Critical Comparison of 3-Particle Correlation and 3-Particle Cumulant Method: *Why I Believe the Former*

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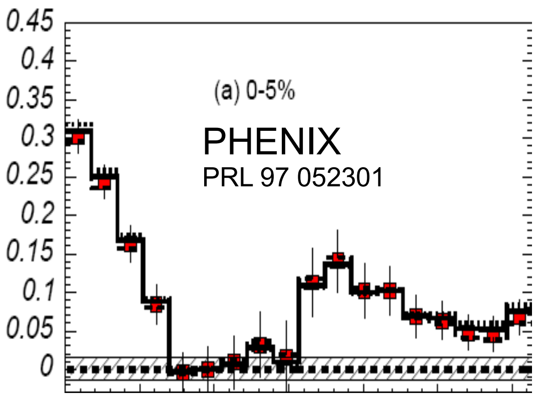
CATHIE-RIKEN Workshop:

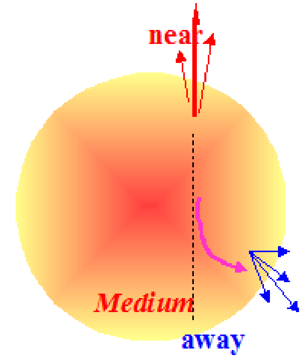
Critical Assessment of Theory and Experiment on Correlations at RHIC

Outline

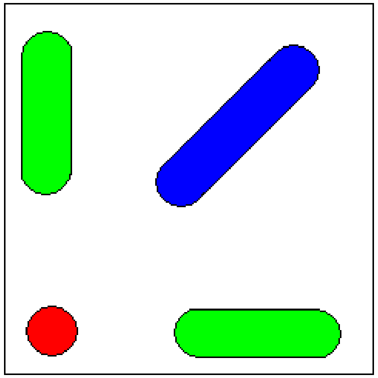
- Introduction and Motivation
- 3-Particle Cumulant
- 3-Particle Correlation
- Comparison
  - Term by term
  - Simulation
- Summary

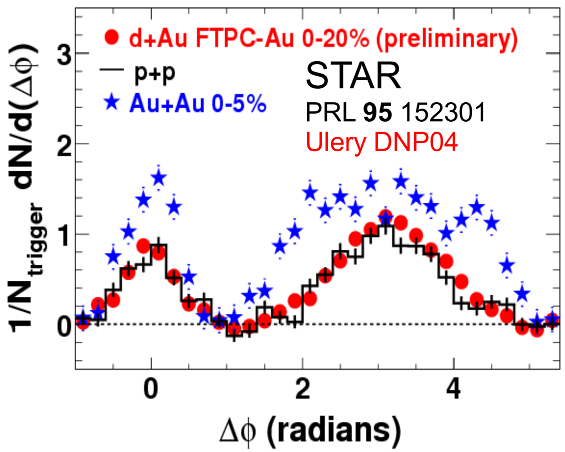
Why Study 3-Particle Correlations?

