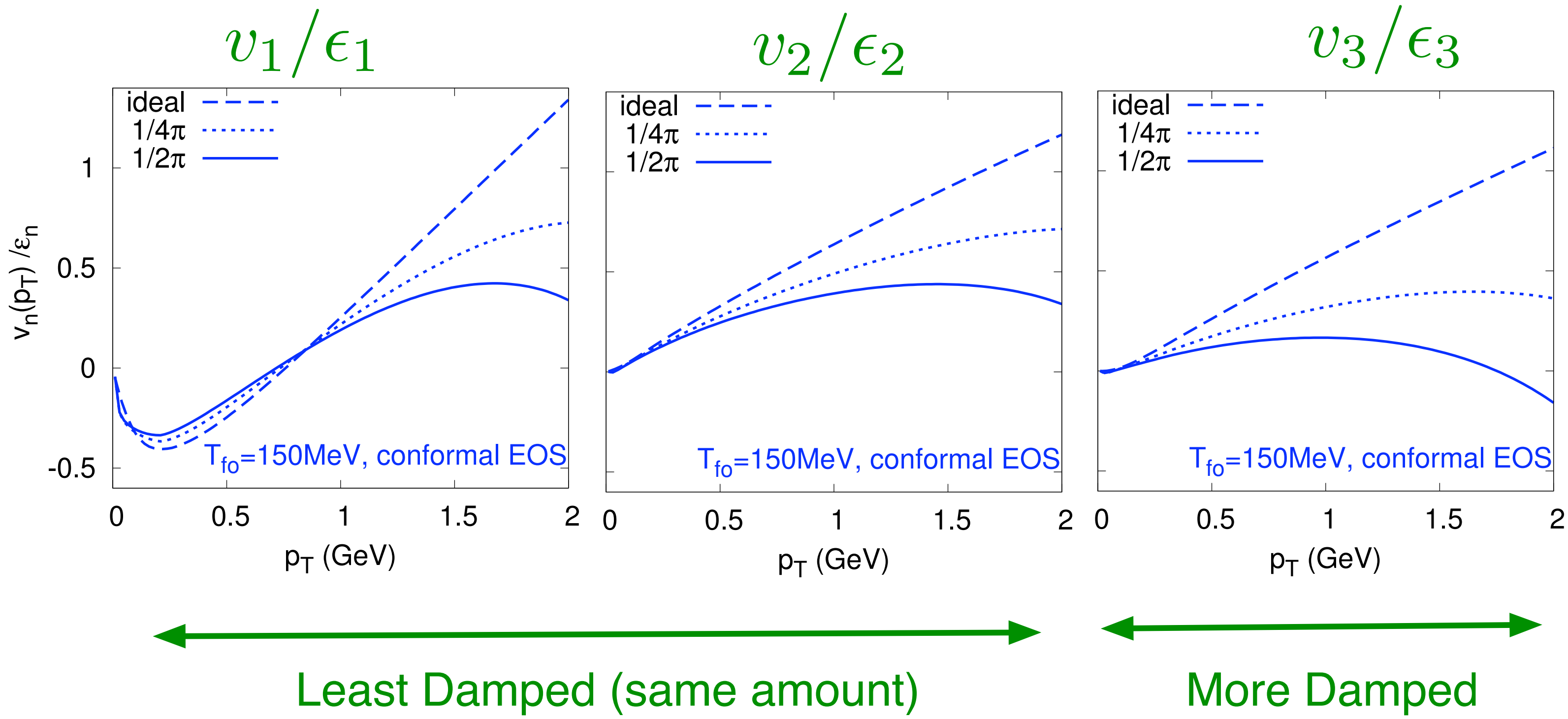


- Small p_T, non-linear response not distinguishable from linear response.
- Large p_T, linear response $\propto p_T$.
- Large p_T, non-linear response $\propto p_T^2$.

So, non-linear response becomes more significant for larger p_T.

Non-linear response dependence on viscosity

Knoweledge from linear response coefficients:



Viscous damping can be qualitatively described by[3] $e^{-\Gamma\tau}$,

$$\text{Damping rate: } \Gamma_{n,m}\tau_{\text{final}} \rightarrow -\frac{\Delta w_n}{w_n^i} \propto \underline{(n+m)^2} \times \frac{\eta}{s}$$

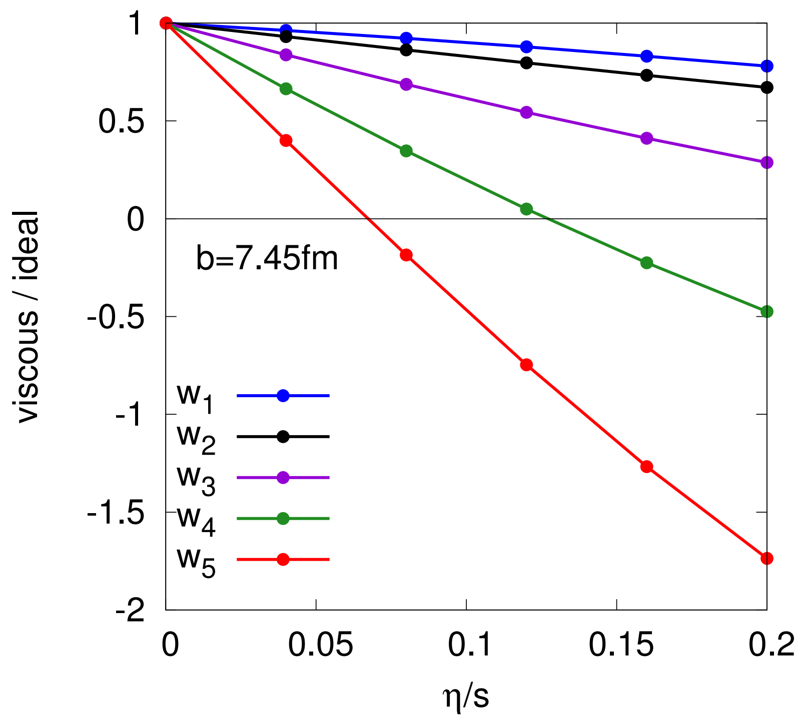
n, m are cumulant indices.

[3] A. Yarom and S. Gubser.

Damping rate as a function of η/s, linear response

One schematic way to understand the damping:

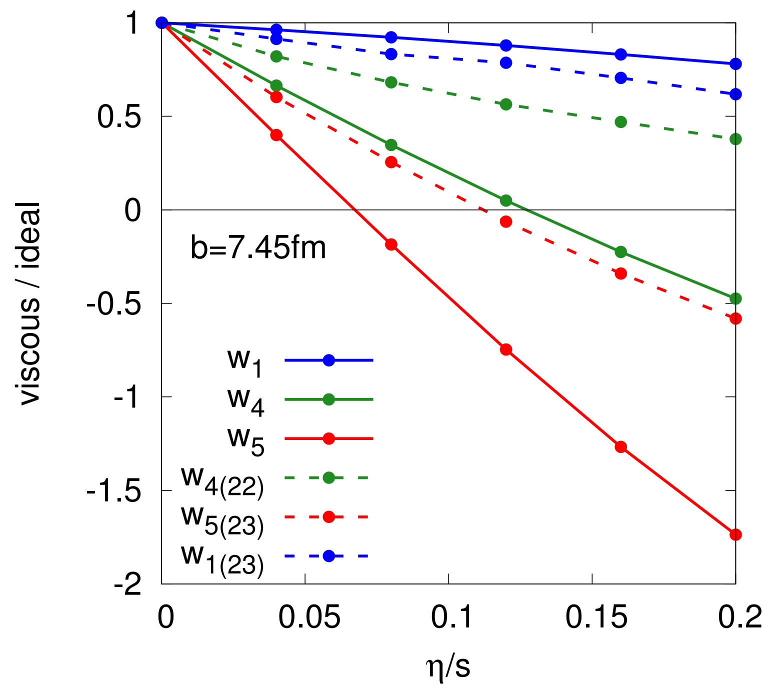
1. Since $\eta/s \propto l_{\mathrm{mfp}}$.
2. Geometric deformation has characterized length l_{ch} for each (n, m).
3. $l_{\mathrm{mfp}} \leq l_{\mathrm{ch}}$ such that hydro. is sensitive to the deformation.



- Damping rate of linear response: $\varepsilon_{1,3} \sim \varepsilon_{2,2} < \varepsilon_{3,3} < \mathcal{C}_{4,4} < \mathcal{C}_{5,5} < \dots$
- Viscous hydro. returns negative result for large η/s.

Damping rate as a function of η/s, non-linear response

Similarly for non-linear response: $e^{-\Gamma_{\text{non-linear}}\tau} = e^{-\Gamma_{\text{linear}}\tau} \times e^{-\Gamma_{\text{linear}}\tau}$



Then these relations are approximately true:

- damping of $w_{4(22)} \simeq 2 \times$ damping of $w_2 <$ damping of w_4.
- damping of $w_{5(23)} \simeq$ damping of w_2 + damping of $w_3 <$ damping of w_5.
- $w_{1(23)}$ is exceptional: constraint from momentum conservation.

Viscosity makes non-linear response more significant than linear response.

$$a^2 + b^2 < (a+b)^2$$

A simple summary on the viscous damping of flow response:

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- ✓ Viscous damping depends on harmonic order $(n+m)^2$.

