Factorization with TMD PDFs:

The status of phenomenological implementations of TMD-factorization:

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- Factorization and Definitions of PDFs.
- TMD PDFs and phenomenology.
- More on Definitions.
- Explicit Determination of TMDs.
- The Sivers Effect.

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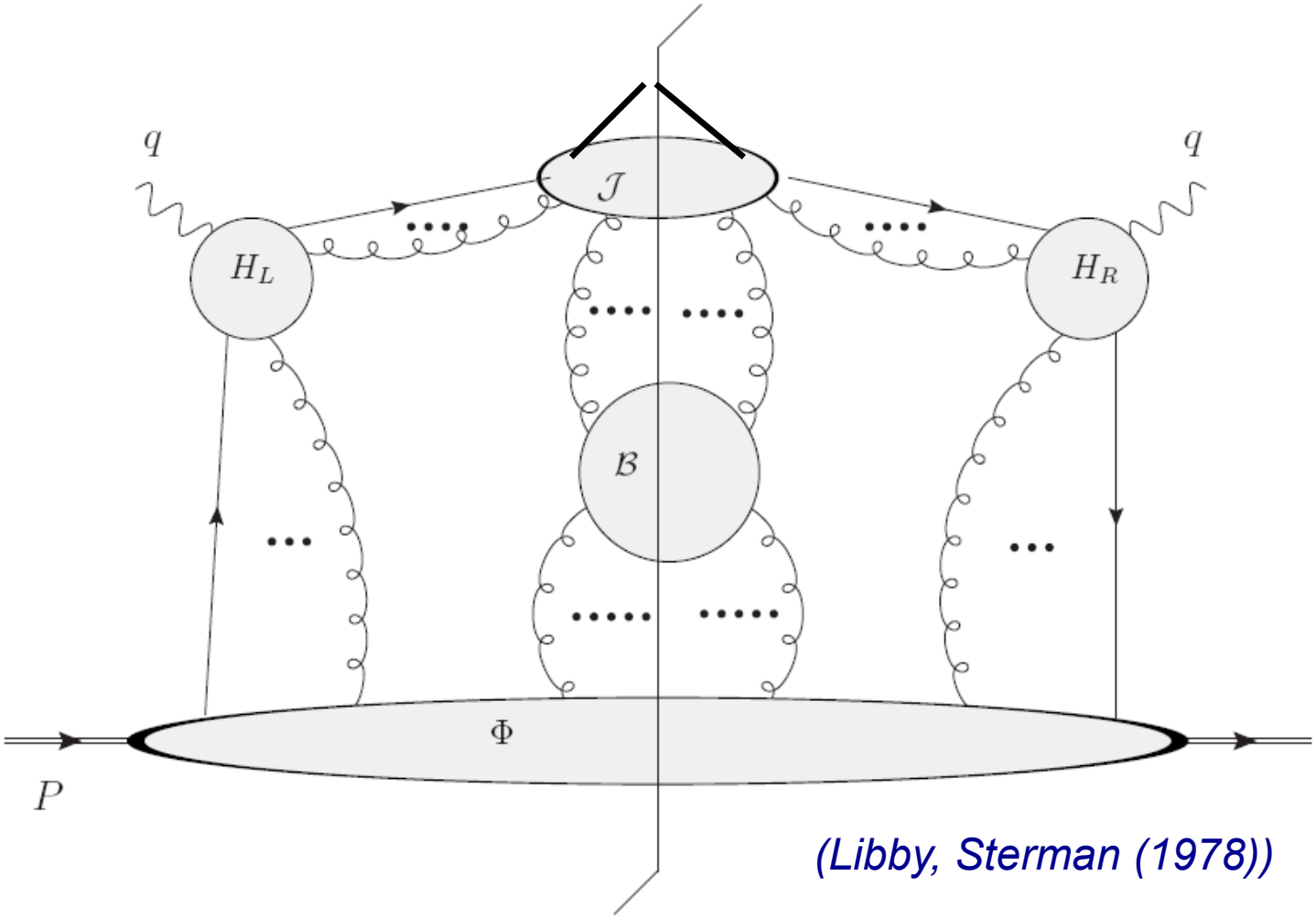
Lunch Seminar, Nov 17, 2011

Part 1: Factorization and Definitions of PDFs

TMD-Factorization:

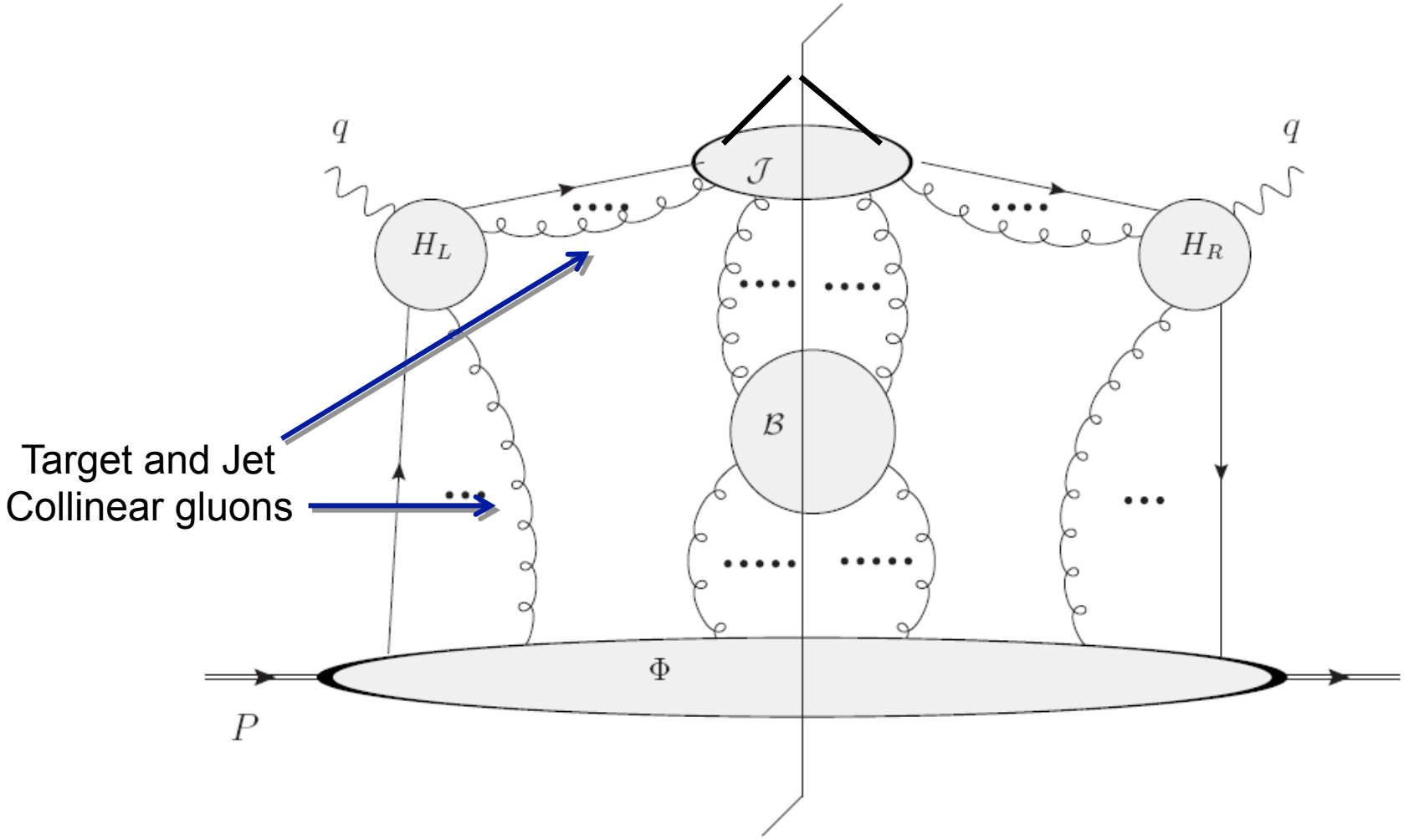
Ex: SIDIS



(Libby, Sterman (1978))

TMD-Factorization:

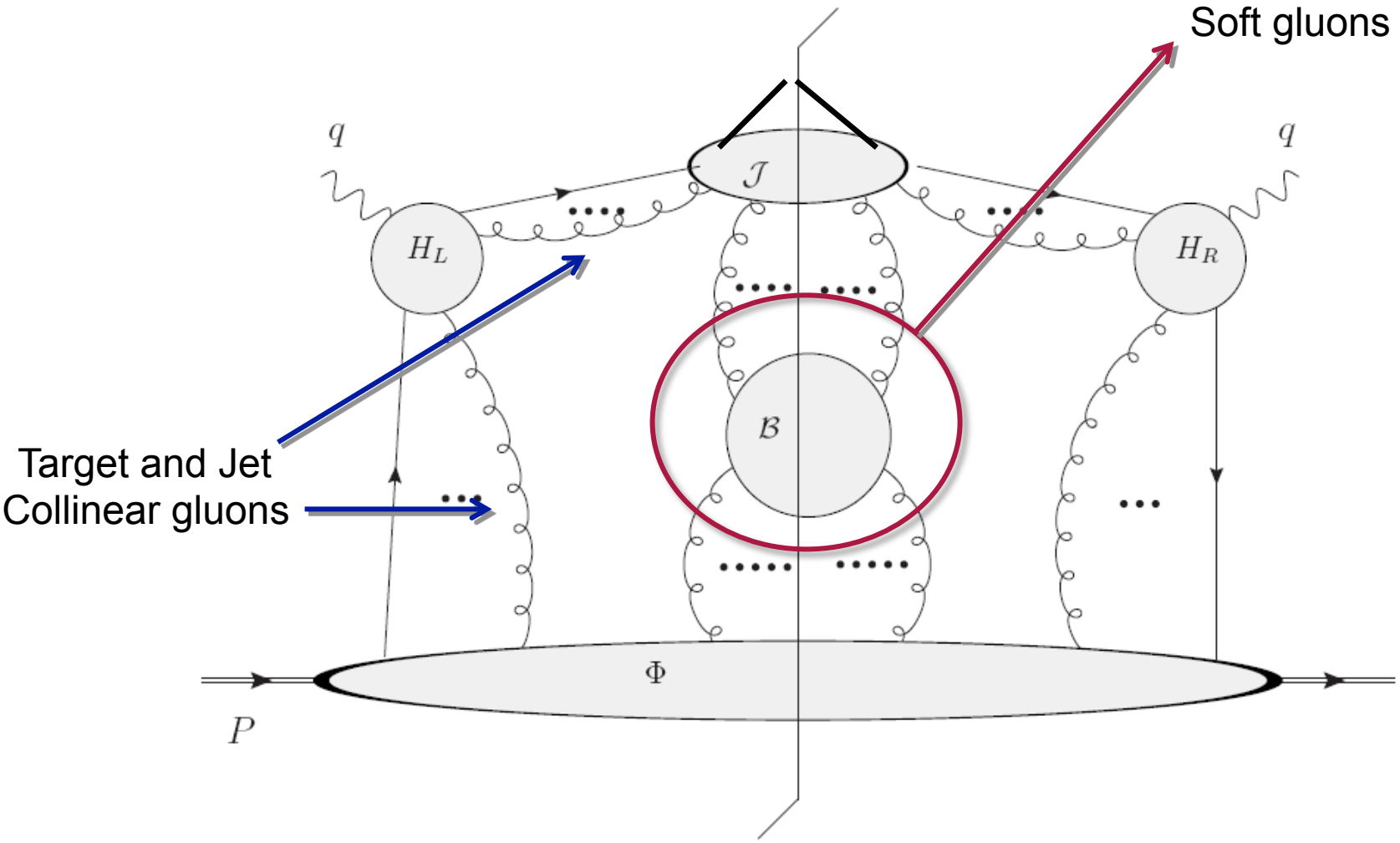
Ex: SIDIS



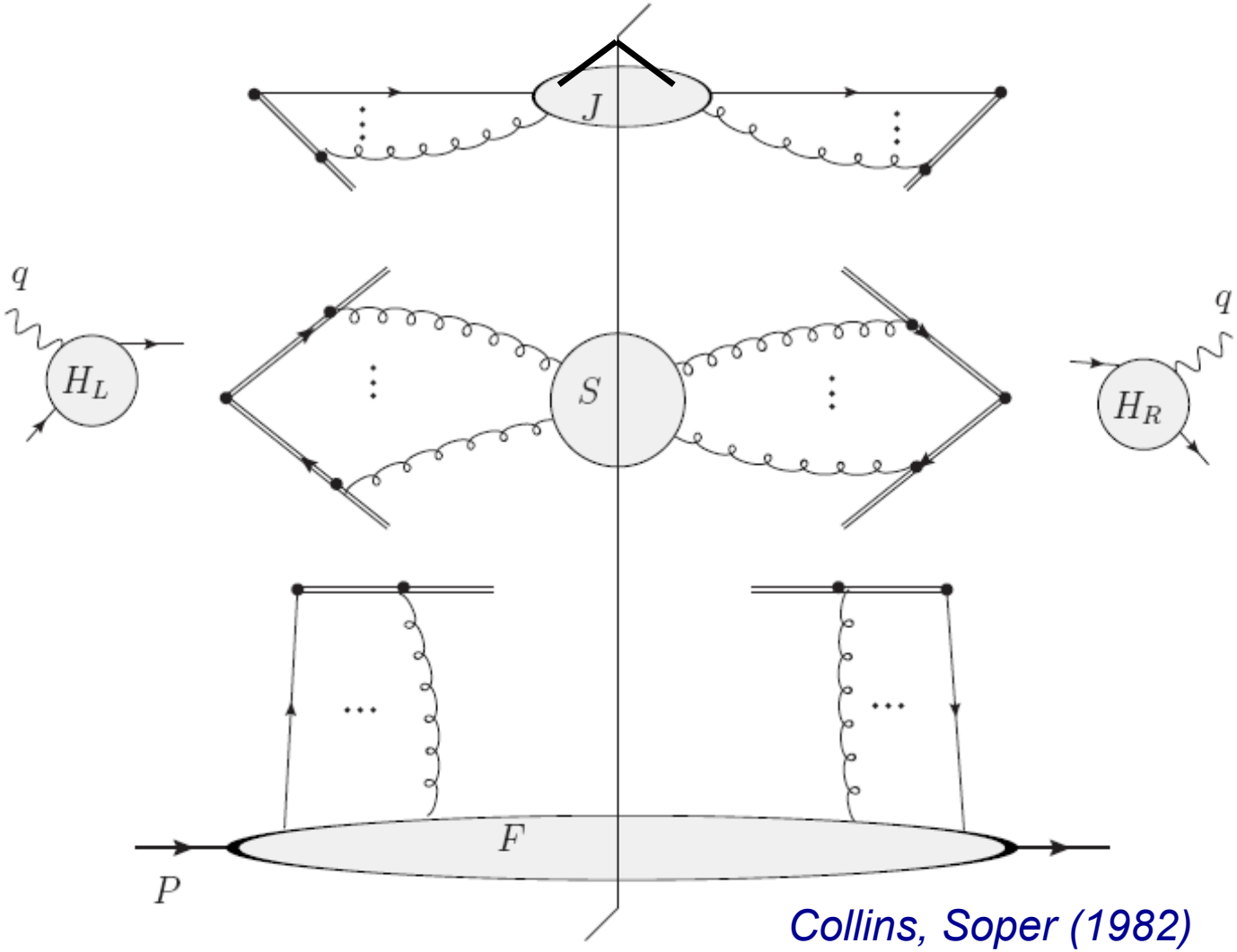
TMD-Factorization:

- Approximations needed to obtain factorization formula.