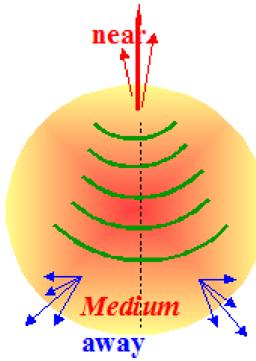




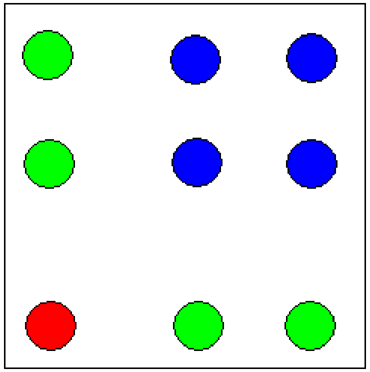
Deflected Jets







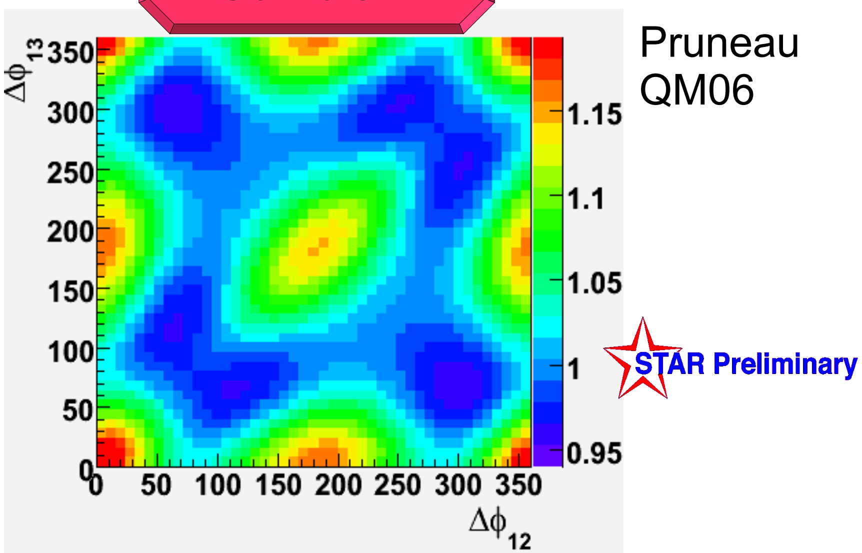
Conical Emission



- Modification to the away-side peak.
- 3-particle can distinguish conical emission from deflected jets.

Methods

Cumulant



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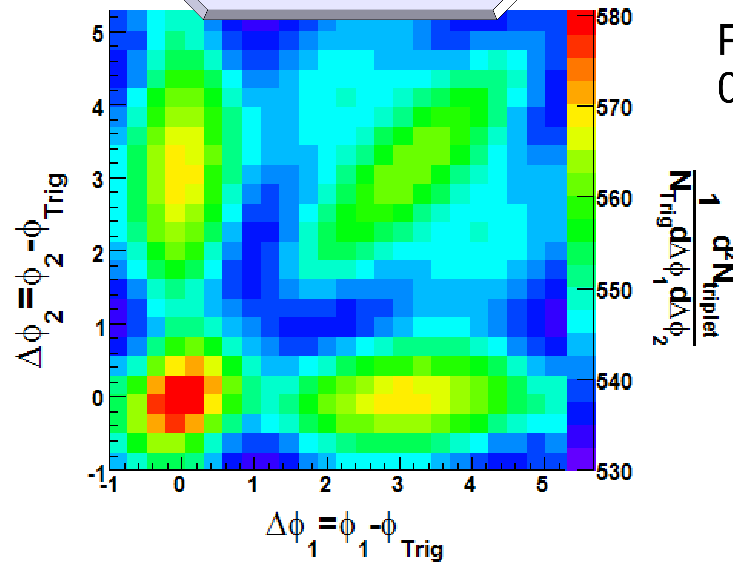
Correlation



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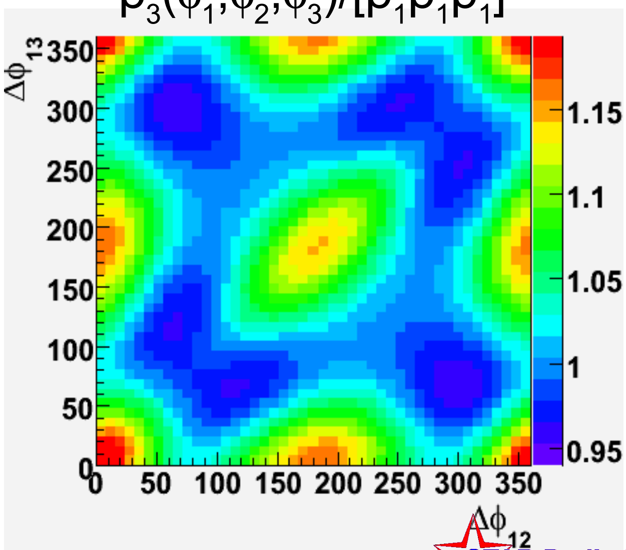
- Model independent
- Unable to interpret without model.
- No normalization
- First order flow cancellation

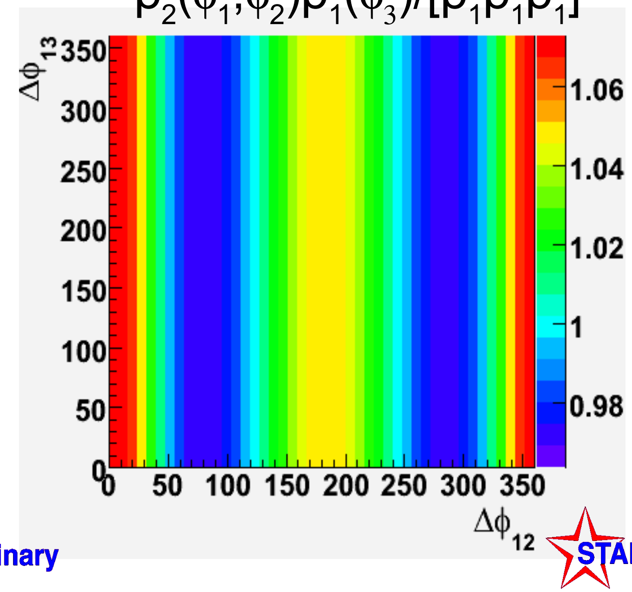
- Model dependent
- Interpretable within the model.
- ZYAM normalization
- Explicit flow subtraction