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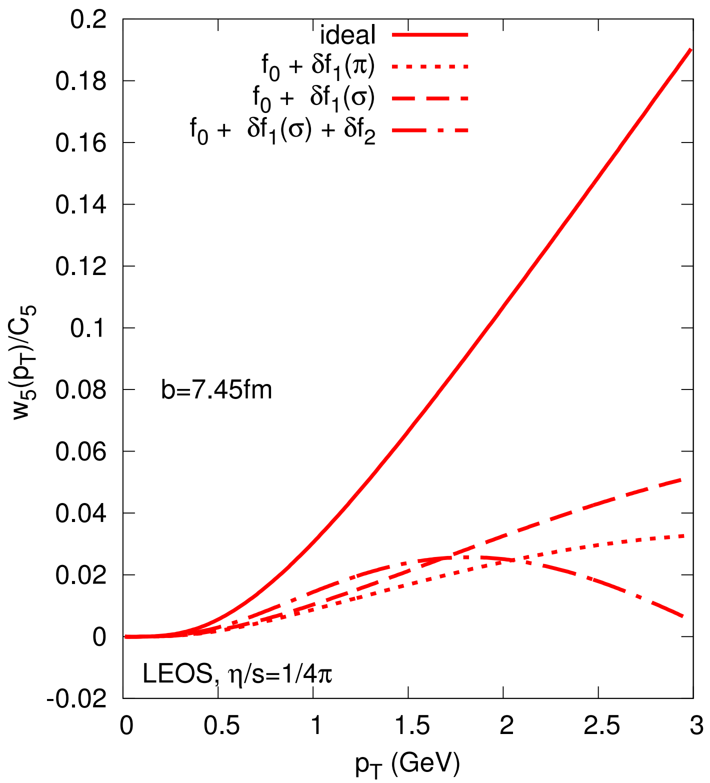


$f_0 =$ equilibrium distribution

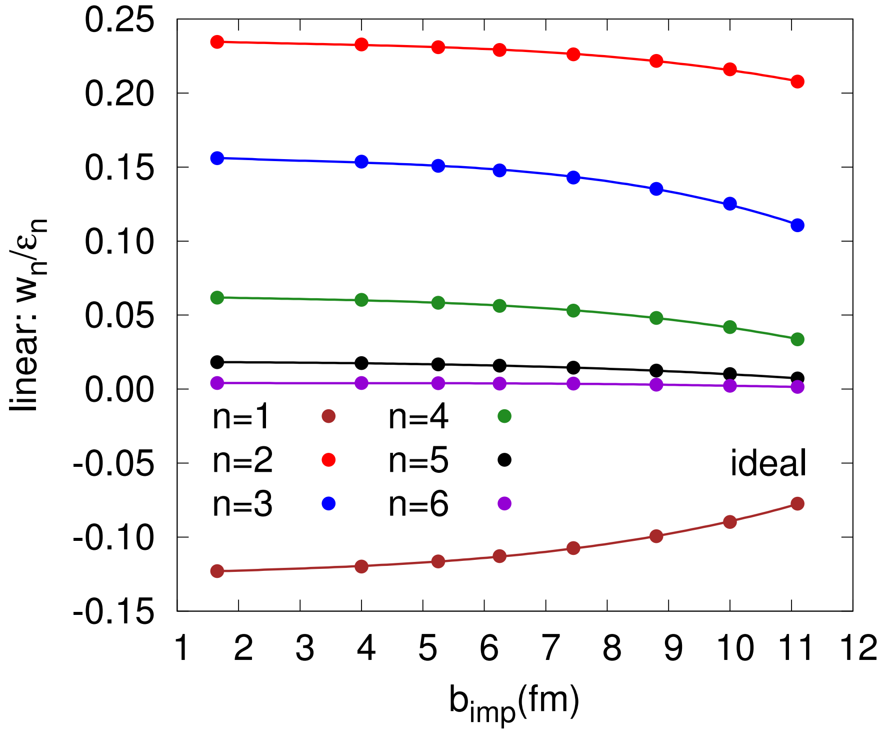
$f_0 + \delta f_1(\pi) \Rightarrow$ quadratic ansatz and $\pi_{\mu\nu}$

$f_0 + \delta f_1(\sigma) \Rightarrow$ quadratic ansatz and $\sigma_{\mu\nu}$

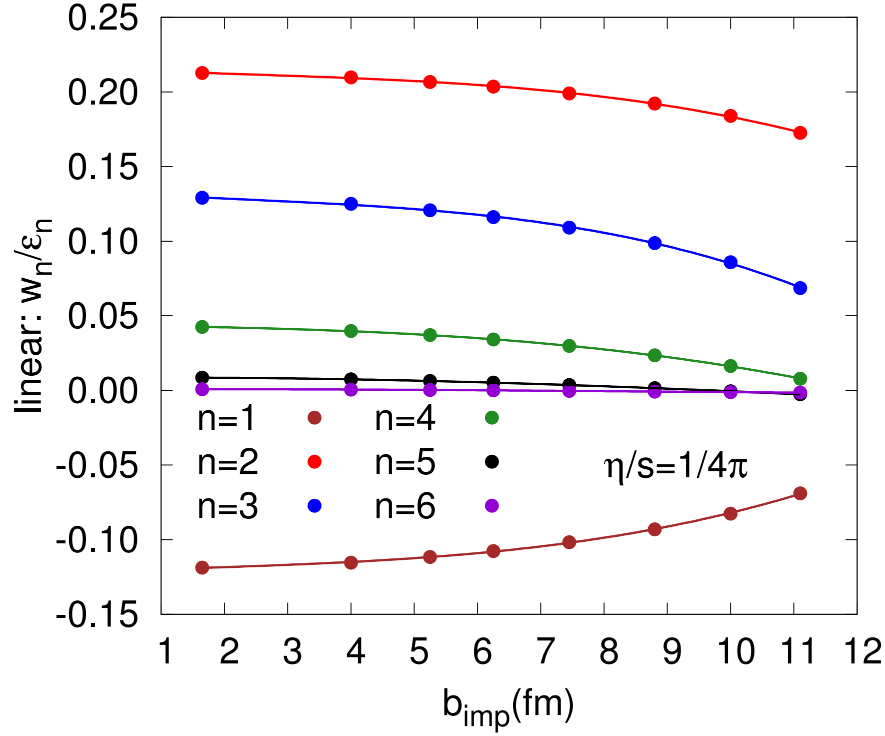
$f_0 + \delta f_1 + \delta f_2 \Rightarrow$ with 2nd order correction

Linear flow response coefficients (vs. centrality)

Ideal hydro.

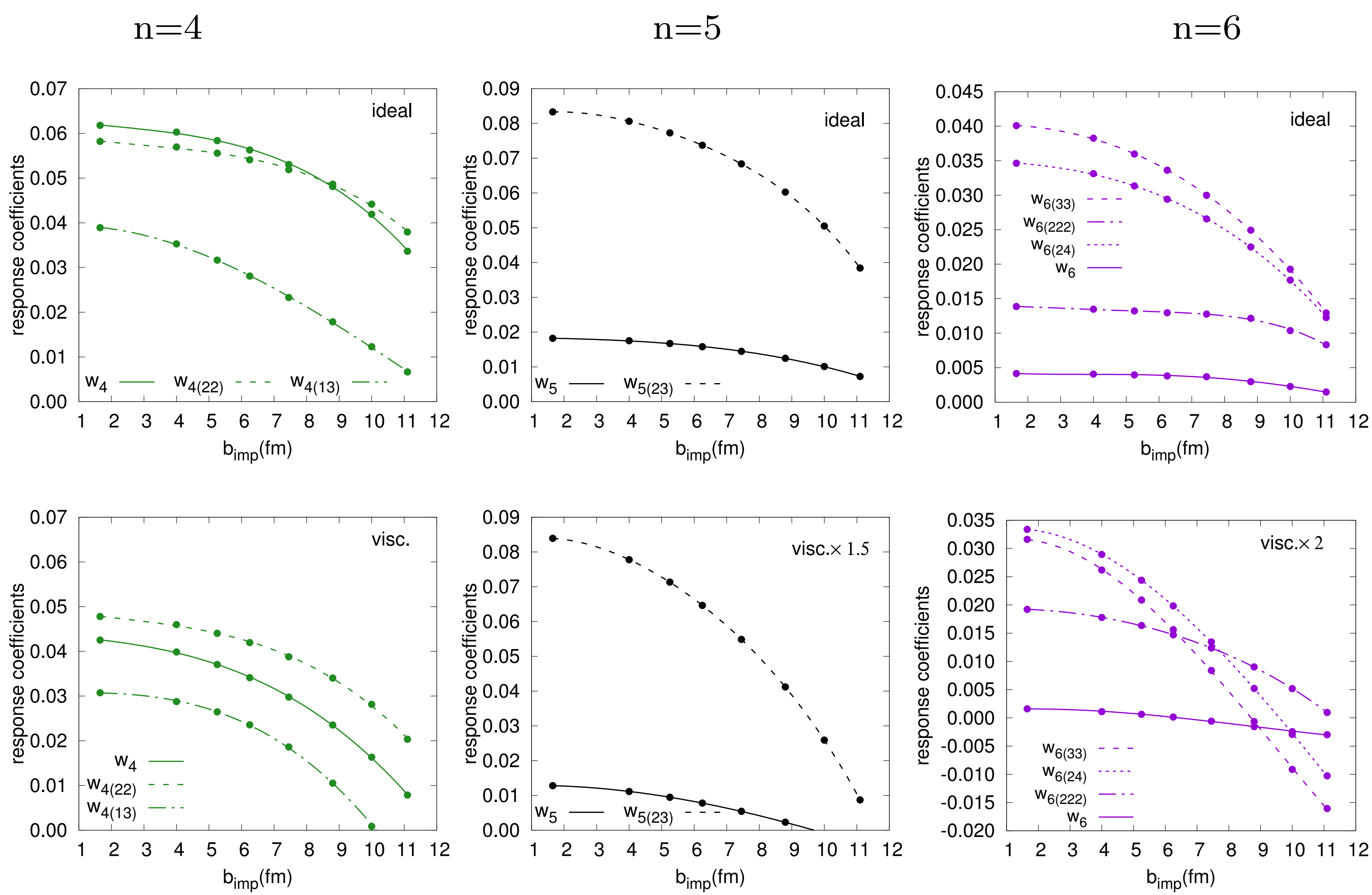


$\eta/s = 1/4\pi$



- Higher order harmonic flow has smaller linear flow response.
- Viscous damping consistent with the Gubser-Yarom rule.
- Linear w_6 almost vanishes, especially from viscous hydro.

Non-linear flow response coefficients (vs. centrality)

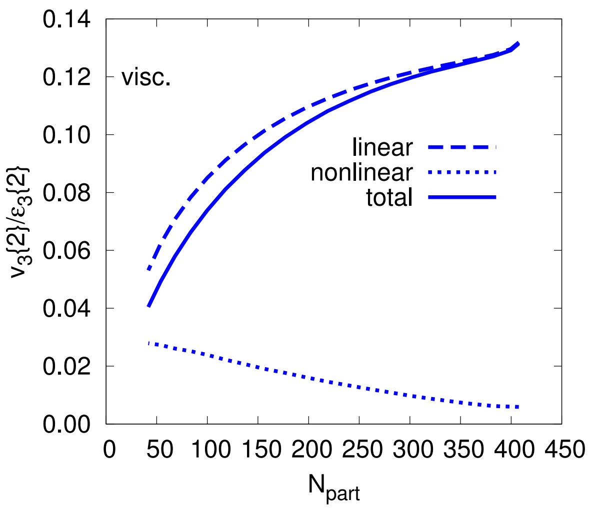


LHC PbPb: ideal hydro. and visc. hydro. ($\eta/s = 1/4\pi$), $T_{\rm fo} = 150$ MeV.