Ex: SIDIS



TMD-Factorization:



Collins, Soper (1982)
Collins, Soper, Sterman (1985)
Ji, Ma, Yuan (2004,2005) (SIDIS)

Recall: Collinear factorization for inclusive DIS

$$d\sigma = \sum_f \int |\underbrace{\mathcal{H}(Q)}|^2 \otimes \underbrace{f(x,Q)}$$

Short distance physics, asymptotic freedom

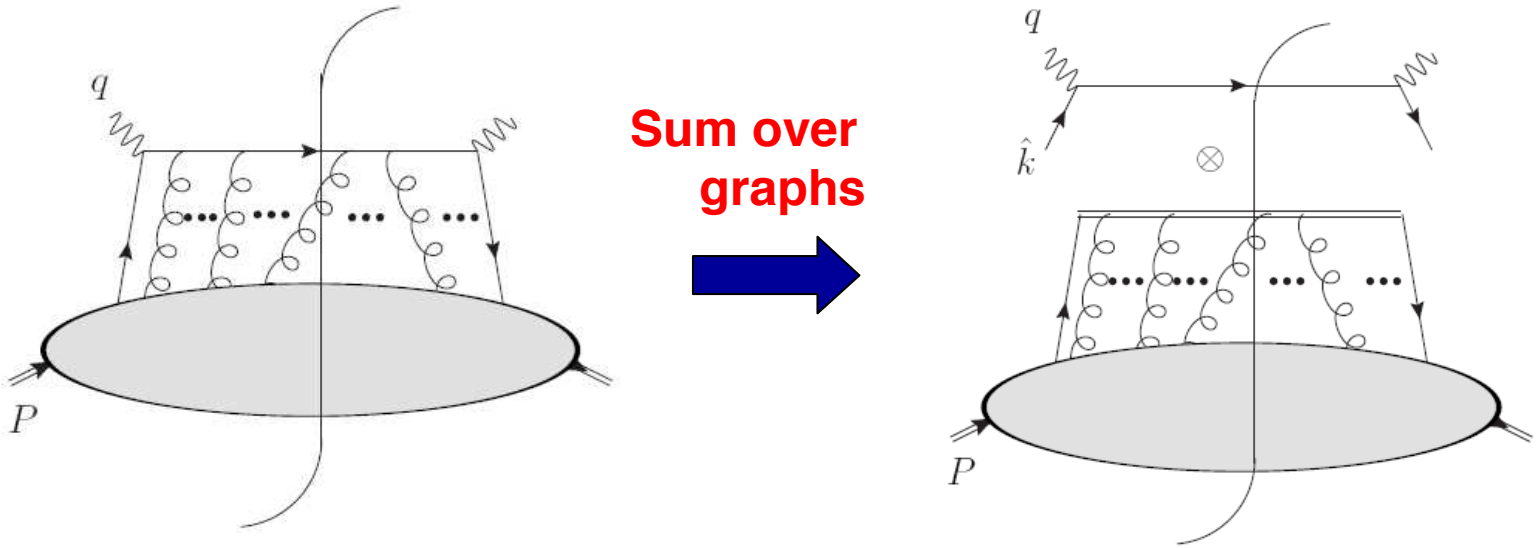
$(\mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots)$

Renormalization group/ DGLAP evolution equations

$(f^{(0)} + f^{(1)} + \cdots)$

- Hard, soft, collinear --- separate factors order-by-order.
- Corrections suppressed by powers of Q.
- Non-perturbative factors are universal.
- Evolution relates different scales.
- Interpretation in terms of parton model concepts.

Ward Identity: Usual (Integrated) PDF



- Eikonal factors, extra Feynman rules.

$$\frac{i}{-l^+ + i\epsilon} \quad , \quad -igt^a u_J^\mu$$

Eikonal Propagator ***Vertex***

Usual (Integrated) PDF

- Operator definition:

$$f(x) = \text{F.T.}\, \langle p|\, \bar{\psi}(0, w^-, \mathbf{0}_t) \underline{V_w^\dagger(u_\text{J})} \gamma^+ \underline{V_0(u_\text{J})} \psi(0) |p\rangle$$

$$u_\text{J} = (0, 1, \mathbf{0}_t)$$

(Example: DIS)

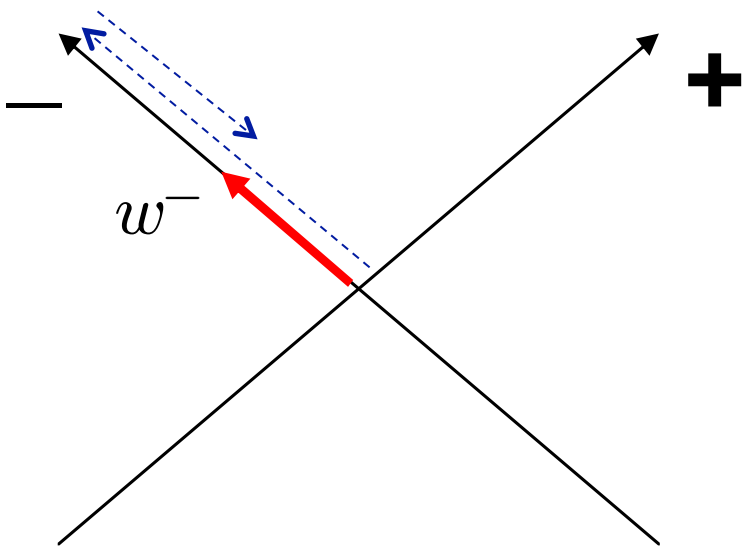
- Wilson lines enforce gauge invariance.

$$\underline{V_w(n)} = P \exp\left(-igt^a \int_0^\infty d\lambda\, n \cdot A^a(w + \lambda n)\right)$$

$$V_w^\dagger(u_J)\, V_0(u_J) = P \exp\left(-igt^a \int_0^{w^-} d\lambda\, u_J \cdot A^a(\lambda u_J)\right)$$

Gauge Links/Wilson Lines

- Paths of Wilson lines in coordinate space:



Standard (Integrated)

Complications with TMD definitions:

- Divergences
  - New kinds!
- Wilson lines / gauge links (what are they?).
- Universality.
- Interface with collinear factorization at large transverse momentum.
- ***Definitions are dictated by the requirements for factorization!***

Gauge Links/Wilson Lines

- Integrated PDF:

$$f(x) = \text{F.T.}\, \langle p|\, \bar{\psi}(0, w^-, \mathbf{0}_t) \underline{V_w^\dagger(u_\text{J})} \gamma^+ \underline{V_0(u_\text{J})} \psi(0) |p\rangle$$

$$u_\text{J} = (0, 1, \mathbf{0}_t)$$

Gauge Links/Wilson Lines

- Fields no longer evaluated along light-like separation.

$$\Phi(x, \mathbf{k}_t) = \quad \text{F.T.} \langle p| \bar{\psi}(0, w^-, \mathbf{w}_t) \quad ?? \quad \psi(0) |p\rangle$$

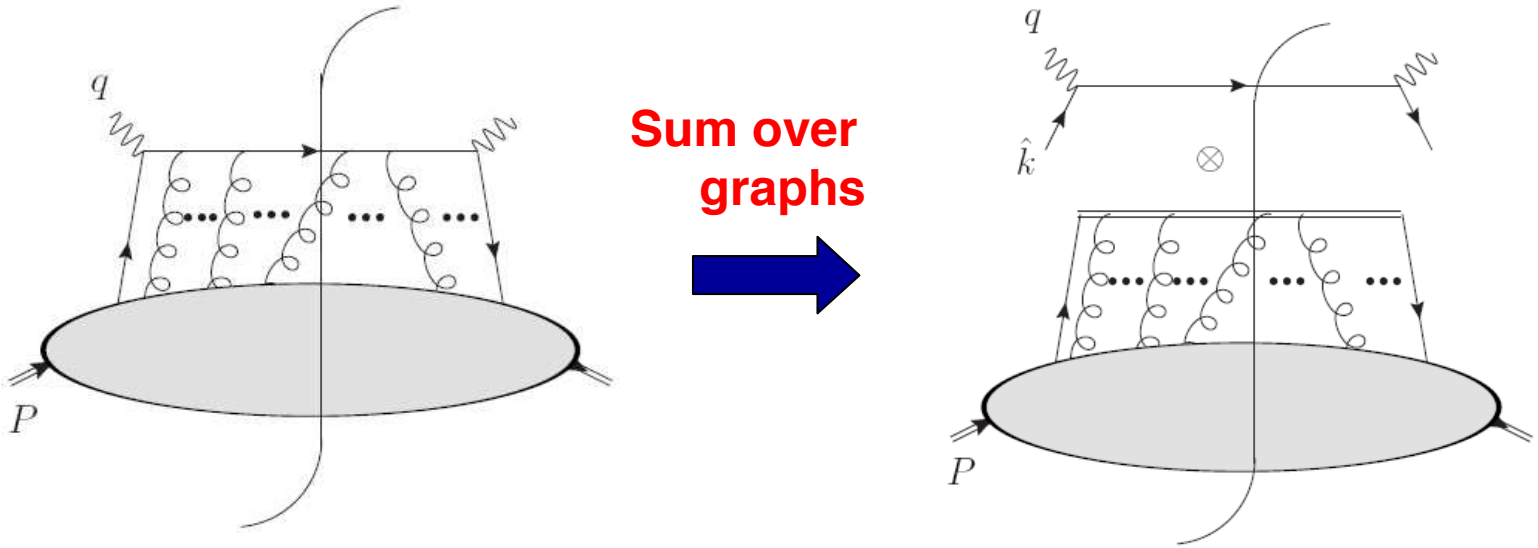
- Extend standard definition *(first try)*:

Link at infinity

$$\Phi^{[+]}(x, \mathbf{k}_t) = \text{F.T.} \langle p| \bar{\psi}(0, w^-, \mathbf{w}_t) \underbrace{V_w^\dagger(u_J) I_{n;w,0} \gamma^+ V_0(u_J)}_{U[0,w]} \psi(0) |p\rangle$$

(Boer, Mulders, Pijlman (2003))
(Belitsky, Ji, Yuan (2003))

Ward Identity: Usual (Integrated) PDF



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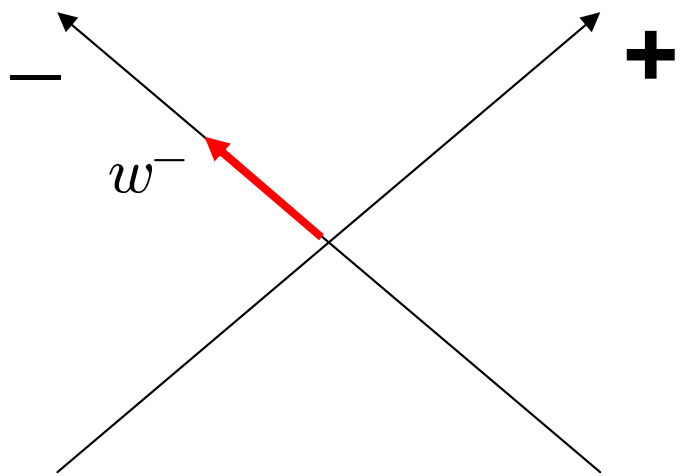
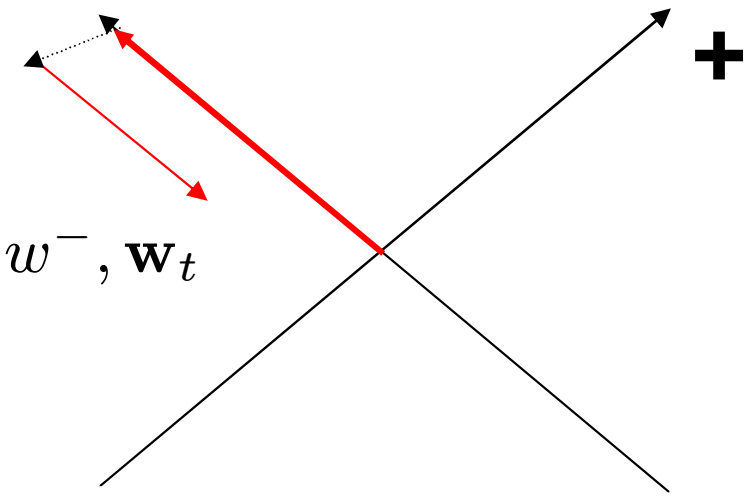
TMD PDFs: Gauge Links/Wilson Lines