3-Particle Cumulant

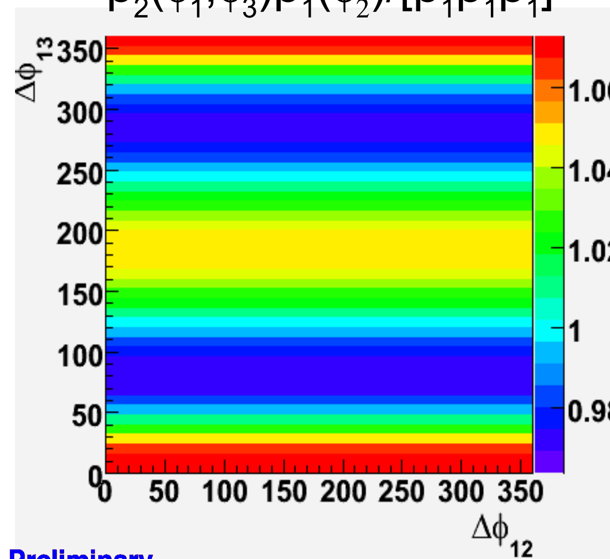
$\rho_3(\phi_1,\phi_2,\phi_3)/[\rho_1\rho_1\rho_1]$



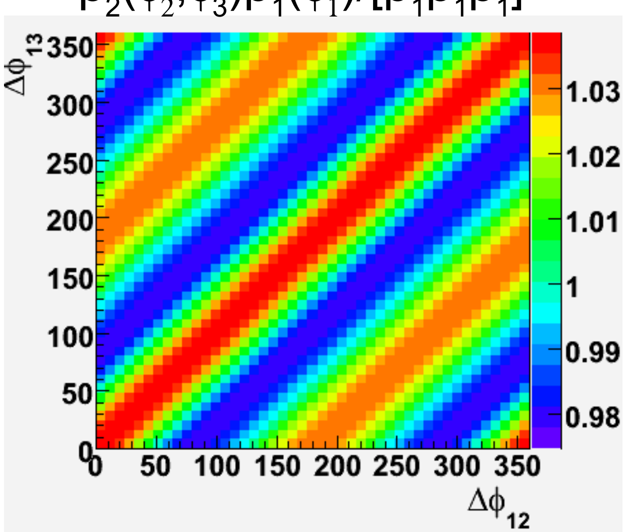
$\rho_2(\phi_1,\phi_2)\rho_1(\phi_3)/[\rho_1\rho_1\rho_1]$



$\rho_2(\phi_1,\phi_3)\rho_1(\phi_2)/[\rho_1\rho_1\rho_1]$



$\rho_2(\phi_2,\phi_3)\rho_1(\phi_1)/[\rho_1\rho_1\rho_1]$



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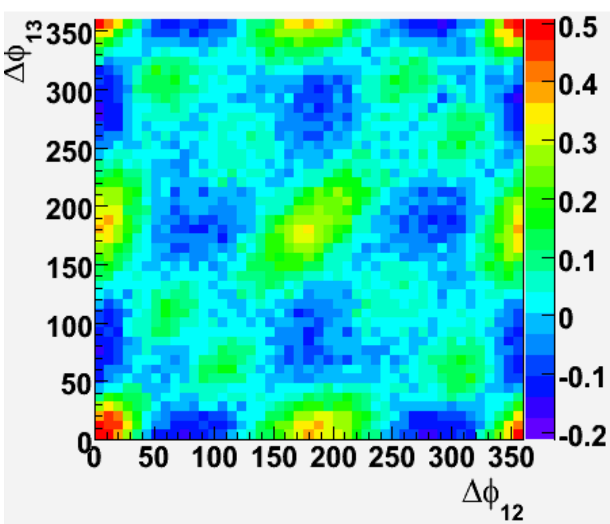
$C_3(\phi_1,\phi_2,\phi_3)$