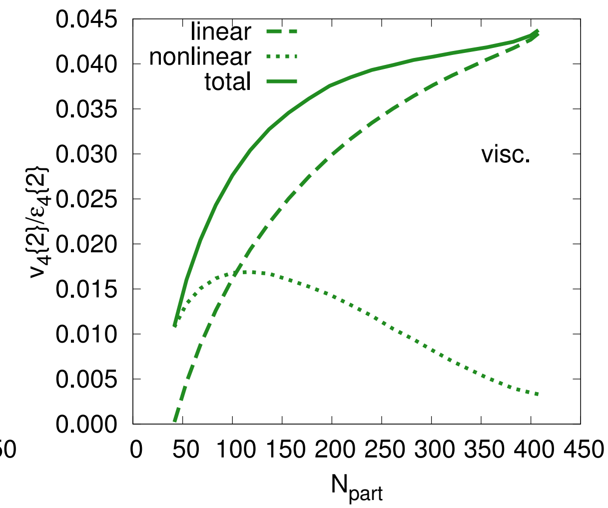
Non-linear response dependence on centrality: integrated $v_n\{2\}$

What actually matters for magnitude is $\underline{\frac{w}{\varepsilon} \times \varepsilon}$.

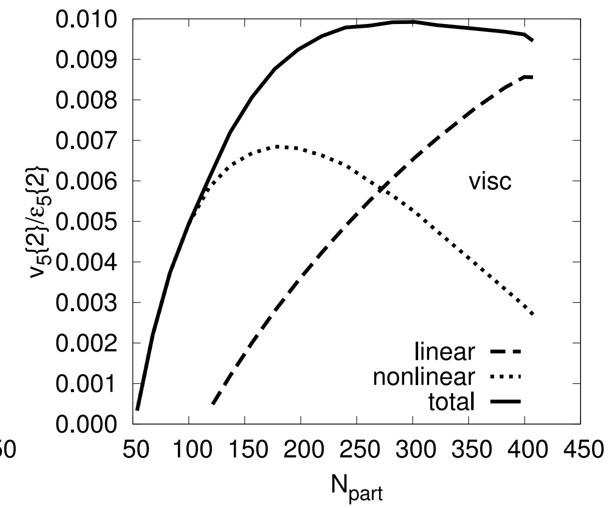
$v_3\{2\} : w_3, w_{3(12)}$



$v_4\{2\} : w_4, w_{4(22)}$



$v_5\{2\} : w_5, w_{5(23)}$

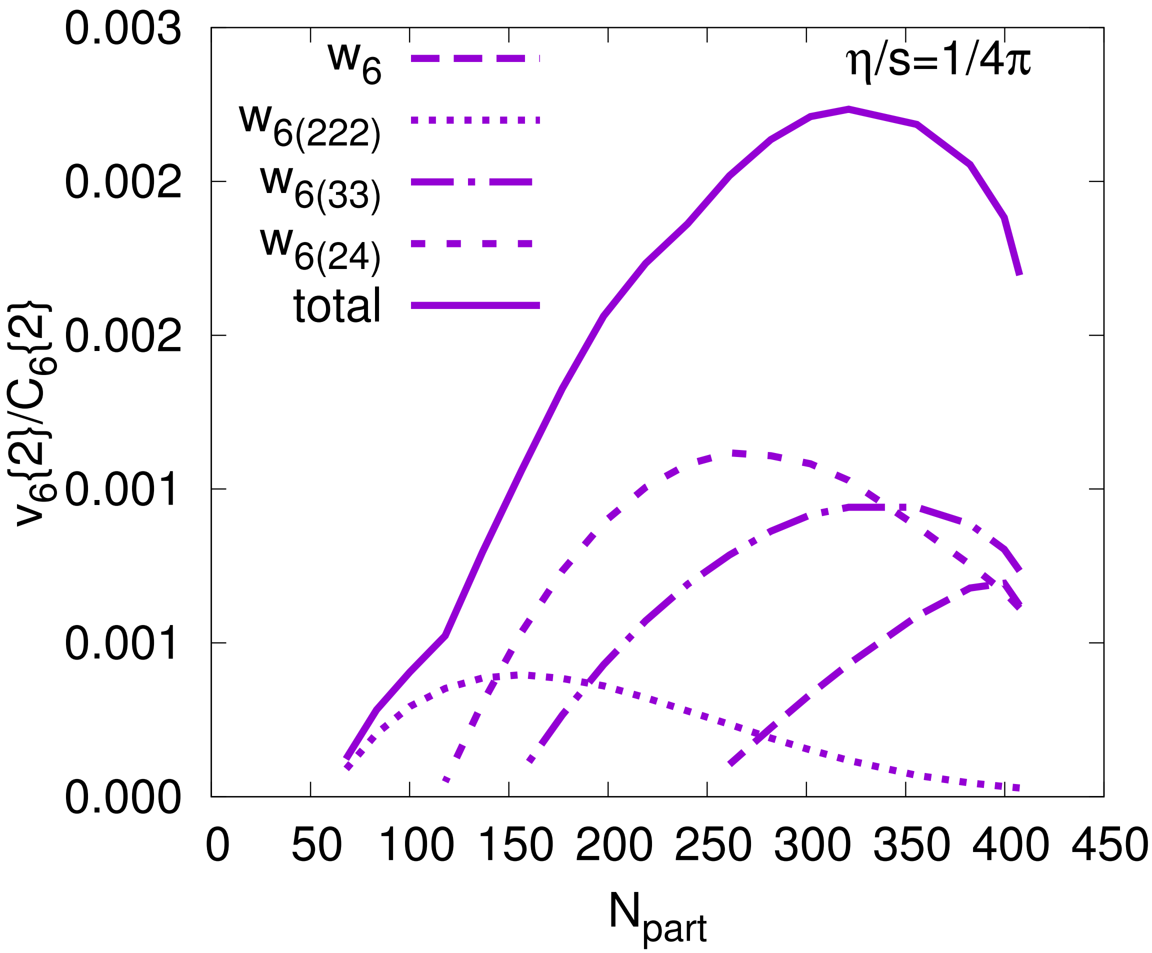


(LHC PbPb, ideal hydro, $T_{\rm fo} = 150$MeV, PHOBOS MC-GLb.)

- Non-linear response is not important for v_3, but crucial for v_4 and v_5.
- Linear response dominates at central bins.
- Non-linear response dominates at peripheral bins.

Non-linear response becomes more important for larger centrality.

$v_6\{2\}$: non-linearity in higher order harmonic flow generation



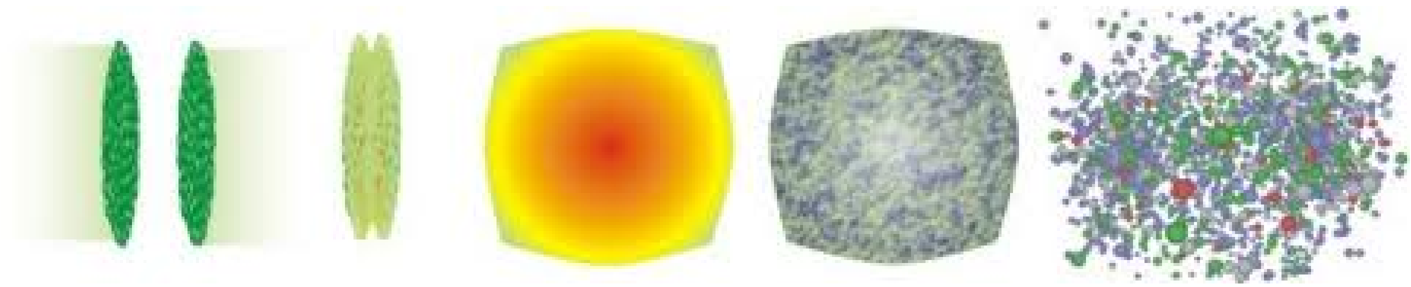
- There are more non-linear terms contributing to v_n, and also observables.

Higher order harmonic flow is more involved in non-linear response formalism.

Discussions on the origins of non-linearity

Three major stages in flow generation,

- Initial state.
- Medium expansion.
- Freeze-out.