- Paths of Wilson lines in coordinate space:



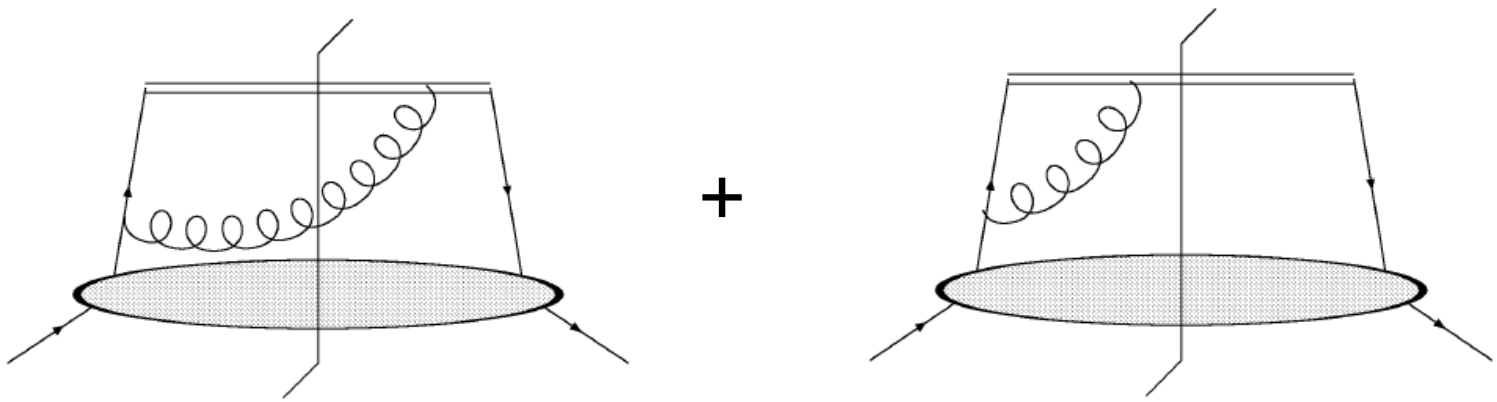
Standard (Integrated)



Unintegrated (First Try)

"Rapidity" Divergences in TMD - Factorization:

- Uncanceled light-cone (rapidity) divergences.



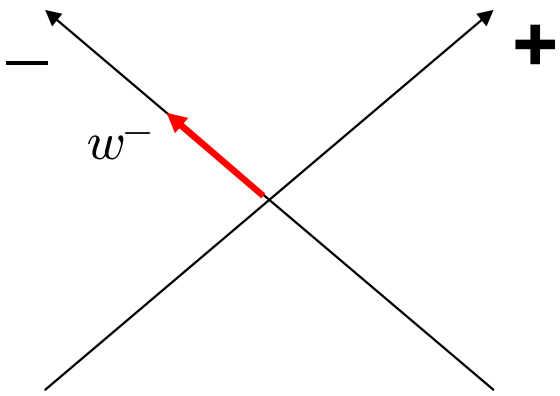
- Use non-light-like Wilson lines.

- Introduces new arbitrary parameter.
  - Predictability recovered with new evolution equations (Collins-Soper equations). ***(Collins, Soper (1983))***

[Exercise: Directly calculate these diagrams with quark/gluon masses
"What exactly is a parton density" (Collins, 2003)]

TMD PDFs: Gauge Links/Wilson Lines

- Paths of Wilson lines in coordinate space:



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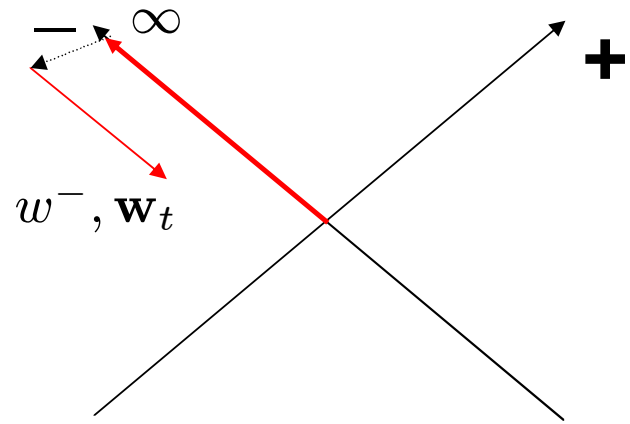

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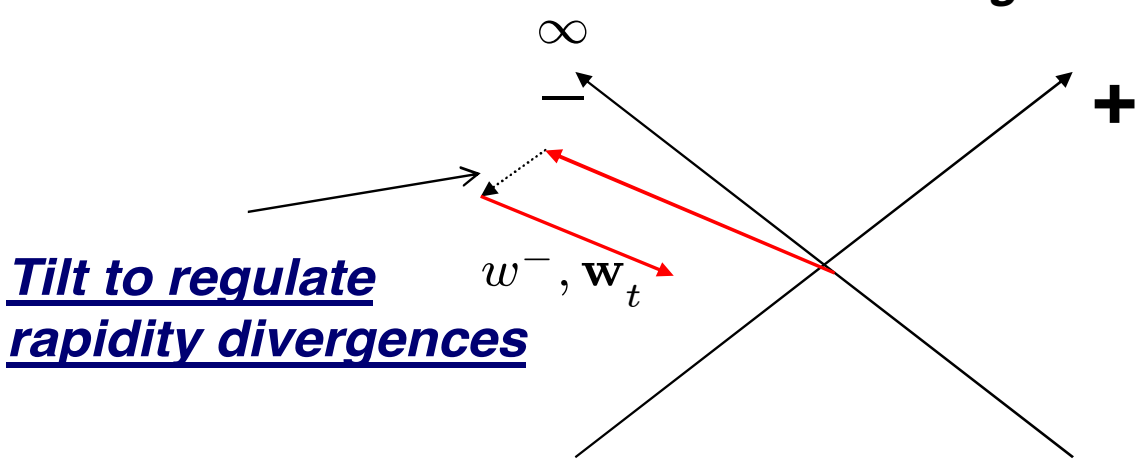
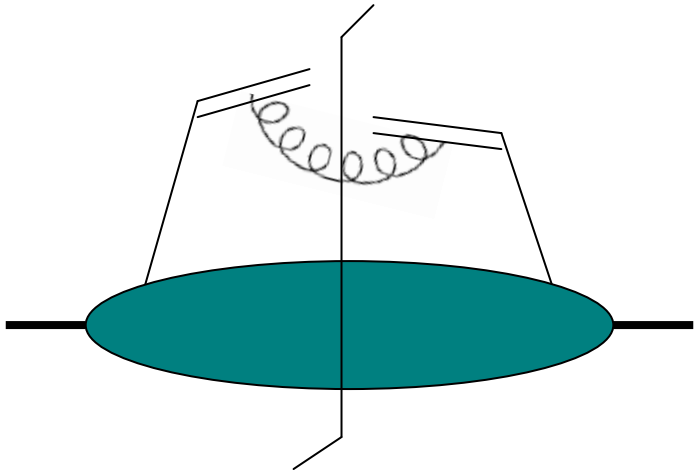


Tilt to regulate rapidity divergences

Unintegrated "tilted" Wilson lines

TMD PDFs: Other Divergences

- Precise definition of TMD PDFs still need some further modification. ***(Cherednikov, Stefanis (2008))***



Nothing to do with original, unfactorized graphs.

- Extra divergences… should cancel in definition.

Part 2: TMD PDFs and Phenomenology

TMD Phenomenology:

- Previously existing implementations:

  – Use existing fixed-scale fits / no evolution.

(Schweitzer, Teckentrup, Metz (2010))
(Anselmino et al., (2009…))
(Collins et al., (2006))
(D'Alesio, Murgia (2008))

Generalized Parton Model

TMD-Factorization

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f \underline{\left|\mathcal{H}_f(Q)^2\right|^{\mu\nu}} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \underline{F_{f/P_2}(x_1, \mathbf{k}_{1T})}\, \underline{F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T})} \times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

Leading order hard part

No evolution

Generalized Parton Model

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  Generalized Parton Model

  – Existing "Old fashioned" implementations of Collins-Soper-Sterman formalism.

  (Landry et al., (2003))
  (Qiu, Zhang, (2001))

Evolved Cross Section:

- Typical appearance of Collins-Soper-Sterman implementation:

(Contrast with GPM picture.)

$$d\sigma \sim \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{q}_T}$$

$$\times \int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P_1}(\hat{x}_1, \mu_b)$$

$$\times \int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{f/j}(x_2/\hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P_2}(\hat{x}_2, \mu_b)$$

$$\times \exp\left[\int_{1/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \left\{\mathcal{A}(\alpha_s(\mu')) \ln \frac{Q^2}{\mu'^2} + \mathcal{B}(\alpha_s(\mu'))\right\}\right]$$

$$\times \exp\left[-g_k(b) \ln \frac{Q^2}{Q_0^2} - g_1(x_1, b) - g_2(x_2, b)\right]$$

TMD definition is given in CSS derivation, but it is difficult to indentify what should by fit!

+ Large q_T term

What is needed?

(Think analogy with collinear factorization.)

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q)^2|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_2}(x_1, \mathbf{k}_{1T})\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}) \times$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

- Using newest definitions:

  (Collins book, 2011)

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+\, Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).$$

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Process dependence in hard part

Universal PDFs with evolution

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$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).$$

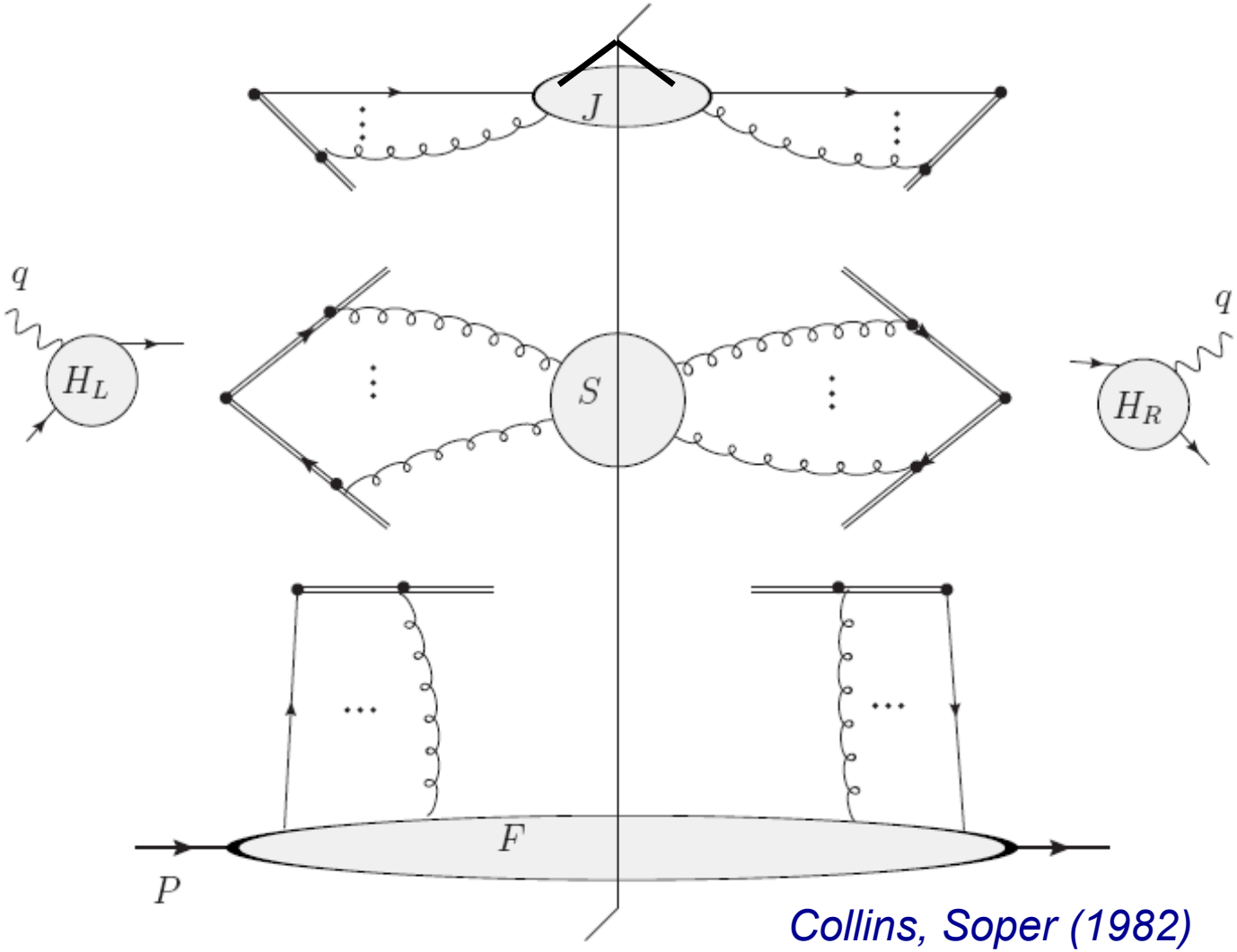
Large Transverse Momentum Correction

Process dependence in hard part

Universal PDFs with evolution

Part 3: More on Definitions

TMD-Factorization:



Collins, Soper (1982)
Collins, Soper, Sterman (1985)
Ji, Ma, Yuan (2004,2005) (SIDIS)

- "Unsubtracted" TMD PDF:

$$\tilde{F}^{\rm unsub}_{f/P^\uparrow}(x, \mathbf{b}_{\rm T}, S; \mu; y_P - y_B)$$
$$= {\rm Tr}_C {\rm Tr}_D \int \frac{dw^-}{2\pi} e^{-ixP^+w^-} \langle P, S|\bar{\psi}_f(w/2) W(w/2, \infty, n_B(y_B))^\dagger \frac{\gamma^+}{2} W(-w/2, \infty, n_B(y_B)) \psi_f(-w/2)|P, S\rangle_c$$

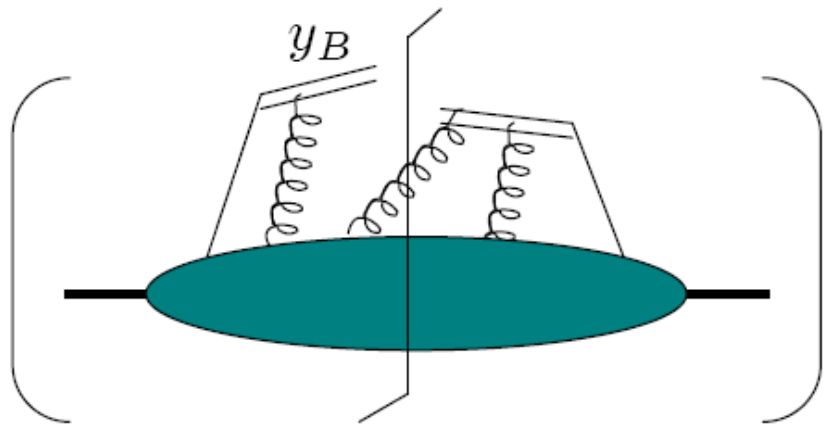
$$n_{\rm A} = (1, -e^{-2y_{\rm A}}, \mathbf{0}_T), \qquad n_{\rm B} = (-e^{2y_{\rm B}}, 1, \mathbf{0}_T)$$

- Soft Factor:

$$\tilde{S}_{(0)}(\mathbf{b}_{\rm T}; y_A, y_B) = \frac{1}{N_c} \langle 0| W(\mathbf{b}_{\rm T}/2, \infty; n_B)^\dagger_{ca} W(\mathbf{b}_{\rm T}/2, \infty; n_A)_{ad} W(-\mathbf{b}_{\rm T}/2, \infty; n_B)_{bc} W(-\mathbf{b}_{\rm T}/2, \infty; n_A)^\dagger_{db} |0\rangle$$

- "Unsubtracted" TMD PDF:

$$\tilde{F}^{\text{unsub}}_{f/P}(x, \mathbf{b}; \mu; y_P - y_B) \sim \text{F.T.}$$



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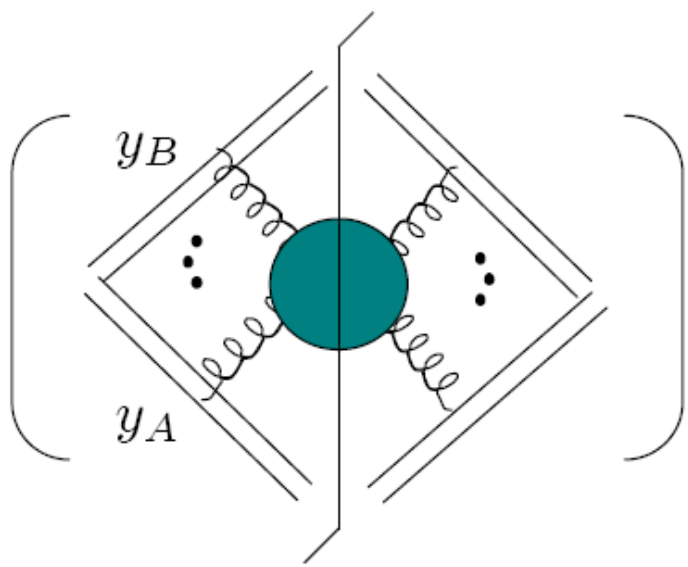

TMD PDF, Complete Definition:

$$\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; y_P - (-\infty)) \sqrt{\frac{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; y_s, -\infty)}} Z_F\, Z_2$$

From *Foundations of Perturbative QCD*, J.C. Collins,
(See also, Collins, TMD 2010 Trento Workshop)

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"Usual" definitions

Standard UV renormalization factor

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$$\zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)}$$

"Usual" definitions

Cuts off soft rapidities

Standard UV renormalization factor

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"Usual" definitions

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Cuts off soft rapidities

Standard UV renormalization factor

Analogous parameter in FF: $\zeta_D = (M_h^2/z^2)e^{2(y_s - y_h)}$

Condition: $\sqrt{\zeta_F \zeta_D} = Q^2$

From *Foundations of Perturbative QCD*, J.C. Collins,
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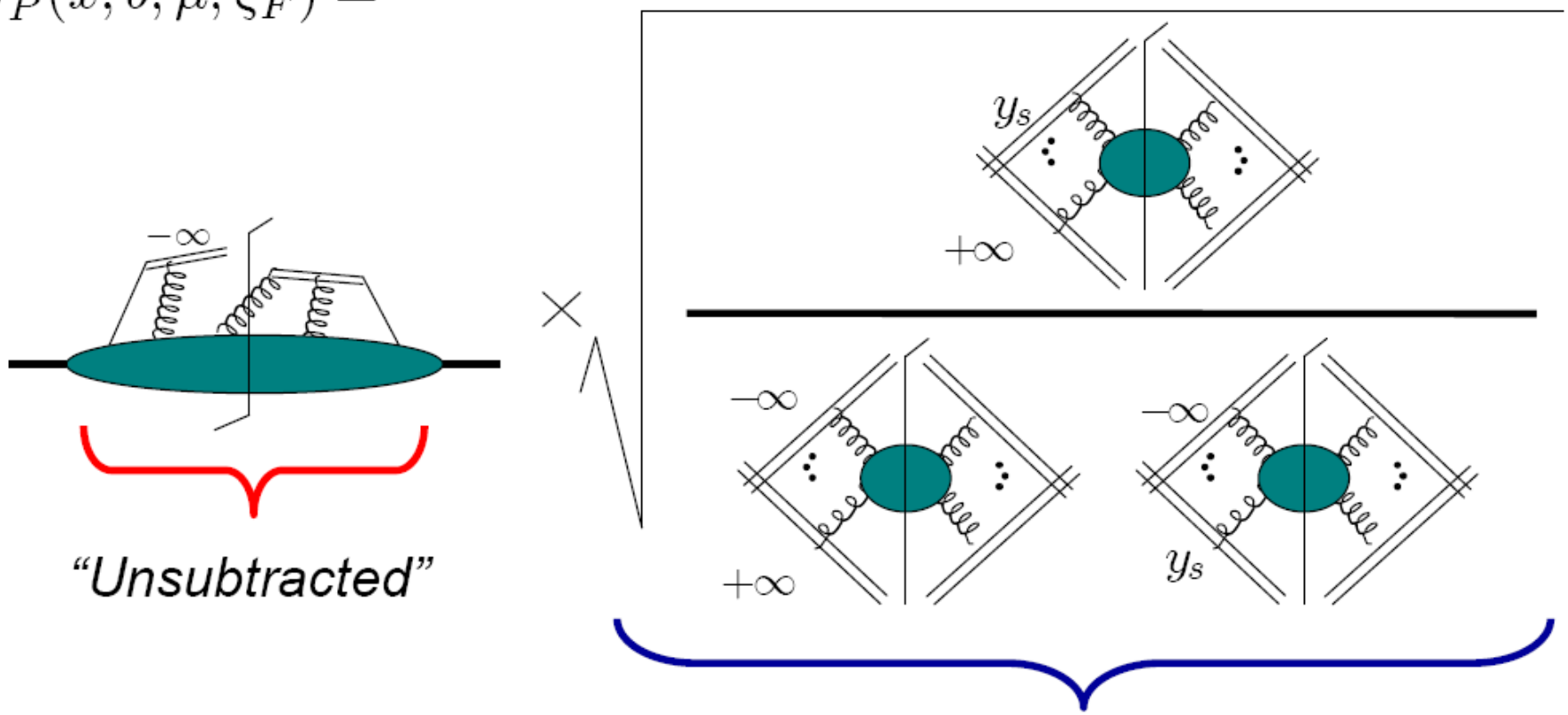
TMD PDF, Complete Definition:

$$F_{f/P}(x, b; \mu; \zeta_F) =$$



"Unsubtracted"

Implements Subtractions/Cancellations

From *Foundations of Perturbative QCD*, J.C. Collins,
(See also, Collins, TMD 2010 Trento Workshop)

Understanding the Definition:

- Start with only the hard part factorized:

Naïve Factorization:

$$d\sigma = |\mathcal{H}|^2 \, \tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2).$$

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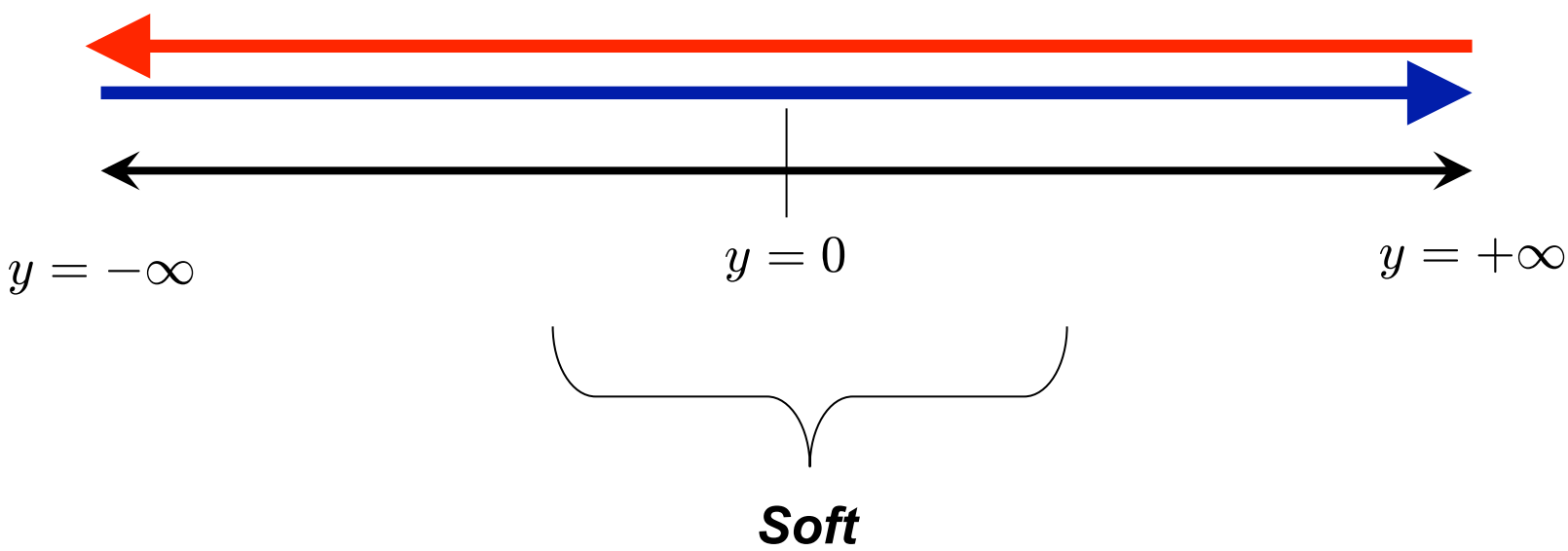

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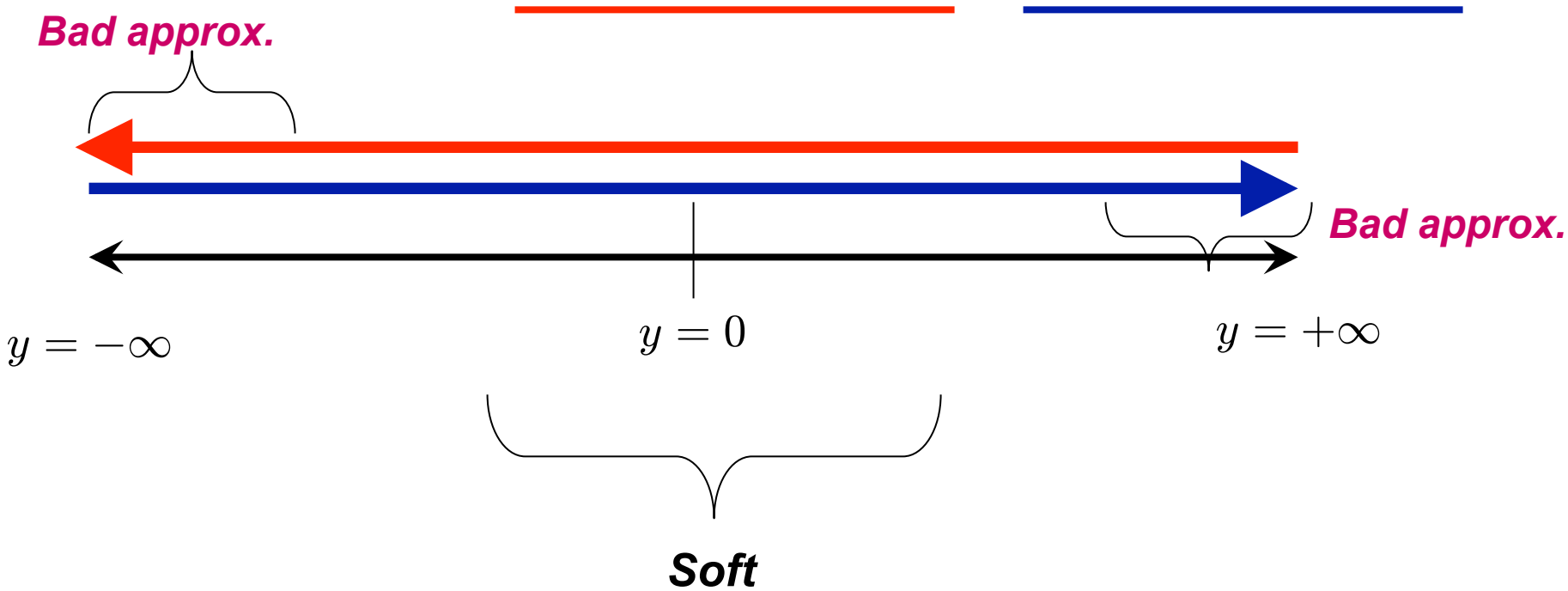