Discussions on the origins of non-linearity

Three major stages in flow generation,

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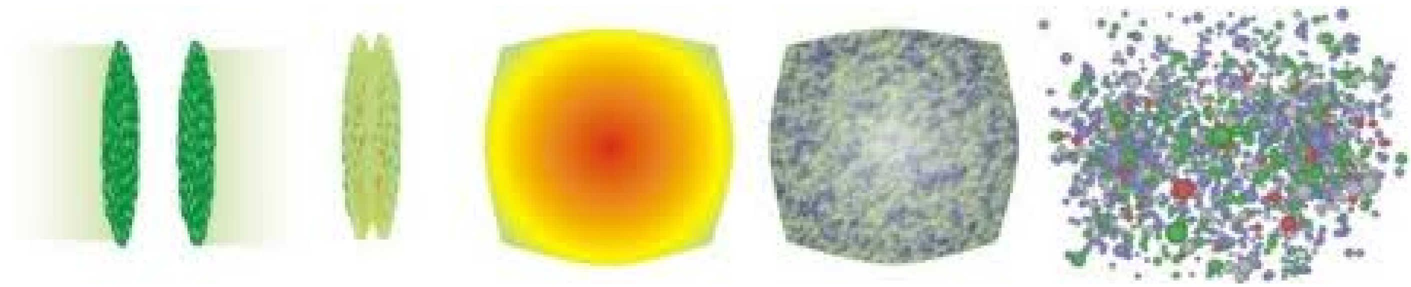
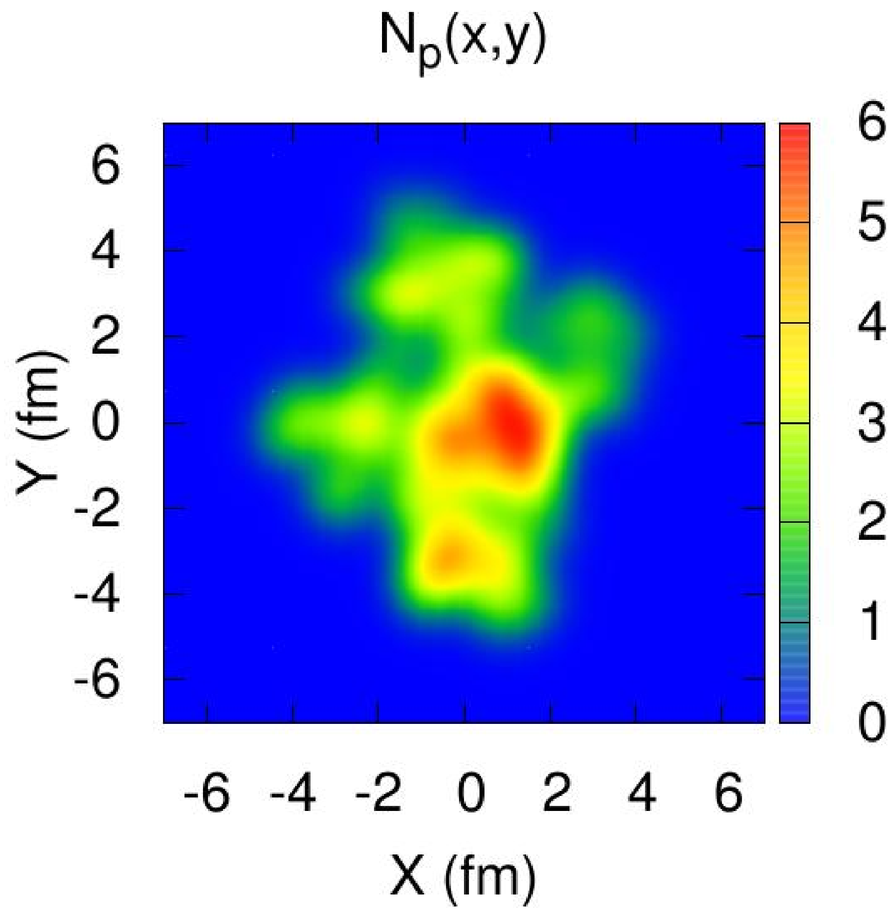
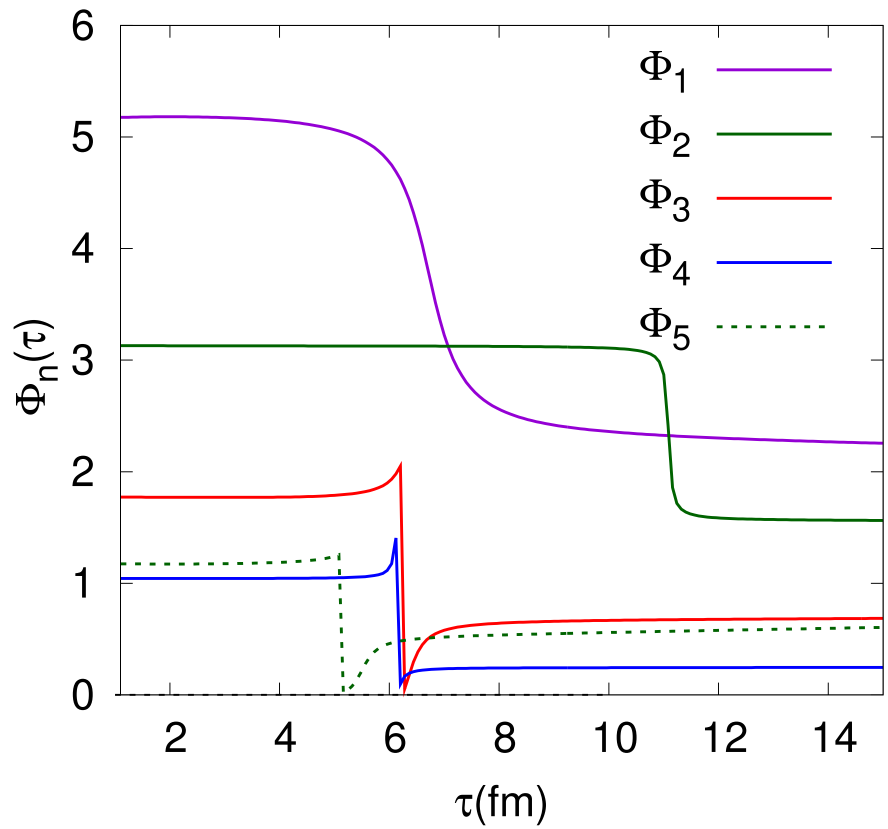






- Shape switch at certain time – $\Delta\Phi_n = |\pi/n|$
- Each of the deformations evolves independently.

Different angular deformations do NOT interact during expansion.

Origins of non-linear response – freeze-out

Cooper-Fryer formula for freeze-out process,

$$E\frac{d^3N}{dp^3} = \int_\Sigma p \cdot d\sigma f(x, \vec{p})$$

Borghini-Ollitrault's argument:(ideal hydro.)

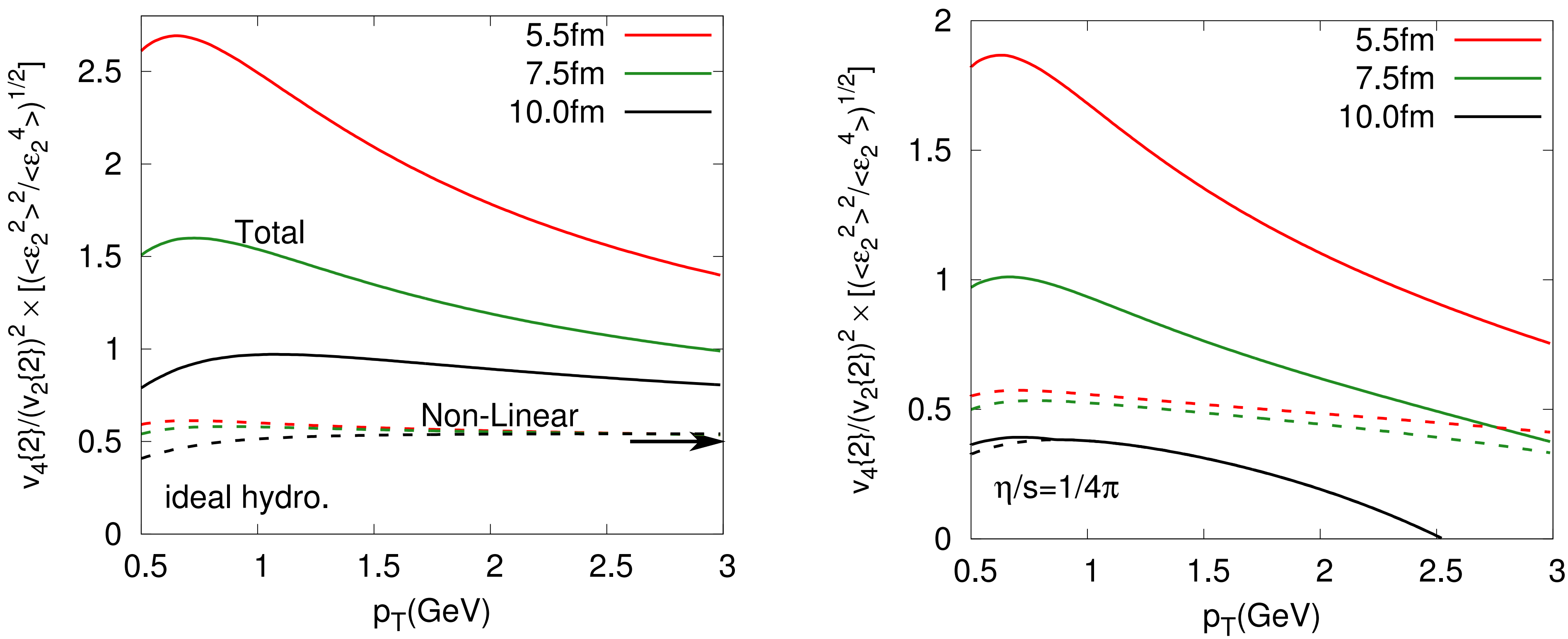
1. Large p_T limit and saddle point approximation: $v_n^{\text{linear}} \propto p_T$
2. The dominant non-linear contributions to v_n flow ($p_T \to \infty$):

$$v_4^{NL} = \frac{1}{2}(v_2^{\text{linear}})^2 \propto p_T^2, \qquad v_5^{NL} = v_2^{\text{linear}} v_3^{\text{linear}} \propto p_T^2$$

So we should expect the scaling behavior of v_4 and v_5.

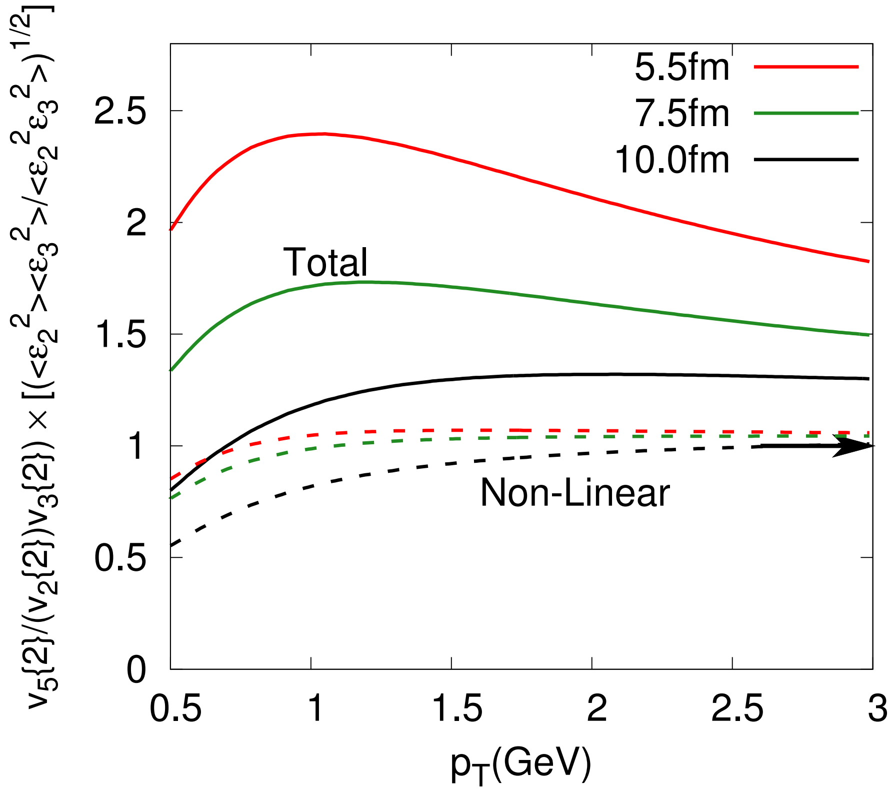
$$\frac{v_4}{v_2^2} \quad \text{and} \quad \frac{v_5}{v_2 v_3}$$

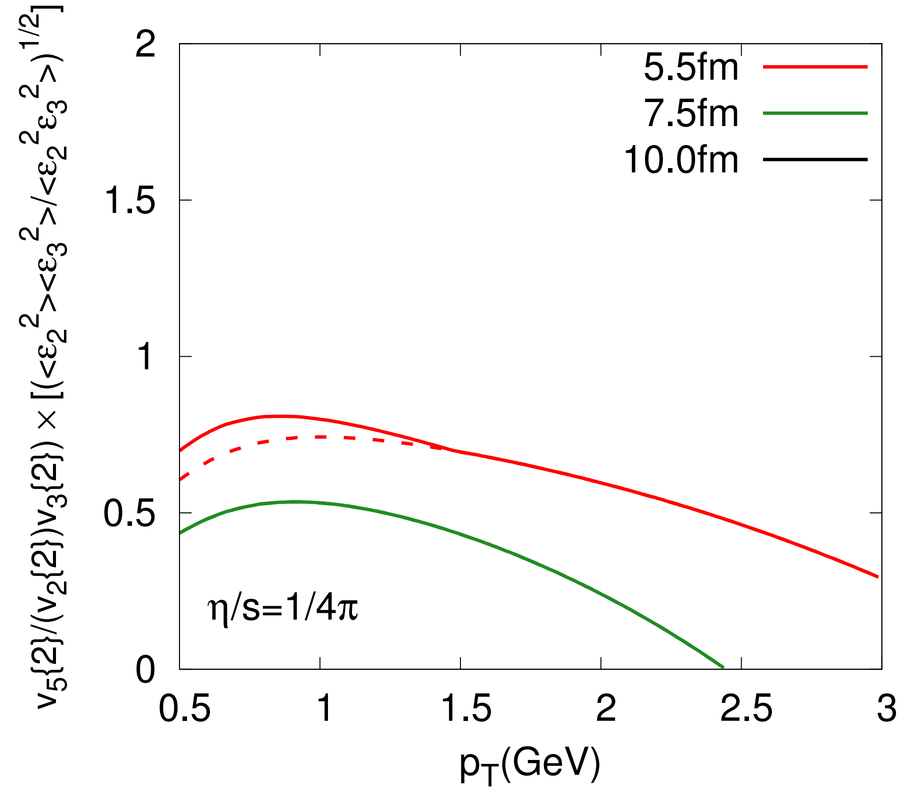
$v_4\{2\}(p_T)/v_2\{2\}(p_T)^2$



- Scaling behavior reproduced for ideal hydro. large p_T limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.

$v_5\{2\}(p_T)/(v_2\{2\}(p_T)v_3\{2\}(p_T))$





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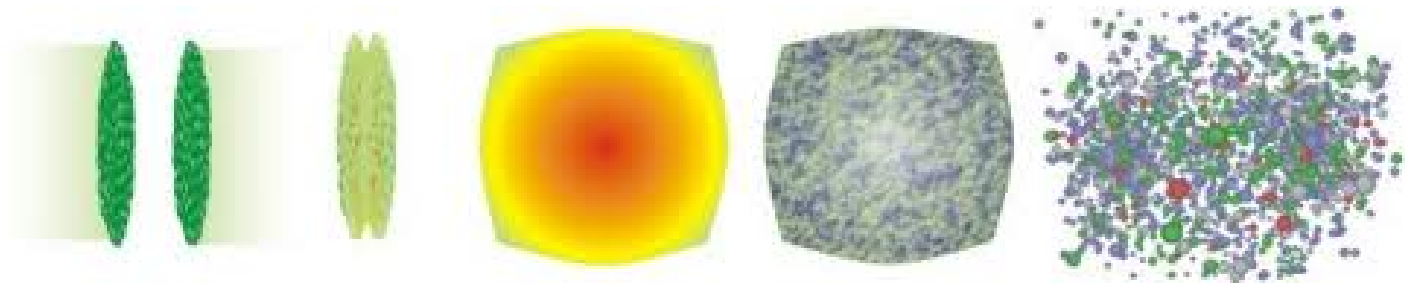
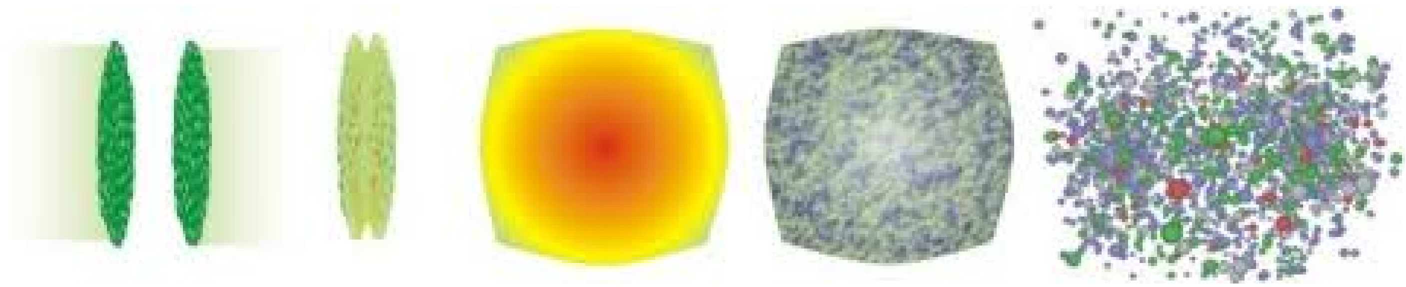
- Initial state. – X (As long as we take cumulants)
- Medium expansion. – X
- Freeze-out. – ✓



Discussions on the origins of non-linearity

Three major steps in flow generation,

- Initial state. – X (As long as we take cumulants)
- Medium expansion. – X
- Freeze-out. – ✓



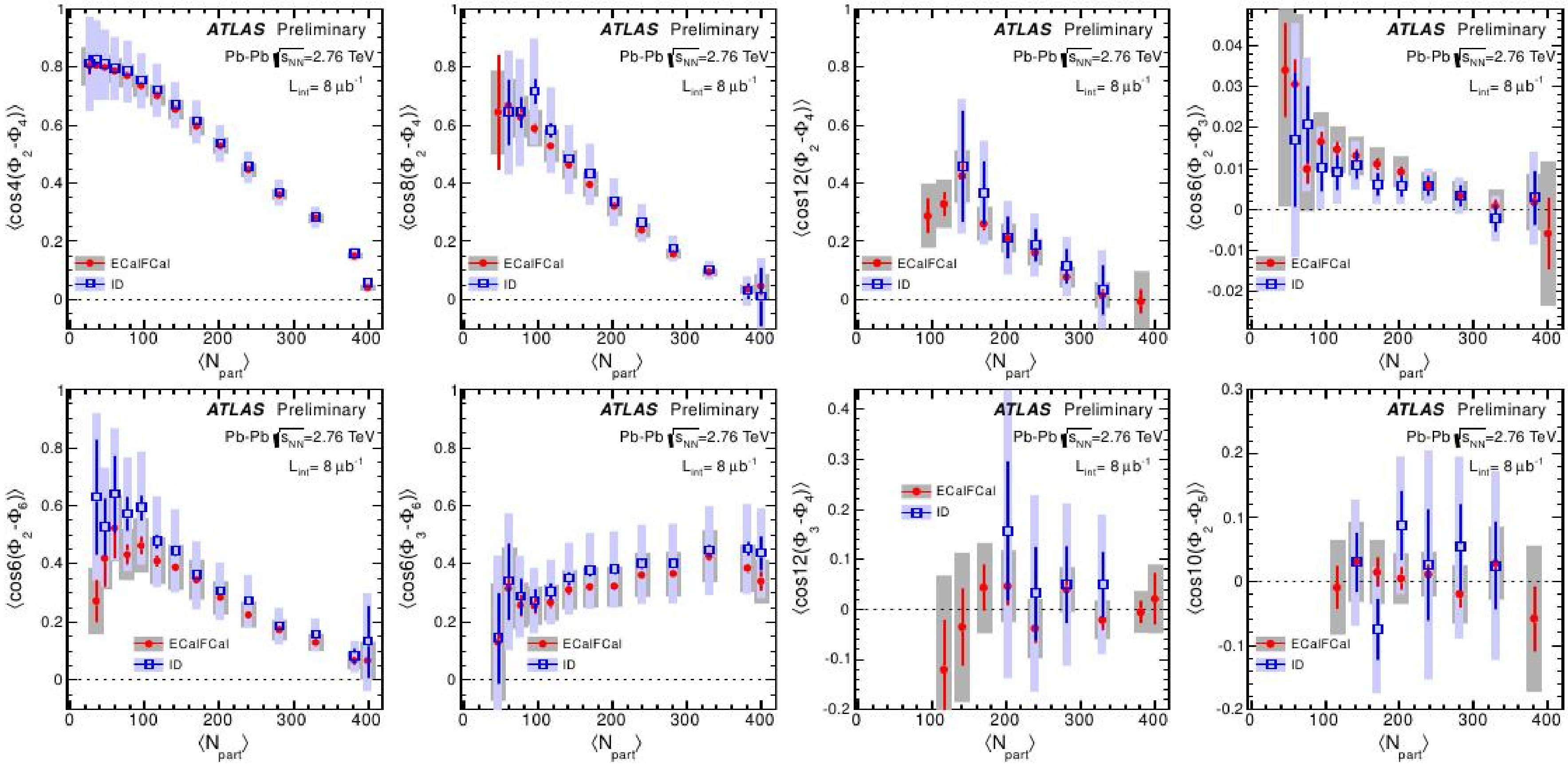
Simple summary:

- 'Complete' single-shot hydro. calculation:

$$\underbrace{\text{(IC with cumulants)}}_{\text{initial state}} + \underbrace{\text{(linear and non-linear response)}}_{\text{Hydrodynamics}}$$

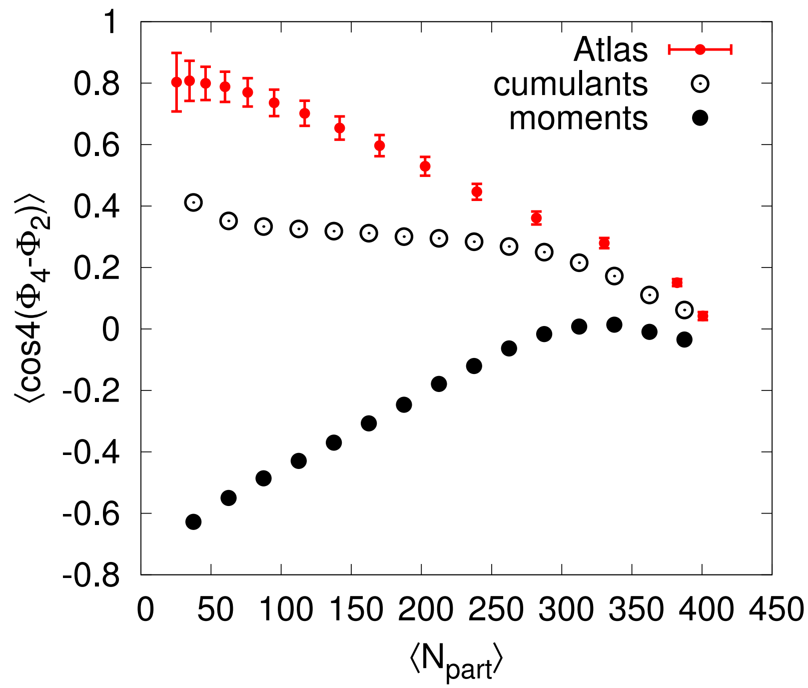
- Some properties of non-linear(and linear) response in hydro.:
  1. Viscous hydro.: non-linearity becomes more important in viscous hydro.
  2. Centrality: non-linearity becomes more important in peripheral bins.
  3. p_T: non-linearity dominates high p_t regime.

Reaction Plane Correlations



Reaction-plane correlations: linear response $(\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)$

(Ψ_4, Ψ_2) correlations:



- Linear flow response not enough to generate reaction plane correlation.
- Deviations (cumulants def.) from experiment data imply NL response.
  1. At central bins(linear dominant): smaller deviations.
  2. At peripheral bins(non-linear dominant): larger deviations.

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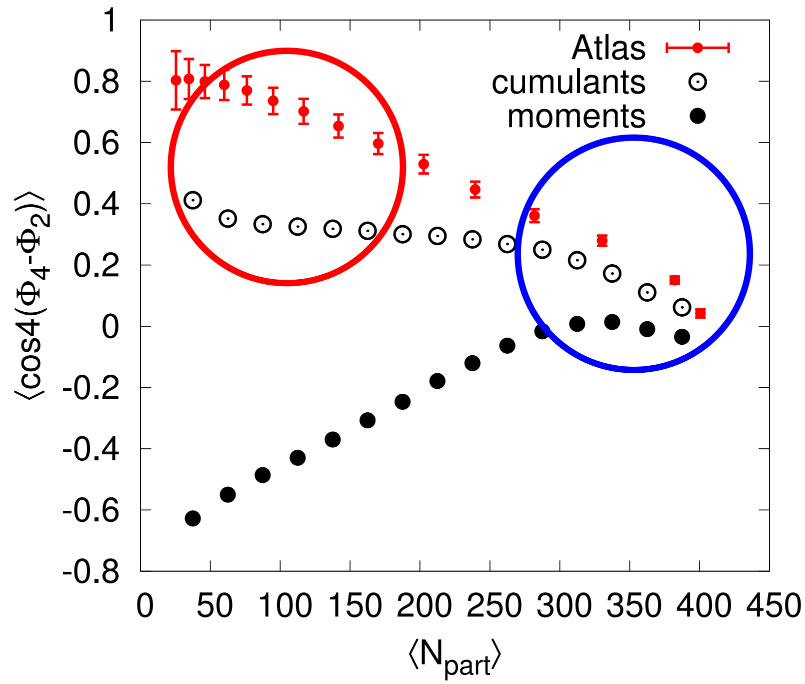

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Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

- $\langle\langle\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5)\rangle\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\langle\cos(2\Psi_2+3\Psi_3-5\Psi_5)\rangle\rangle = \left\langle\left\langle \frac{\overbrace{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) w_5}^{\text{linear}} + \overbrace{w_{5(23)}}^{\text{Non-linear}}}{|w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(2\Phi_2+3\Phi_3)}|} \right\rangle\right\rangle$$

- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\color{blue}{\text{Linear response limit}}} + \underbrace{\text{NL correlations}}_{\color{red}{\text{NL response limit}}} \rangle\rangle$$

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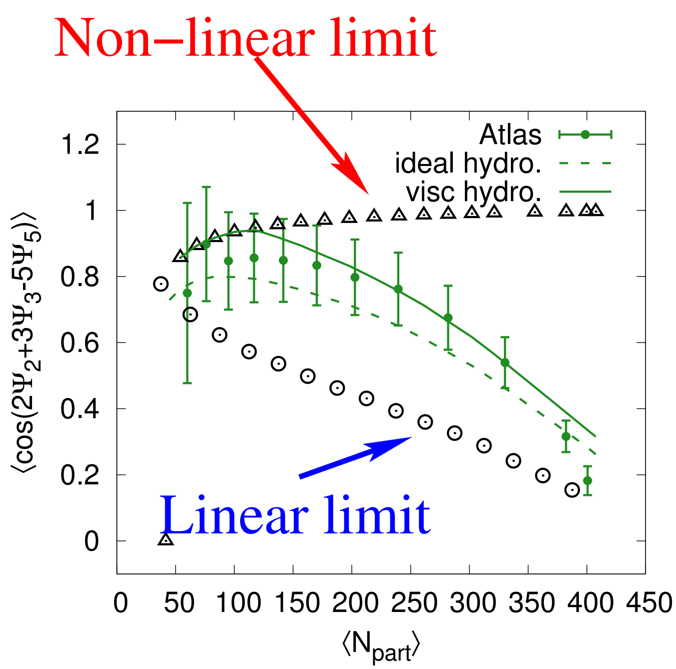

2–3–5 correlation

Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

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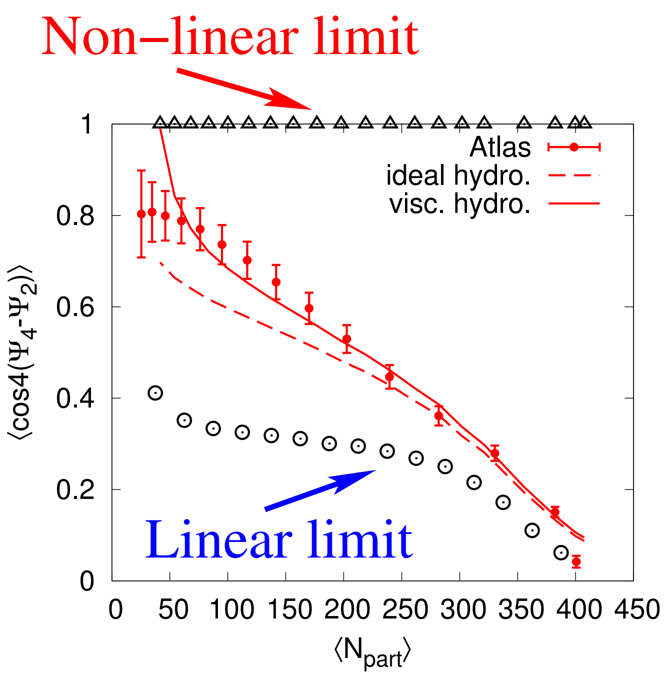
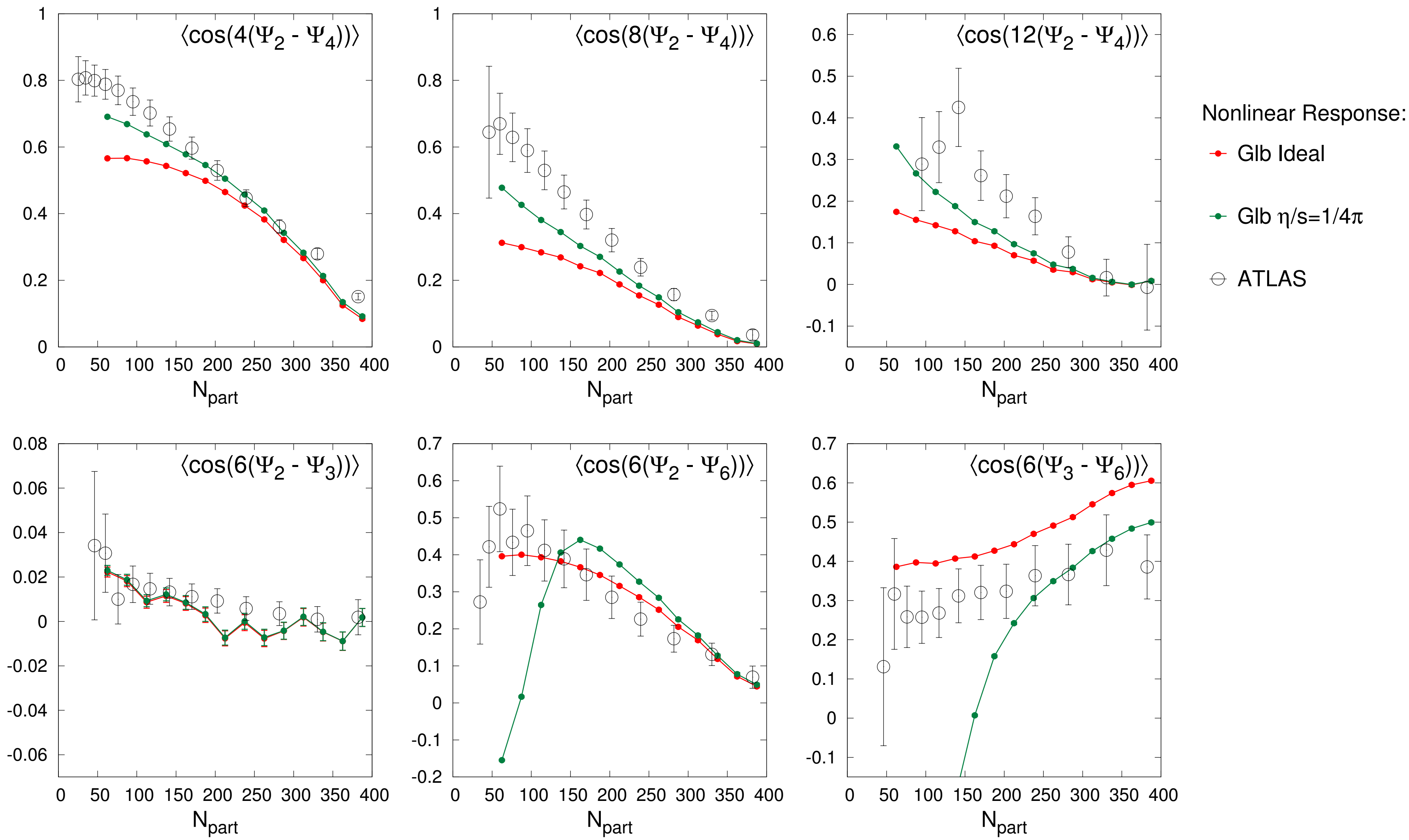