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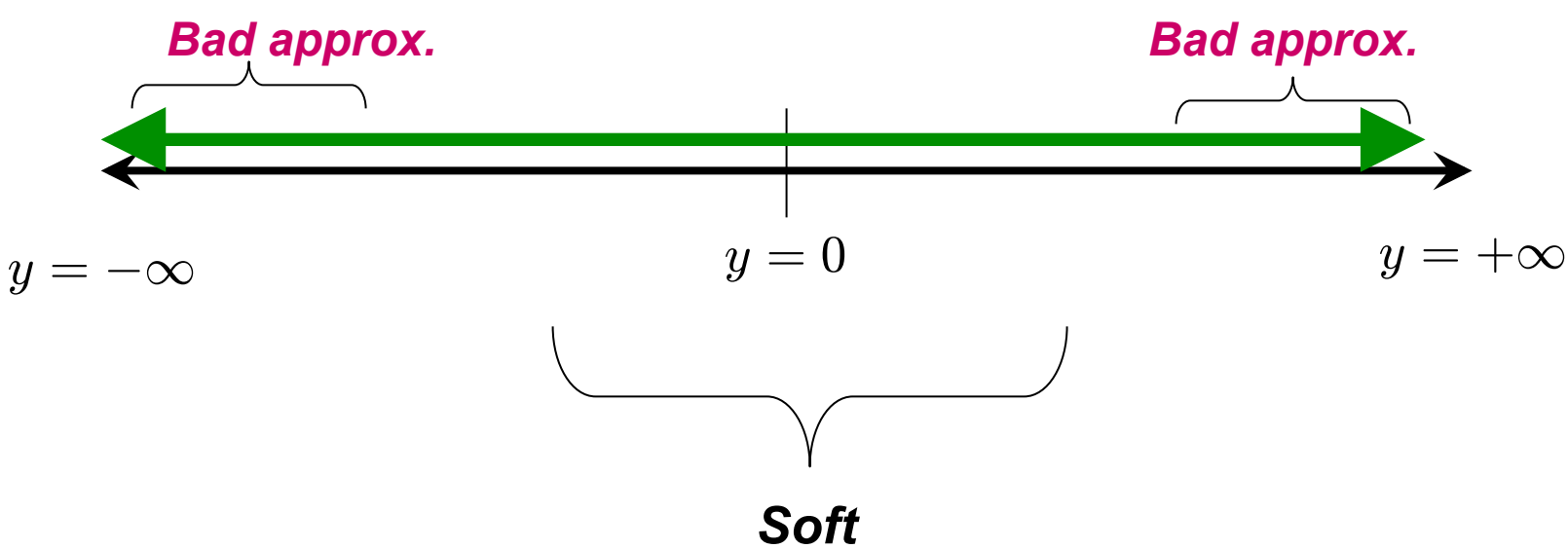
$$d\sigma = |\mathcal{H}|^2 \, \tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2).$$

$\tilde{S}(+\infty, -\infty)$



Understanding the Definition:

- Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

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- Separate soft part:

$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\text{unsub}}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

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- Multiply by: $\dfrac{\sqrt{\tilde{S}(+\infty, y_s)\, \tilde{S}(y_s, -\infty)}}{\sqrt{\tilde{S}(+\infty, y_s)\, \tilde{S}(y_s, -\infty)}}$

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- Rearrange factors: $d\sigma = |\mathcal{H}|^2 \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\dfrac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\}$

$$\times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

Understanding the Definition:

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- Rearrange factors: $d\sigma = |\mathcal{H}|^2 \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\dfrac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\}$

$$\times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

Summary: Complete Definition:

- No more pathologies or unregulated divergences.
  - Non-perturbative models?? Lattice Calcs??

    (See, e.g., Musch, Hagler (2011))
- Universal (for factorizable processes).

  Remember Sivers sign flip
- Evolution understood.
- Follows from TMD-factorization derivation.
  - Consistent matching to large-q_T collinear treatment that preserves factorization point-by-point in q_T.
- No *explicit* soft factor in the TMD factorization formula.

What is needed?

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q)^2|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_2}(x_1, \mathbf{k}_{1T})\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}) \times$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

- Using newest definitions:

(Collins book, 2011)

$$W^{\mu\nu} = \sum_f |\underline{\mathcal{H}_f(Q;\mu)^2}|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \underline{F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)}\, \underline{F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)}$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).$$

Large Transverse Momentum Correction

Process dependence in hard part

Universal PDFs with evolution

Evolution

- Collins-Soper Equation:
  - $$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Perturbatively calculable from definition at small b.

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

- RG:
  - $$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_K(g(\mu))$$
  - $$\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

Perturbatively calculable, from definitions

Implementing Evolution

- Small b_T:

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \overbrace{\tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b))}^{\textit{Perturbatively calculable coefficient functions}} \overbrace{f_{j/H}(x, \mu_b)}^{\textit{Standard (int) PDF}} \times$$

$$\left.\boxed{b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*}\right\} \textit{CSS matching procedure}$$

Not Unique!

Implementing Evolution

- Evolution (valid at all b_T):

Perturbatively calculable

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

Implementing Evolution

- Matching to non-pertubative b_T-dependence:

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ g_{j/H}(x, b_T) + \underbrace{g_K(b_T)}_{\text{Universal}} \ln \frac{\sqrt{\zeta}}{Q_0} \right\}$$

Non-Pertubative b_T dependence

Universal
(same for PDFs, FFs etc…)

Implementing Evolution

- After evolution:

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times \Bigg\} \text{ A}$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \Bigg\} \text{ B}$$

$$\times \exp\left\{ g_{j/H}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta}}{Q_0} \right\} \Bigg\} \text{ C}$$

$$\boxed{b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*}$$

CSS matching procedure

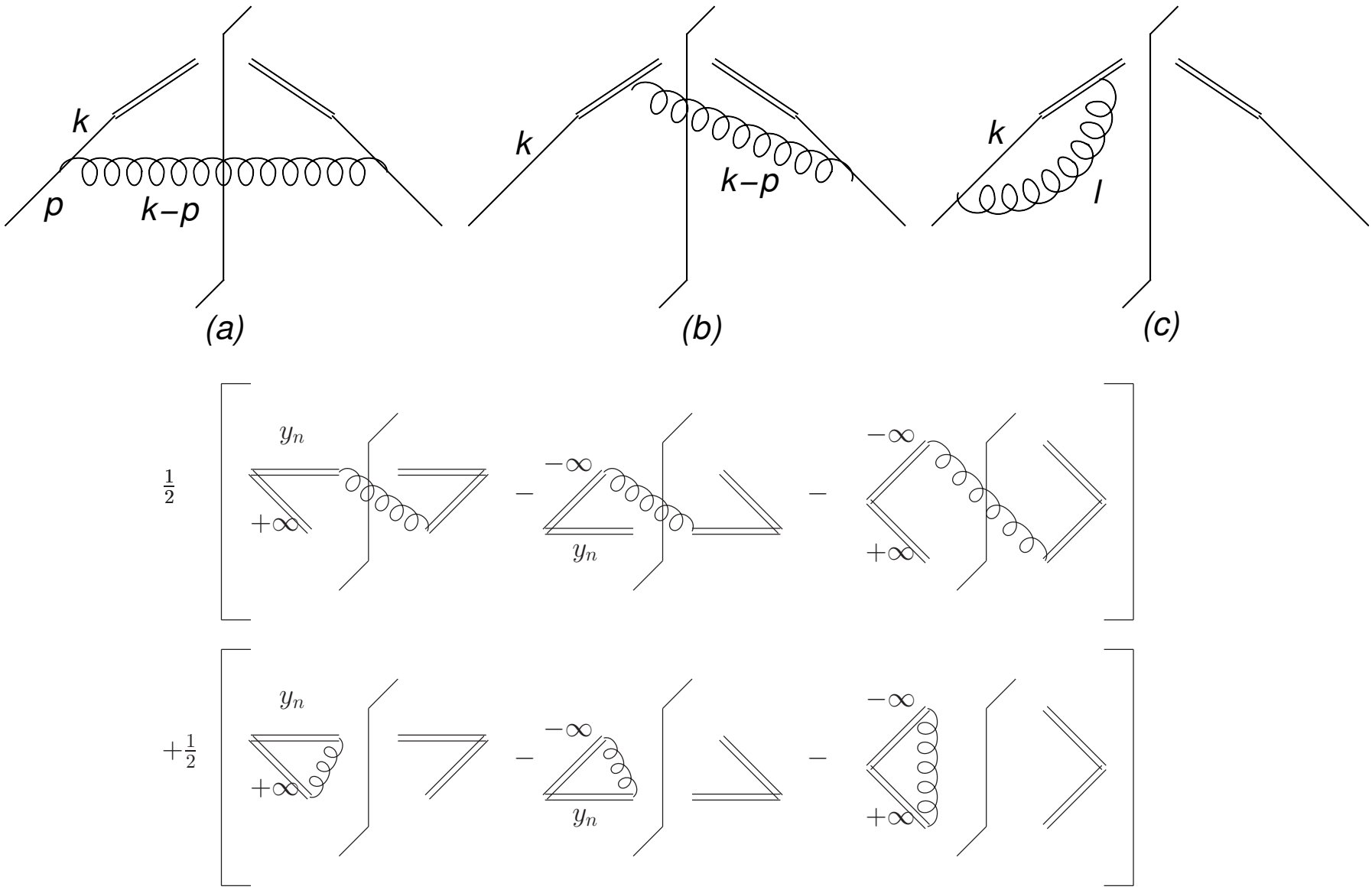
Not Unique!

Part 4: Explicit determination of TMDs

Calculate

$$\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; y_P - (-\infty)) \sqrt{\frac{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; y_s, -\infty)}} Z_F \, Z_2$$

Calculate


k
p
k−p
(a)
k
k−p
(b)
k
l
(c)
$\frac{1}{2}$
y_n
$+\infty$
$-$
$-\infty$
y_n
$-$
$-\infty$
$+\infty$
$+\frac{1}{2}$
y_n
$+\infty$
$-$
$-\infty$
y_n
$-$
$-\infty$
$+\infty$

Calculate

$$\tilde{C}_{j'/j}(x,\mathbf{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{2\pi}\left\{2\left[\ln\left(\frac{2}{\mu b_{\rm T}}\right)-\gamma_{\rm E}\right]\left[\left(\frac{2}{1-x}\right)_+ - 1 - x\right] + 1 - x + \right.$$

$$\left. +\delta(1-x)\left[-\frac{1}{2}\left[\ln\left(b_{\rm T}^2\mu^2\right) - 2(\ln 2-\gamma_{\rm E})\right]^2 - \left[\ln(b_{\rm T}^2\mu^2) - 2(\ln 2-\gamma_{\rm E})\right]\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2).$$

$$\gamma_{\rm F}(\mu;\zeta_F/\mu^2) = \alpha_s\frac{C_{\rm F}}{\pi}\left(\frac{3}{2} - \ln\left(\frac{\zeta_F}{\mu^2}\right)\right) + \mathcal{O}(\alpha_s^2)$$

$$\tilde{K}(\mu,b_T) = -\frac{\alpha_s C_F}{\pi}\left[\ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_{\rm E}\right] + \mathcal{O}(\alpha_s^2).$$

$$\gamma_K(\mu) = 2\frac{\alpha_s C_F}{\pi} + \mathcal{O}(\alpha_s^2)$$

Implementing Evolution

- After evolution:

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times \Big\} \text{ A}$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \Big\} \text{ B}$$

$$\times \exp\left\{ g_{j/H}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta}}{Q_0} \right\} \Big\} \text{ C}$$

$$\boxed{b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*}$$

CSS matching procedure

Not Unique!

Momentum Space

$$F(x, \mathbf{k}_T; \mu; \zeta) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T \, e^{-i\mathbf{k}_T \cdot \mathbf{b}_T} \, \tilde{F}(x, \mathbf{b}_T; \mu; \zeta)$$

What is needed?

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q)^2|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_2}(x_1, \mathbf{k}_{1T})\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}) \times$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

- Using newest definitions:

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)$$

$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).$$

Large Transverse Momentum Correction

Process dependence in hard part

Universal PDFs with evolution

Our strategy up to now:

- Use evolution to combine existing fits into unified/global fits that include evolution.

  (S.M. Aybat, TCR (2011))

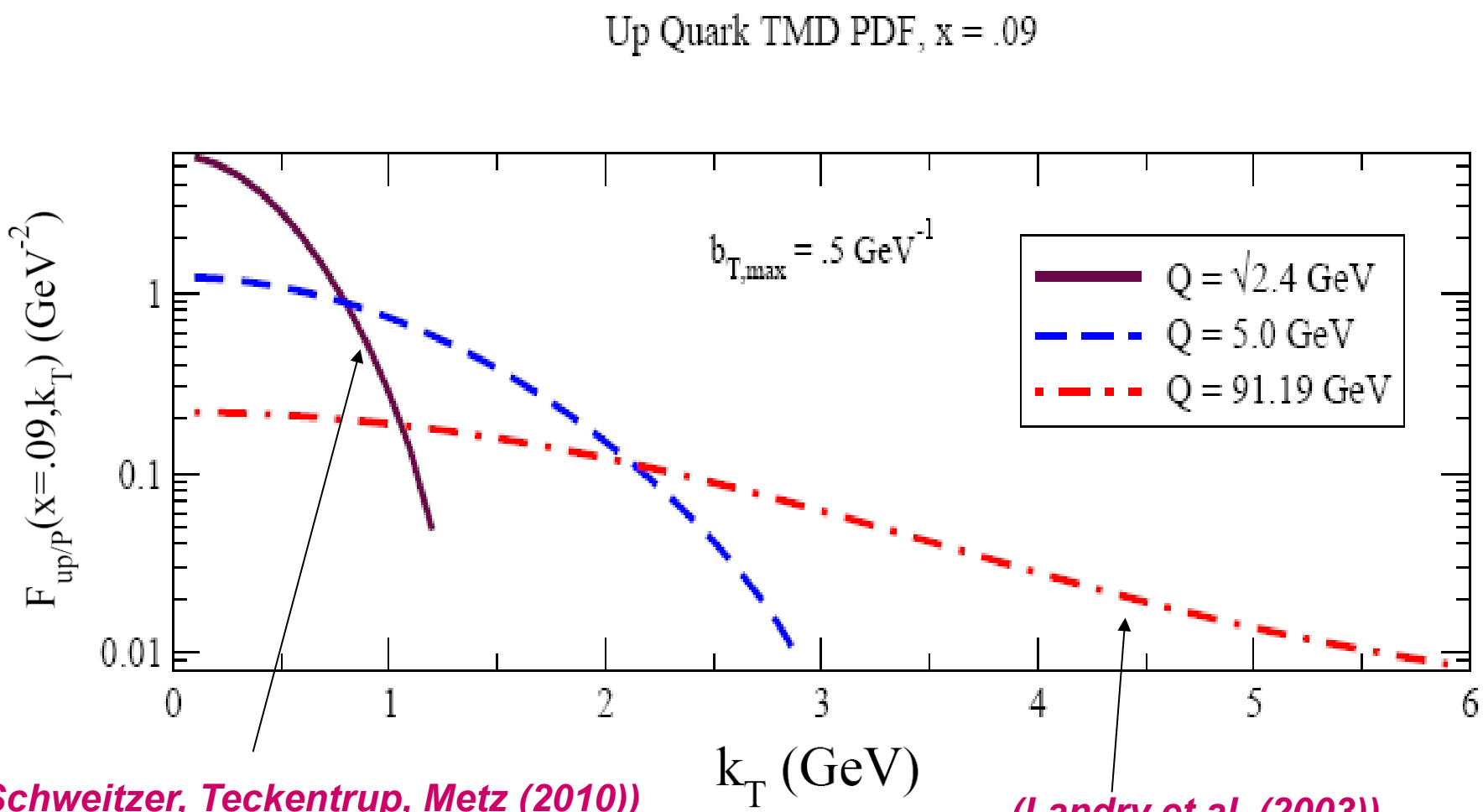
  – PDFs:
    - Start with DY:

      (Landry et al, (2003); Konychev, Nadolsky (2006)) ***(BLNY)***
    - Modify to match to SIDIS:

      (Schweitzer, Teckentrup, Metz (2010)) ***(STM)***

- Can now supply explicit, evolved TMD PDF fit.

Evolving TMD PDFs



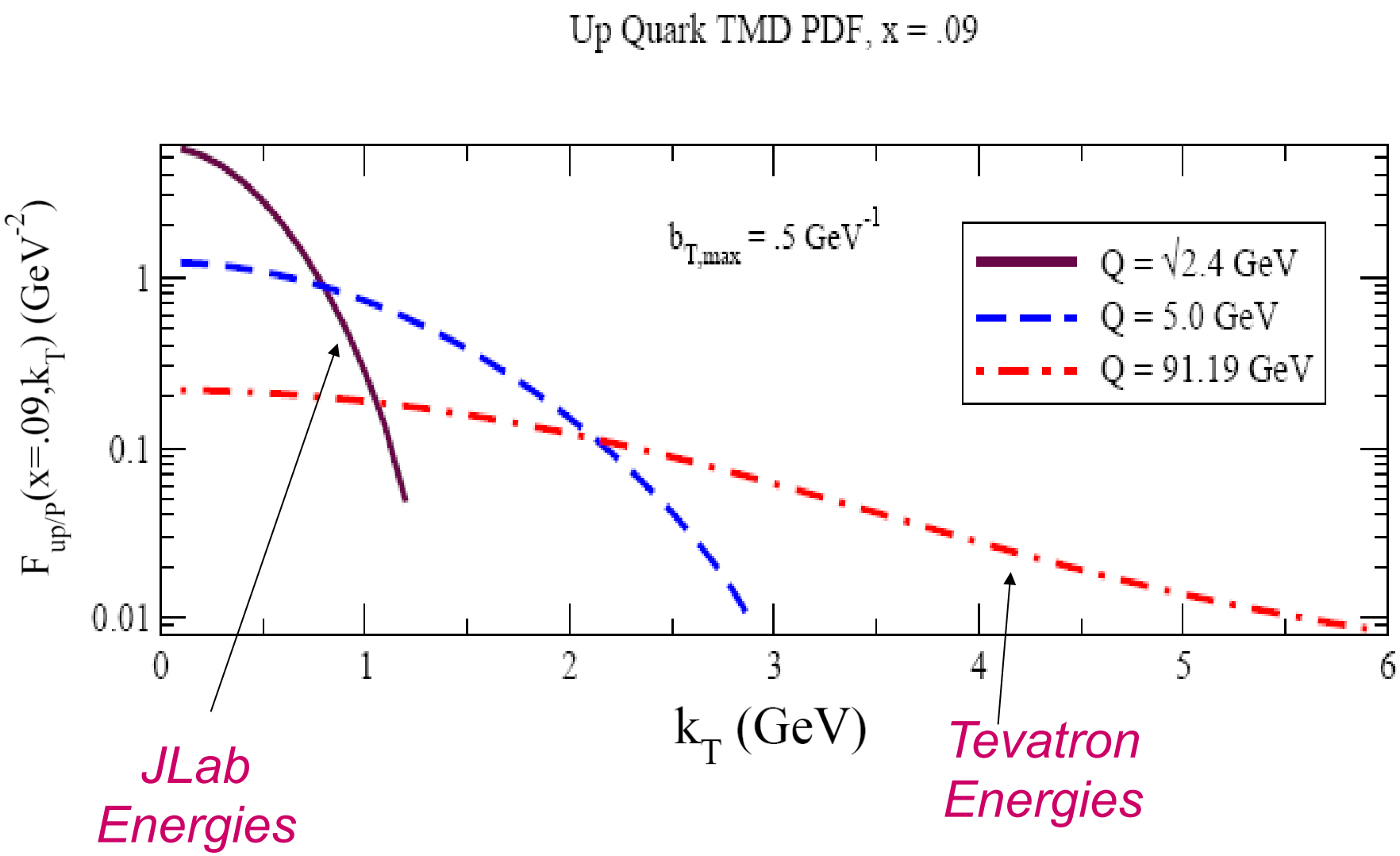
(Schweitzer, Teckentrup, Metz (2010))

(SIDIS)

(Landry et al, (2003))

(Drell-Yan)

Evolving TMD PDFs



Implementing Evolution

- After evolution:

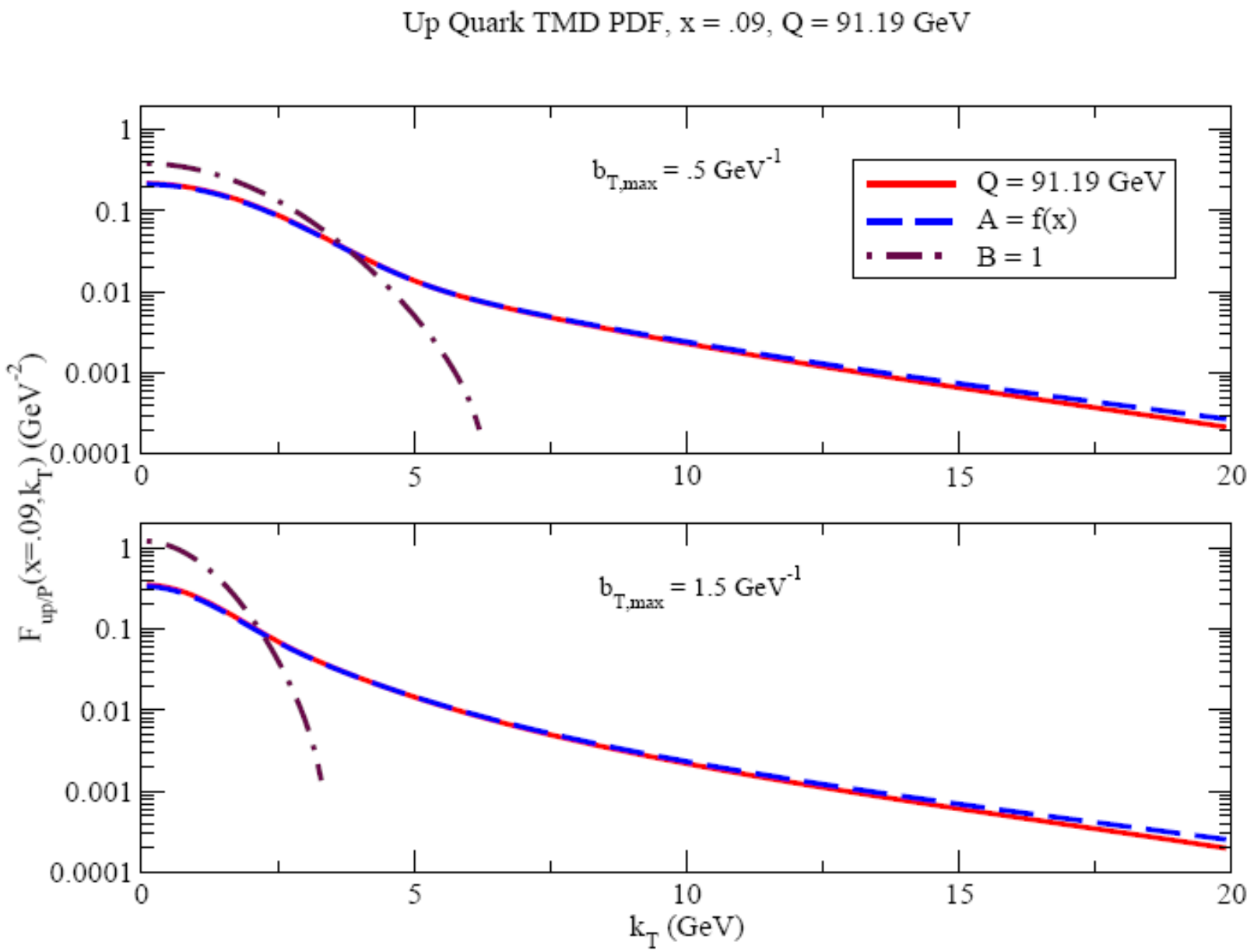
$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times \Bigg\} \text{ A}$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \Bigg\} \text{ B}$$