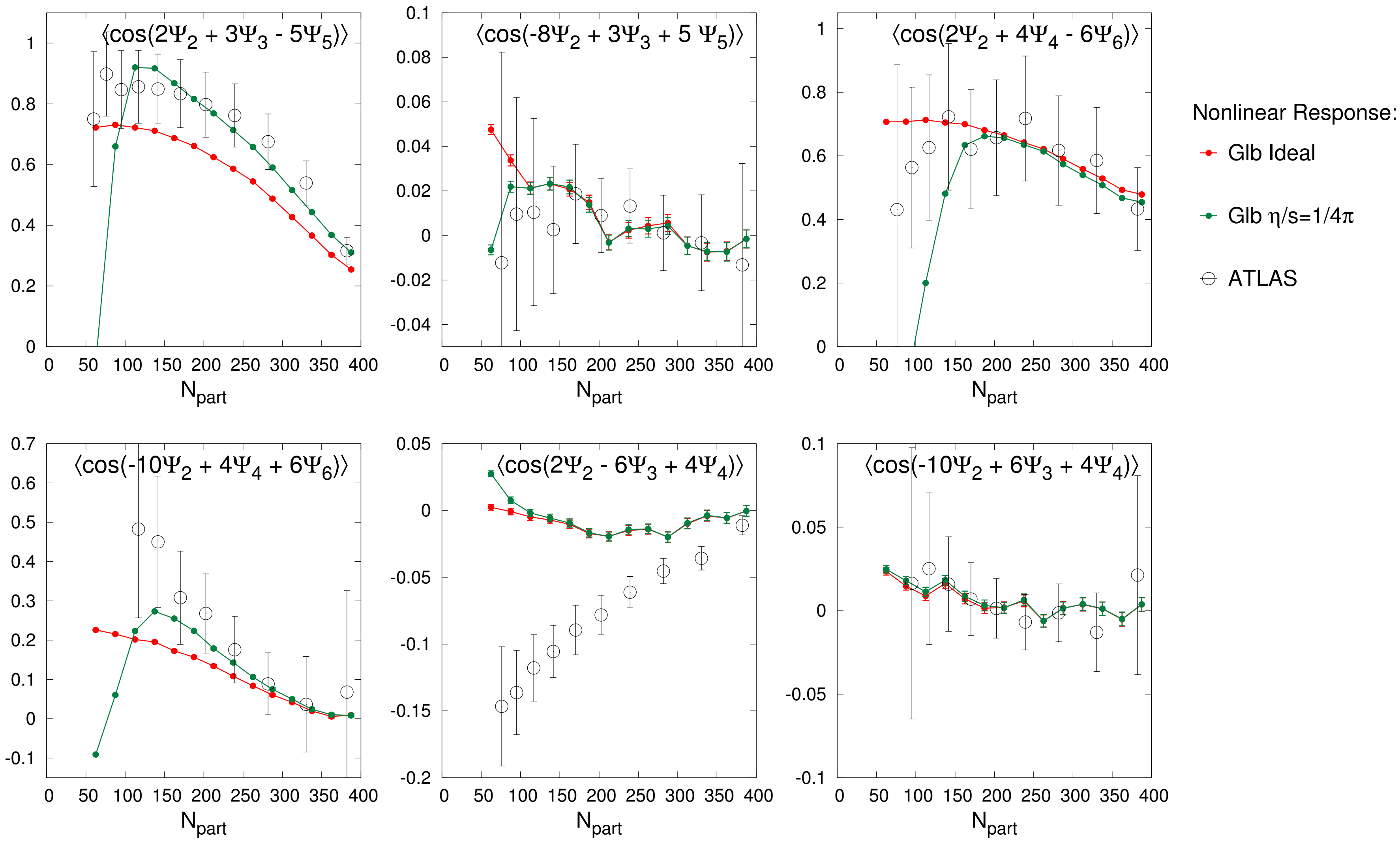
2–4 correlation

Two-plane correlations



- $v_3(3,12), v_4(4,22), v_5(5,23), v_6(33,24,222)$
- (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150\text{MeV}$, PHOBOS MC-GLb.)

Three-plane correlations



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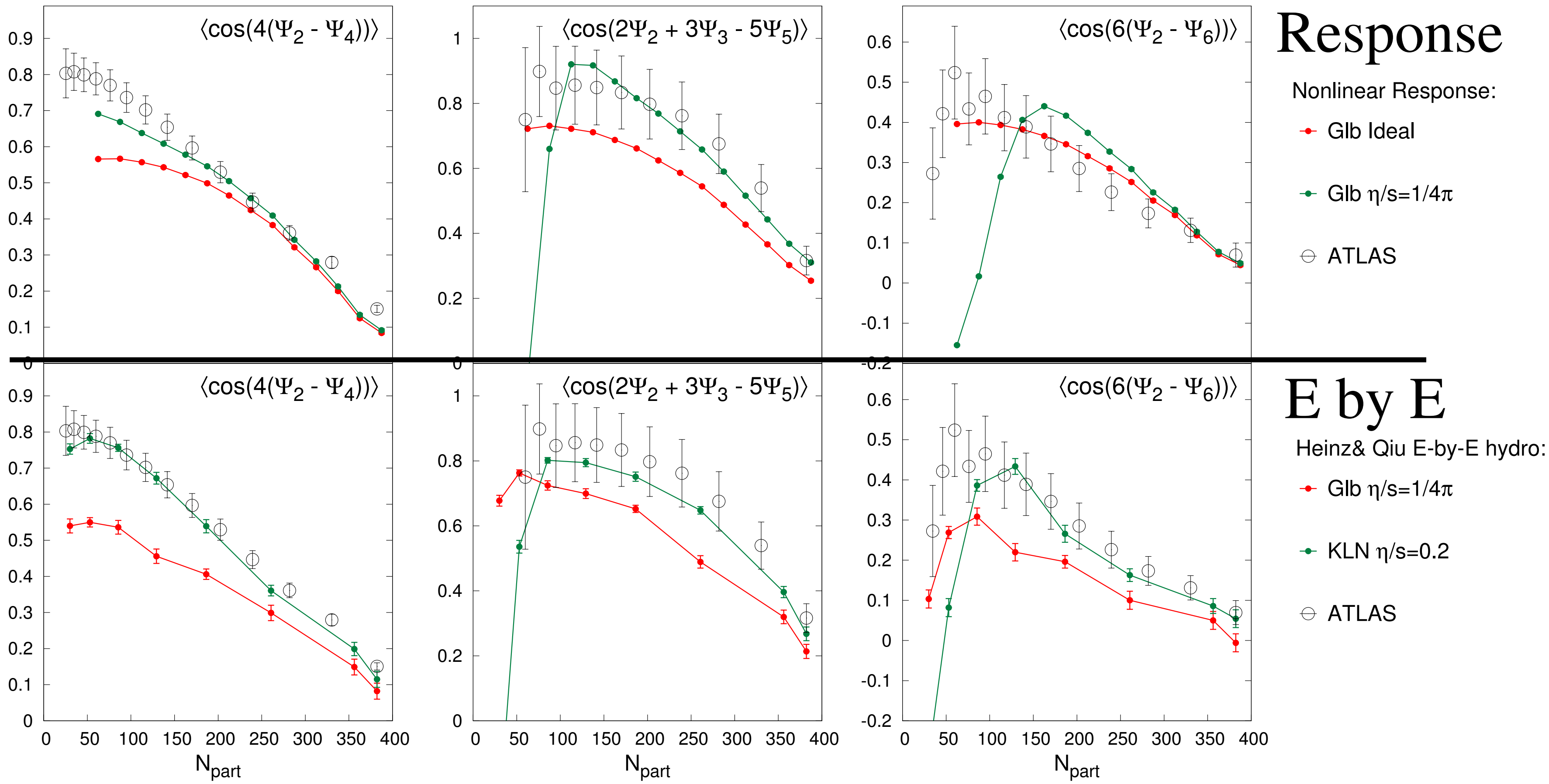
Not fair comparison with E-By-E hydro



Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

$$\underbrace{\text{Initial correlations}}_{\text{cumulants}} + \underbrace{\text{Flow response}}_{\text{Linear \& \underline{Non-linear}}} = \underbrace{\text{Reaction plane correlations}}_{\text{Final state}}$$

- Ingredients:
  1. We use cumulant formalism to classify initial fluctuations.
  2. We take linear *and* non-linear response in hydro calculations,

     non-linear response *vs.* (p_T, η/s, centrality).
  3. Predictions are consistent to final state observables (E-B-E hydro).