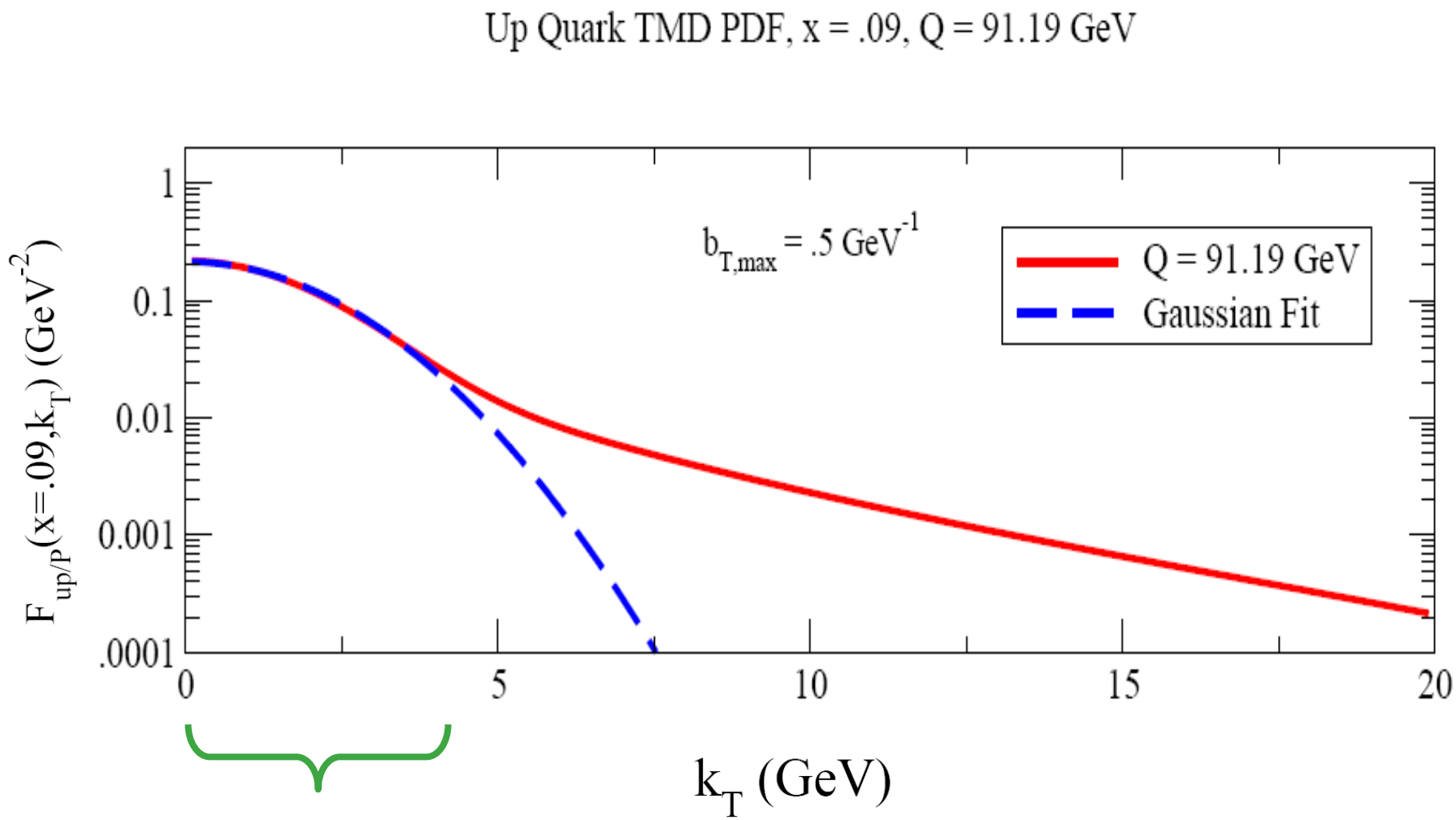
$$\times \exp\left\{ g_{j/H}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta}}{Q_0} \right\} \Bigg\} \text{ C}$$

$$\boxed{b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*}$$

CSS matching procedure


Up Quark TMD PDF, x = .09, Q = 91.19 GeV
$b_{T,max}$ = .5 GeV^{-1}
Q = 91.19 GeV
A = f(x)
B = 1
$b_{T,max}$ = 1.5 GeV^{-1}
$F_{up/P}(x=.09,k_T)$ (GeV^{-2})
k_T (GeV)
1
0.1
0.01
0.001
0.0001
0
5
10
15
20

Evolving TMD PDFs



Gaussian fit good at small k_T.

Unambiguous Hard Part

- Higher orders follow systematically from definitions:

$$W^{\mu\nu} = |\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu}\, F_{f/P_1} \otimes F_{f/P_2}$$

$$\Rightarrow \quad |\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

Unambiguous Hard Part

- Definition:

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

- Drell-Yan:

Unambiguous Hard Part

- Definition:

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

- Drell-Yan: $\overline{\text{MS}}$

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} =$$

$$e_f^2 |H_0^2|^{\mu\nu} \left(1 + \frac{C_F \alpha_s}{\pi} \left[\frac{3}{2} \ln\left(Q^2/\mu^2\right) - \frac{1}{2} \ln^2\left(Q^2/\mu^2\right) - 4 + \frac{\pi^2}{2}\right]\right) + \mathcal{O}(\alpha_s^2)$$

Unambiguous Hard Part

- Definition:

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

- Drell-Yan: $\overline{\text{MS}}$

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} =$$

$$e_f^2 |H_0^2|^{\mu\nu} \left(1 + \frac{C_F \alpha_s}{\pi} \left[\frac{3}{2} \ln\left(Q^2/\mu^2\right) - \frac{1}{2} \ln^2\left(Q^2/\mu^2\right) - 4 + \frac{\pi^2}{2}\right]\right) + \mathcal{O}(\alpha_s^2)$$

- SIDIS

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} =$$

$$e_f^2 |H_0^2|^{\mu\nu} \left(1 + \frac{C_F \alpha_s}{\pi} \left[\frac{3}{2} \ln\left(Q^2/\mu^2\right) - \frac{1}{2} \ln^2\left(Q^2/\mu^2\right) - 4\right]\right) + \mathcal{O}(\alpha_s^2)$$

Part 5: The Sivers Effect

Newest results: Sivers Function

(S.M. Aybat, J.C. Collins, J.W. Qiu, TCR:* arXiv:1110.6428*)

- $$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - \underbrace{F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)}_{\textit{Sivers}} \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

- Relevant transformations:

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = -2\pi \int_0^\infty dk_T\, k_T^2 J_1(k_T b_T) F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)$$

$$F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^\infty db_T\, b_T J_1(k_T b_T) \underline{\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F)}$$

See Boer, Gamberg, Musch, Prokudin (2011)

Newest results: Sivers Function

- $$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - \underbrace{F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)}_{\textbf{Sivers}} \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

- Recall the definition:

$$\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; y_P - (-\infty)) \sqrt{\frac{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathbf{b}_\mathrm{T}; y_s, -\infty)}} Z_F\, Z_2$$

Evolution equations apply to this.

Newest results: Sivers Function

- $$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - \underbrace{F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)}_{\textbf{Sivers}} \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

F.T.

$$= i \frac{1}{M_P} \frac{b_T^i}{b_T} \tilde{F}_{1T}^{\prime\, \perp\, f}(x, b_T; \mu, \zeta_F) \epsilon_{ij} S_T^j$$

- $\tilde{F}_{1T}^{\prime\, \perp\, f}(x, b_T; \mu, \zeta_F)$ obeys evolution equations. Same anomalous dimensions and CS kernal as in unpolarized case.

Evolved Sivers Function

- Small b_T:

(Kang, Xiao,Yuan (2011))

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}^{\rm Sivers}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$

Qiu-Sterman Function

Evolved Sivers Function

- Small b_T:

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}^{\rm Sivers}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g^{\rm Sivers}_{j/P}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}$$

Evolved Sivers Function

- Consider $k_T \ll Q \quad , \quad \sim \Lambda_{\rm QCD}$

Input distribution

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = \boxed{\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu_0, Q_0^2)}$$

$$\times \ \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right.$$

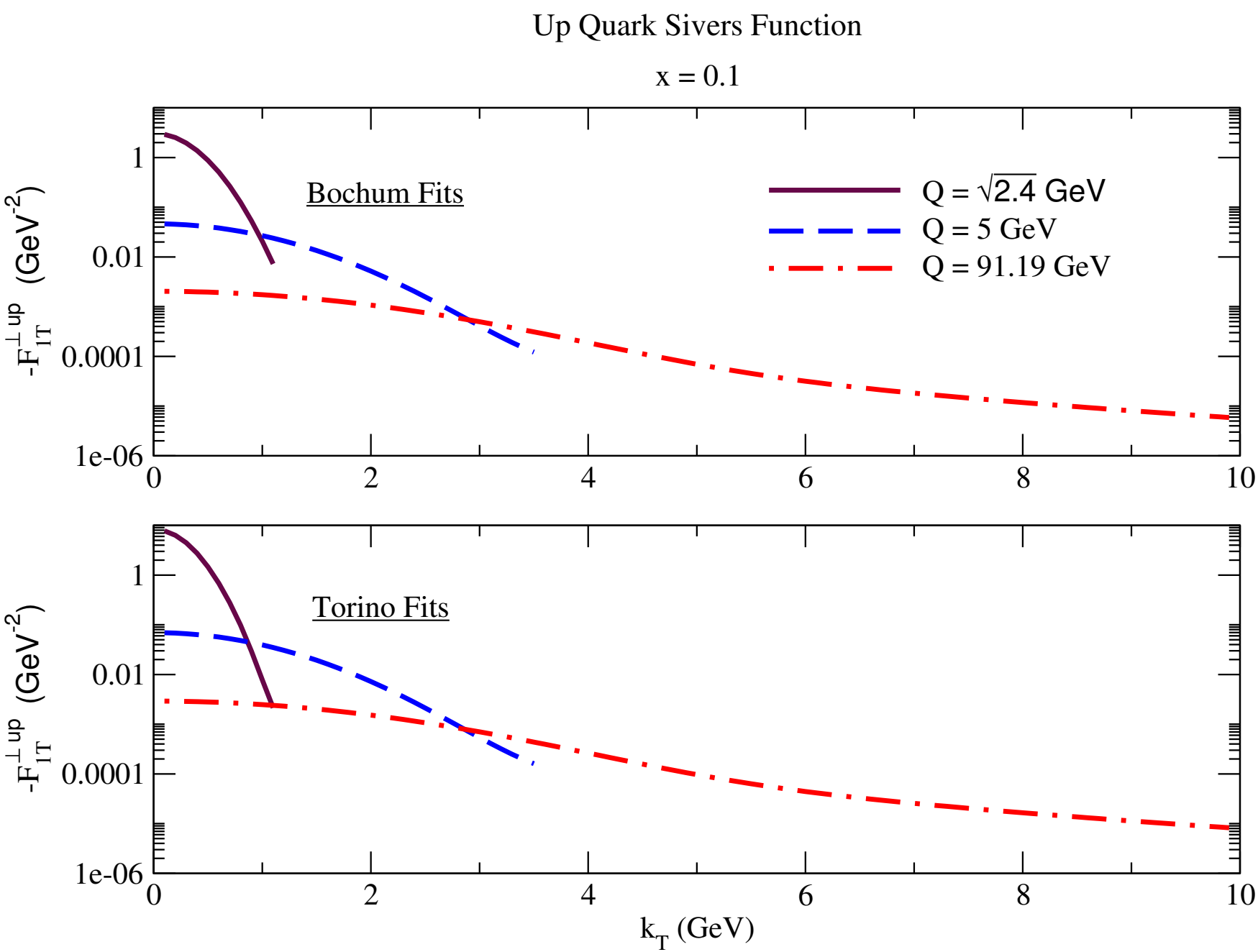
$$\left. + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}$$

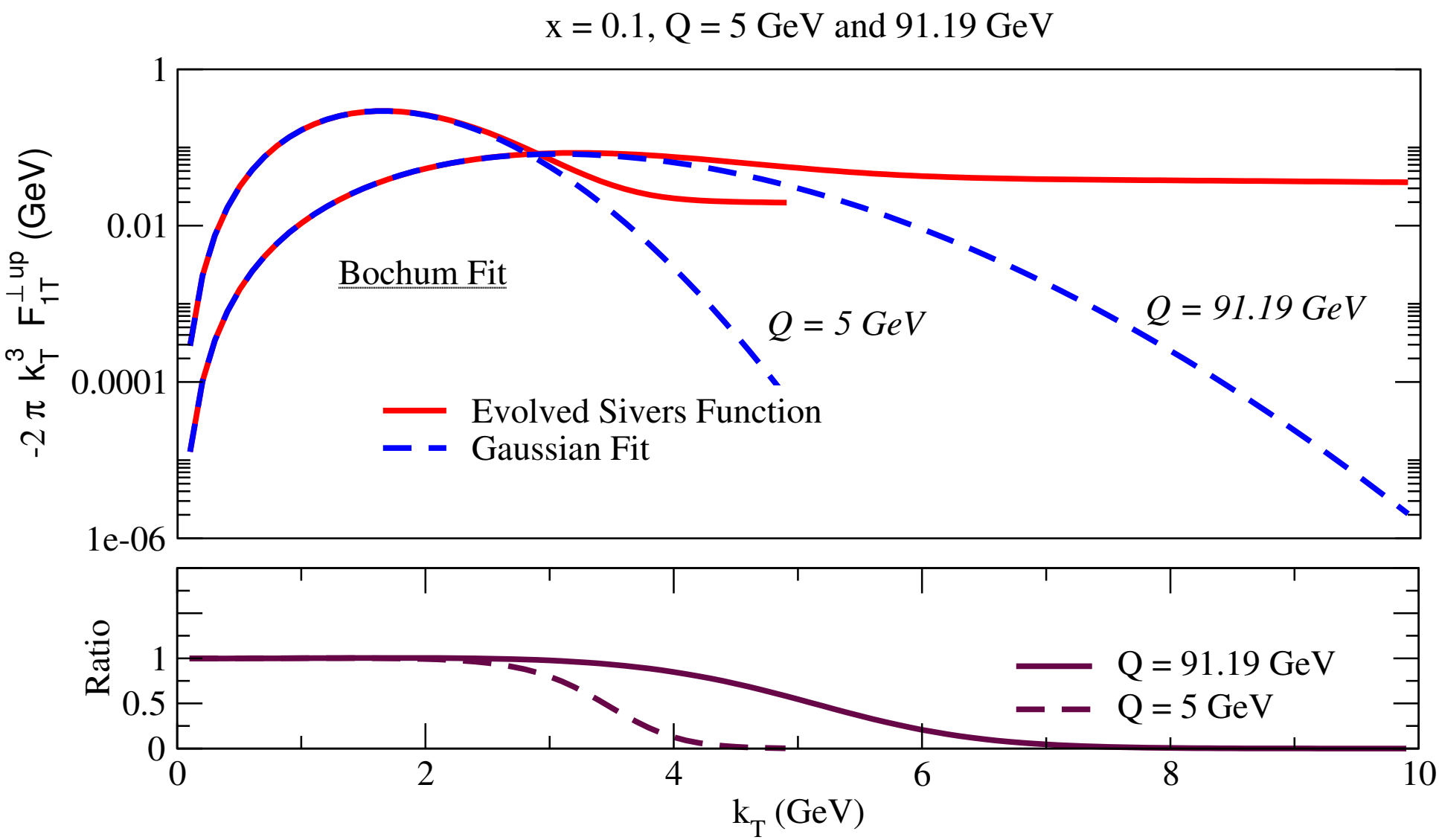
Evolve previously existing fixed-scale Gaussian fits

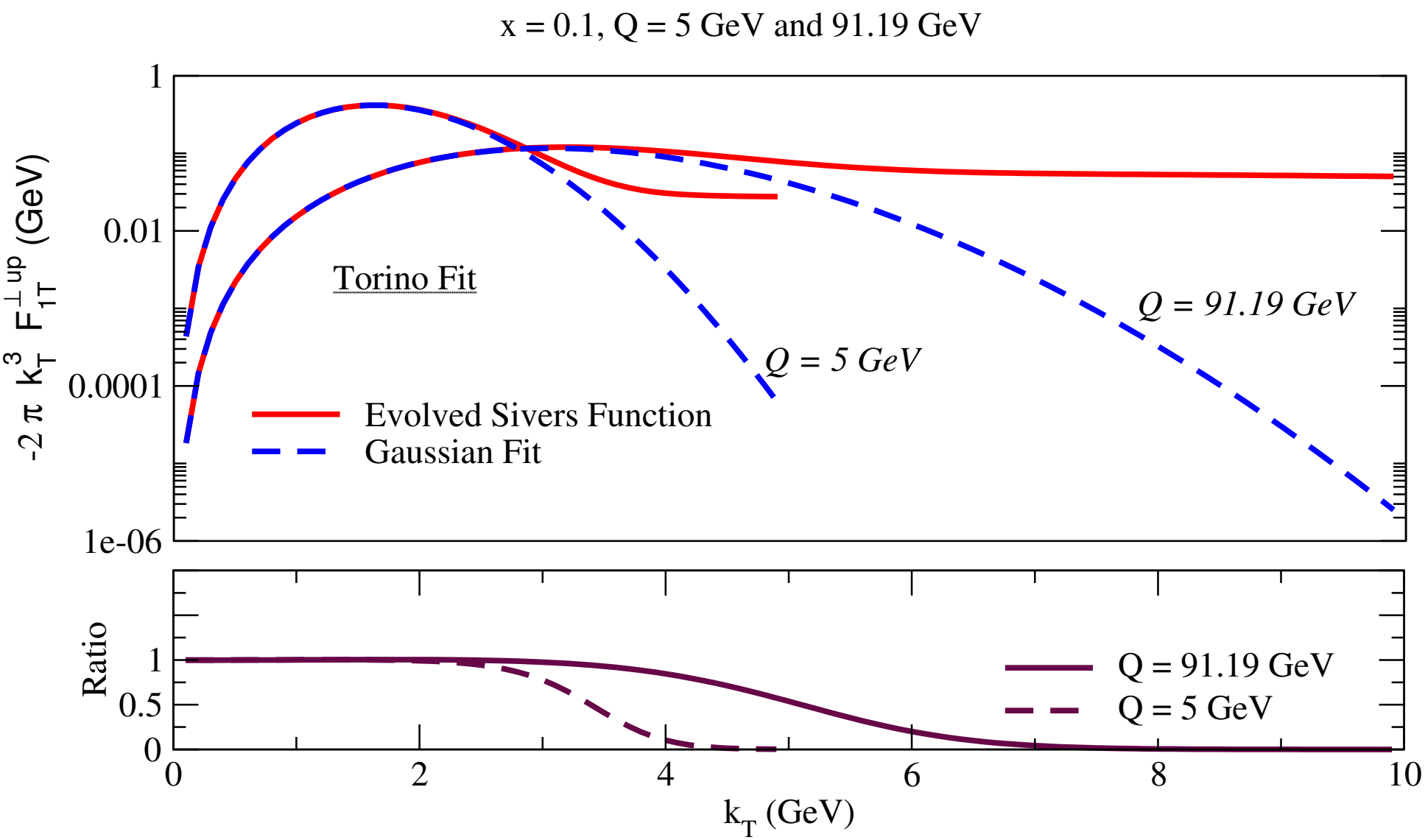
- Gaussian Fits.
  - $$F^{\perp}_{1T,\,0\,f}(x,k_T) = \frac{f^{\perp\,f}_{1T}(x)}{\langle k_T^2\rangle^f_0\pi}\exp\left[-k_T^2/\langle k_T^2\rangle^f_0\right]$$
  - Corresponds to:

$$\tilde{F}'^{\perp\,f}_{1T,\,0}(x,b_T) = -\frac{\langle k_T^2\rangle_0 f^{\perp}_{1T}(x)b_T}{2}\exp\left[-\langle k_T^2\rangle_0 b_T^2/4\right]$$

- Bochum Fits: **(Collins et al., (2006))**
- Torino Fits: **(Anselmino et al., (2009…))**


Up Quark Sivers Function
x = 0.1
Bochum Fits
Torino Fits
Q = √2.4 GeV
Q = 5 GeV
Q = 91.19 GeV
-F1T⊥ up (GeV-2)
kT (GeV)
1
0.01
0.0001
1e-06
0
2
4
6
8
10


x = 0.1, Q = 5 GeV and 91.19 GeV
-2 π k_T^3 F_1T^⊥up (GeV)
1
0.01
0.0001
1e-06
Bochum Fit
Q = 5 GeV
Q = 91.19 GeV
Evolved Sivers Function
Gaussian Fit
Ratio
1
0.5
0
Q = 91.19 GeV
Q = 5 GeV
0
2
4
6
8
10
k_T (GeV)

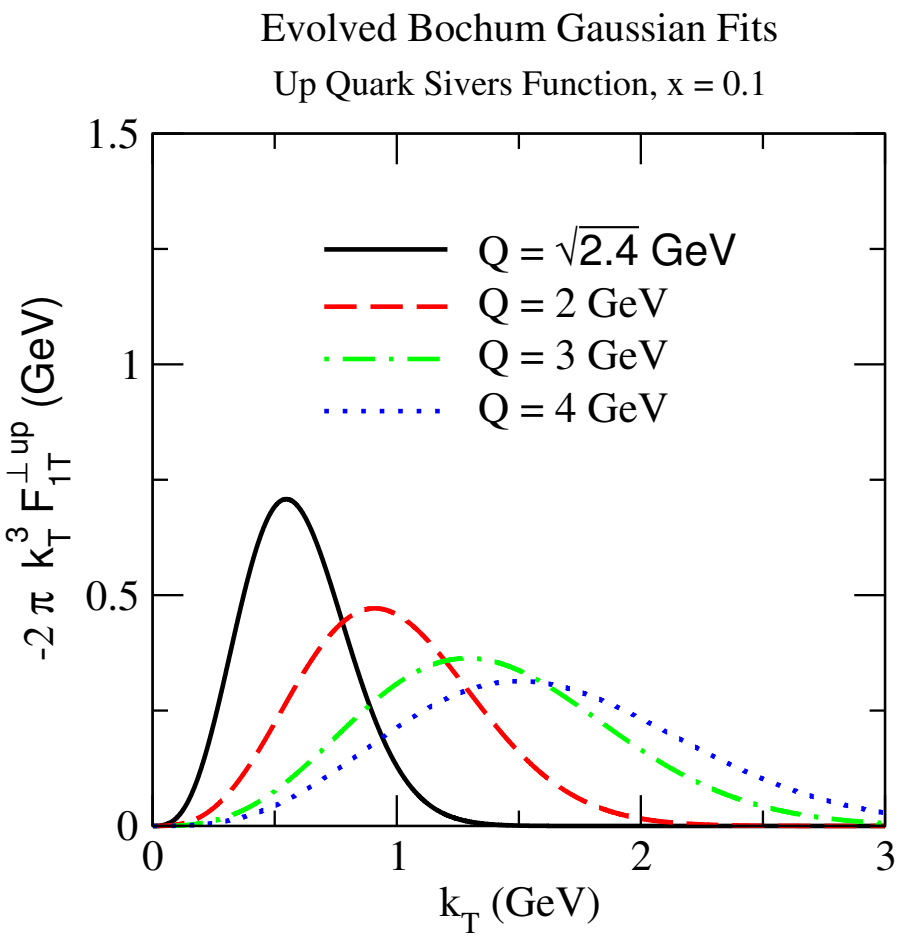

x = 0.1, Q = 5 GeV and 91.19 GeV
1
0.01
0.0001
1e-06
-2 π k_T^3 F_1T^⊥ up (GeV)
Torino Fit
Q = 91.19 GeV
Q = 5 GeV
Evolved Sivers Function
Gaussian Fit
Ratio
1
0.5
0
Q = 91.19 GeV
Q = 5 GeV
0
2
4
6
8
10
k_T (GeV)

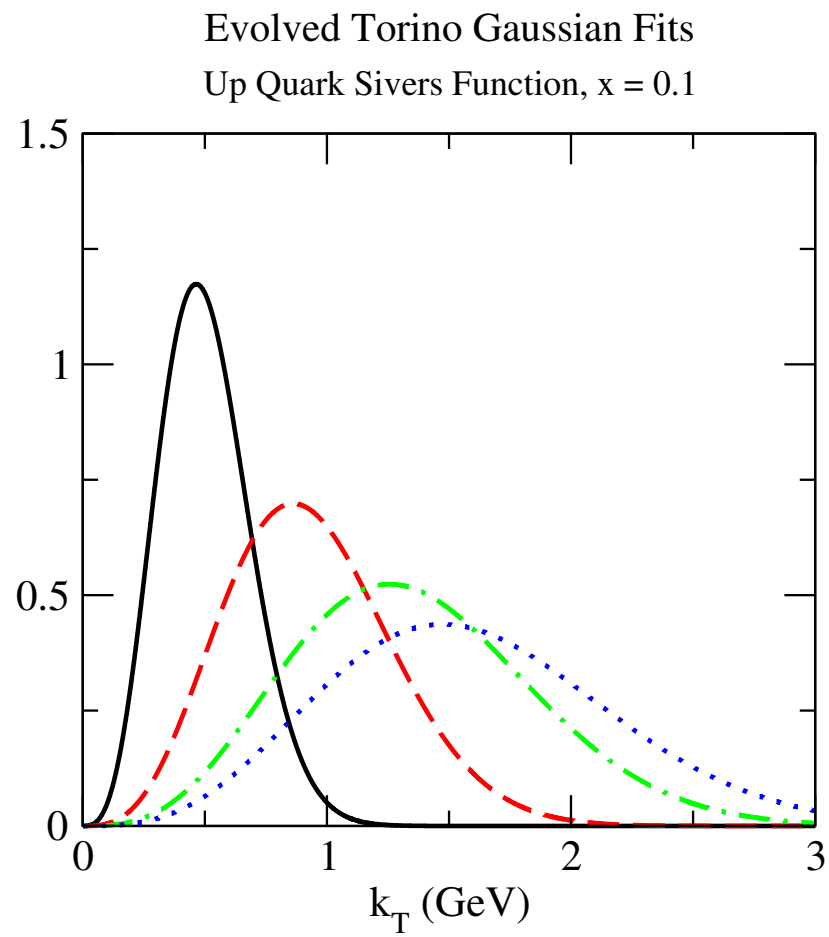
Gaussians fit to evolved TMD PDF

$$x\Delta^N F^{\rm fit}_{f/P}(x=0.1,k_T) = a^{\rm fit}_f k_T e^{-b^{\rm fit} k_T^2}$$

Q (GeV)	$b^{\rm Bochum}$ (GeV^{-2})	$b^{\rm Torino}$ (GeV^{-2})	$a^{\rm Bochum}_{\rm up}$ (GeV^{-3})	$a^{\rm Torino}_{\rm up}$ (GeV^{-3})	$k^{\rm Bochum}_{T,\rm max}$ (GeV)	$k^{\rm Torino}_{T,\rm max}$ (GeV)
$\sqrt{2.4}$	4.9999	6.9382	6.5570×10^{-1}	1.7763×10^{0}	...	...
2.0	1.8251	2.0329	9.5506×10^{-2}	1.6661×10^{-1}	...	...
2.5	1.1726	1.2552	4.1658×10^{-2}	6.7105×10^{-2}	2.36	2.29
3.0	0.9067	0.9555	2.5716×10^{-2}	4.0138×10^{-2}	2.56	2.50
3.5	0.7604	0.7945	1.8430×10^{-2}	2.8276×10^{-2}	2.70	2.65
4.0	0.6668	0.6929	1.4329×10^{-2}	2.1745×10^{-2}	2.80	2.76
4.5	0.6013	0.6225	1.1718×10^{-2}	1.7649×10^{-2}	2.89	2.85
5.0	0.5526	0.5705	9.9179×10^{-3}	1.4854×10^{-2}	2.96	2.92
10.0	0.3562	0.3637	3.9881×10^{-3}	5.8409×10^{-3}	3.39	3.36

Gaussians fit to evolved TMD PDF





Long-Term Goals:

- Repository of improved TMD fits with evolution based on well-understood operator definitions.

https://projects.hepforge.org/tmd/

- Create new fits for unpolarized TMDs (PDFs and fragmentation functions), spin dependent TMDs, SSAs, gluon TMDs.

- Find specific factorization breaking effects??

List of items:

- Begin to implement new fits (starting with unpolarized case) using the full TMD-factorization treatment.
  (In progress…)

- Calculate higher orders in hard part, anomalous dimensions, coefficient functions. Include Y-terms.

- Extend to other polarization dependent functions (Boer-Mulders, etc…). Calculate their hard coefficient functions to relate to higher twist collinear factorization treatments.

- Full TMD-factorization valid for all q_T.

- TMD gluon distribution. *(Higgs…)*

Thanks!