- Single-shot hydro. compare to event-by-event hydro.

$$\lim_{\text{all terms}} \text{Single-shot hydro.} \to \text{E-By-E hydro.}$$

Summary and conclusions

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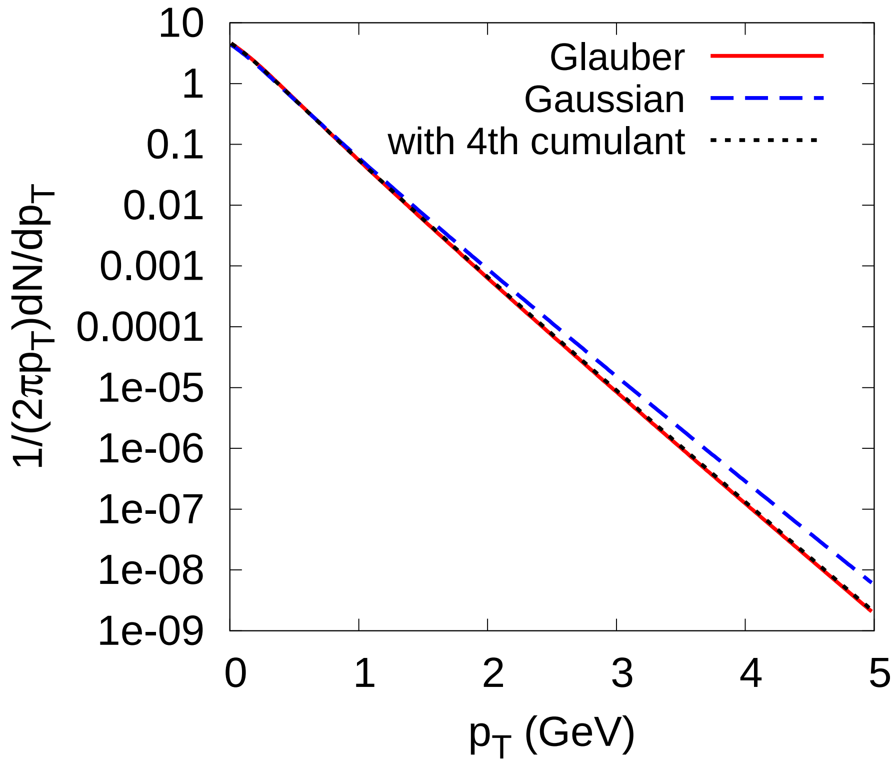
$$\lim_{\text{all terms}} \text{Single-shot hydro.} \rightarrow \text{E-By-E hydro.}$$

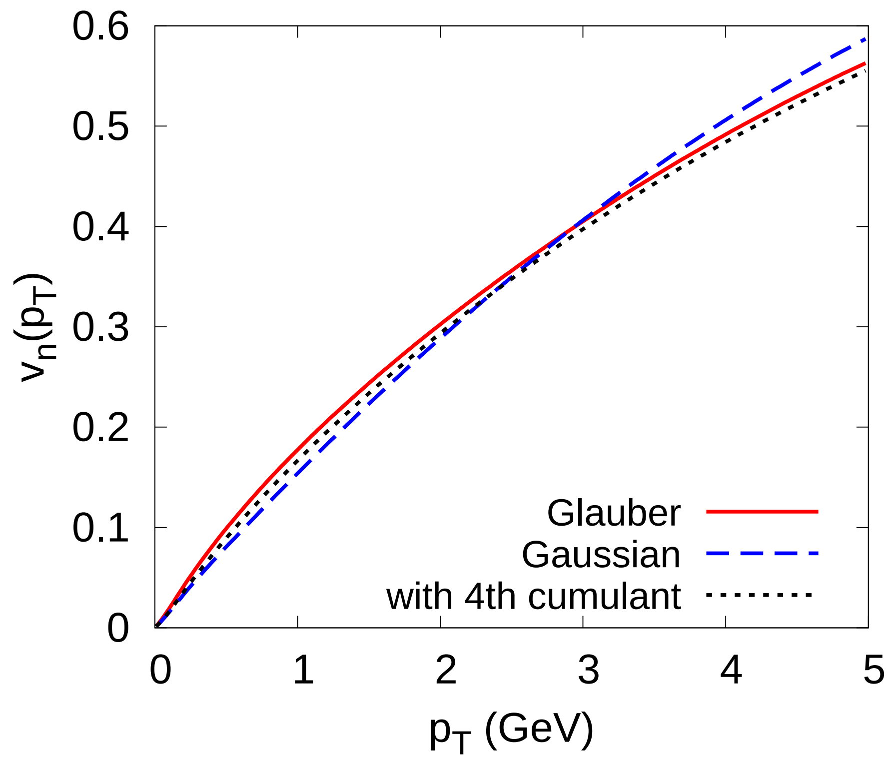
Thank you.

Back-up slides

Cumulant expansion – validity check

For distribution from smooth Glauber model, at $b = 7.6fm$





For any distribution $\rho(x, y)$, with $W_{n,m}$ as inputs, generally,

$$\rho_{n,m}(x, y) = W_{n,m} \left[\frac{\partial}{\partial \mathbf{x}} \right]^m \text{Gaussian}$$

- Cumulant expansion as approximation,

$$\rho(\vec{x}_\perp) \approx \underbrace{\rho_{0,2} + \rho_{2,2}}_{\text{Gaussian}} + \rho_{0,4}$$

Predictions for $v_n\{2\}(p_T)$ – differential flow

- Conventionally defined for 2-particle correlations,

$$v_n\{2\}(p_T) = \frac{\langle\langle v_n(p_T) v_n^{\rm int} \rangle\rangle}{v_n\{2\}}$$

- Compare with our spectrum decomposition, (v_5 for example)

$$v_5 e^{-i5\Psi_5} = w_1 e^{-i5\Phi_5} + w_{5(23)} e^{-i(3\Phi_3+2\Phi_2)}$$
$$v_5(p_T) e^{-i5\Psi_5(p_T)} = w_1(p_T) e^{-i5\Phi_5} + w_{5(23)}(p_T) e^{-i(3\Phi_3+2\Phi_2)}$$

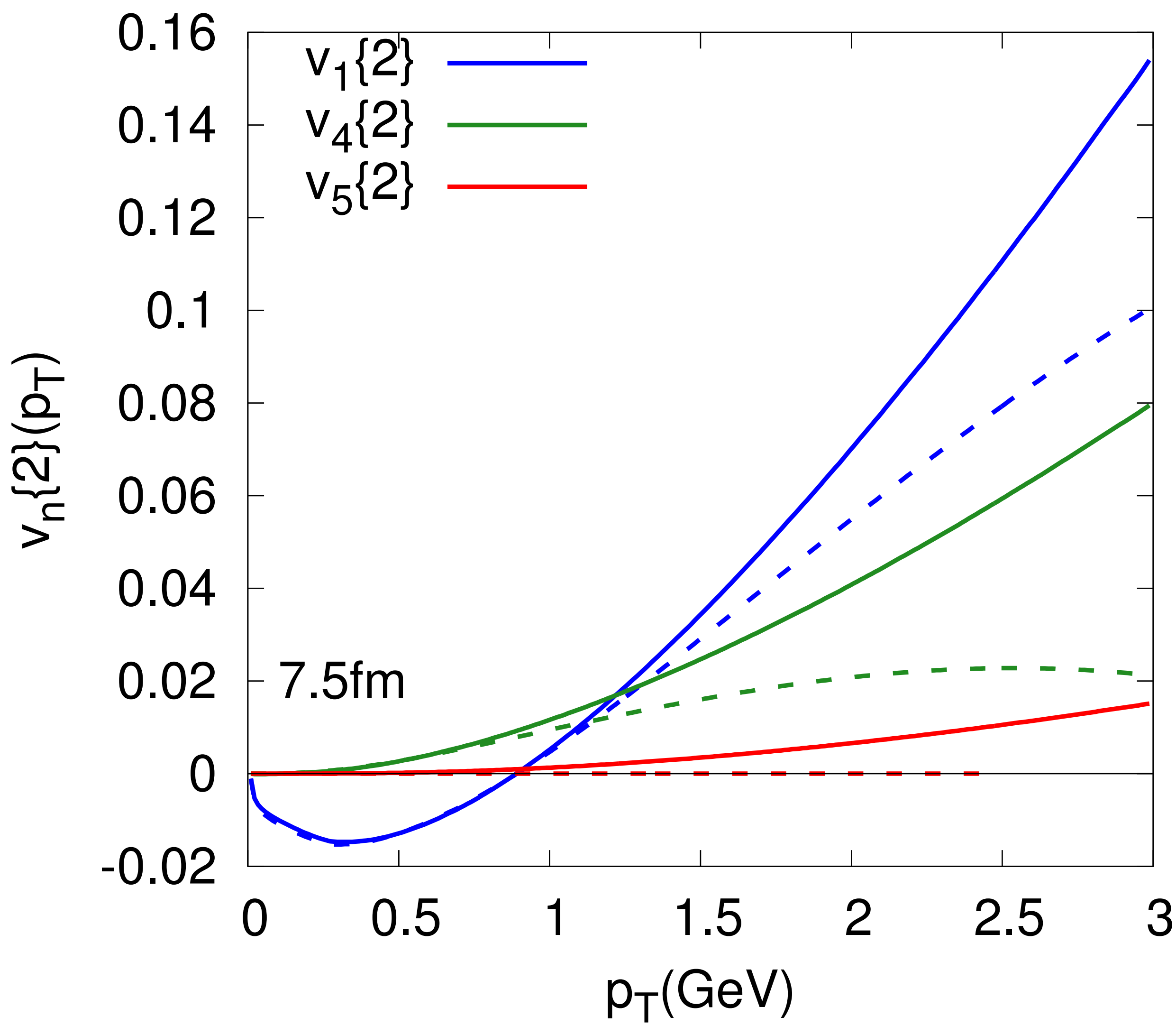
Actually we find

$$v_5\{2\}(p_T) = \frac{\langle\langle v_5(p_T) v_5^{\rm int} \cos 5(\Psi_5(p_T) - \Psi_5) \rangle\rangle}{v_5\{2\}}$$

Thus for p_T dependence,

$$v_n\{2\}(p_T) \propto \underbrace{\text{Linear response}}_{\sim p_T} + \underbrace{\text{Non-linear response}}_{\sim p_T^2}$$

Differential flow at mid-central bin



- Non-linear correction in v_1 not significant, compare to v_4 and v_5.
- Viscous damping at large p_T region.