3-Particle Correlation: Hard-Soft

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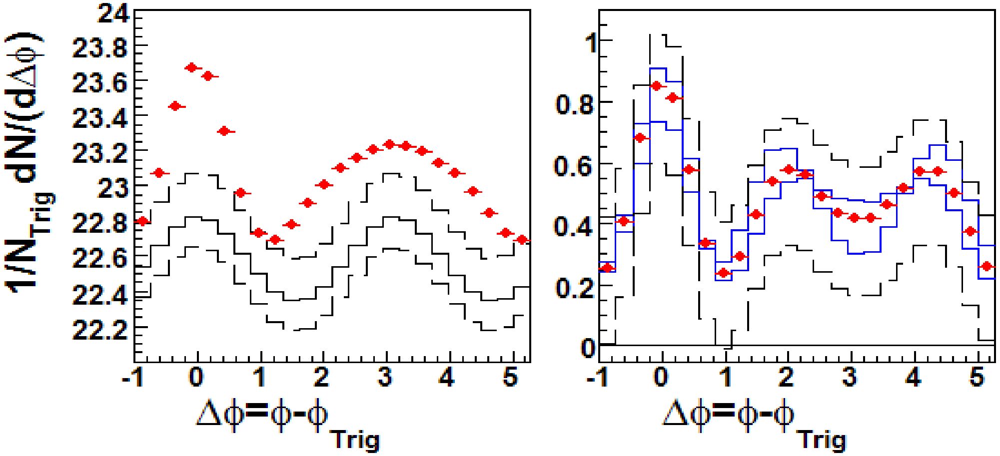

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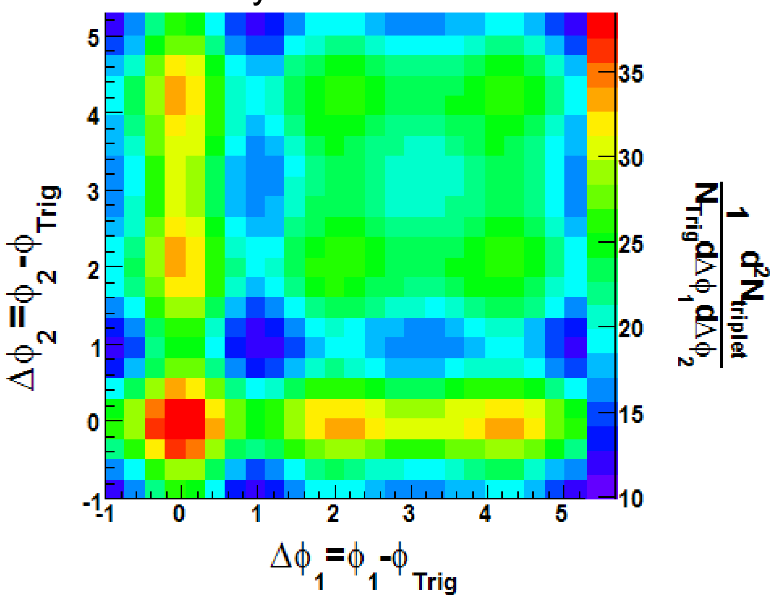

- Normalized mixed event 2-particle background folded with the 2-particle jet-like correlation. (norm. factor a)
- Contains the non-flow 2-particle correlations.
- Note: done in 6 bins of trigger particle relative to EP

3-Particle Correlation:Soft-Soft and Flow

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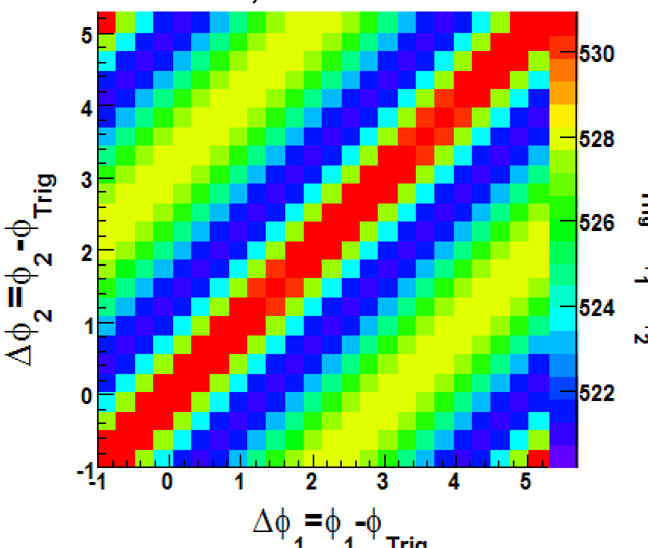
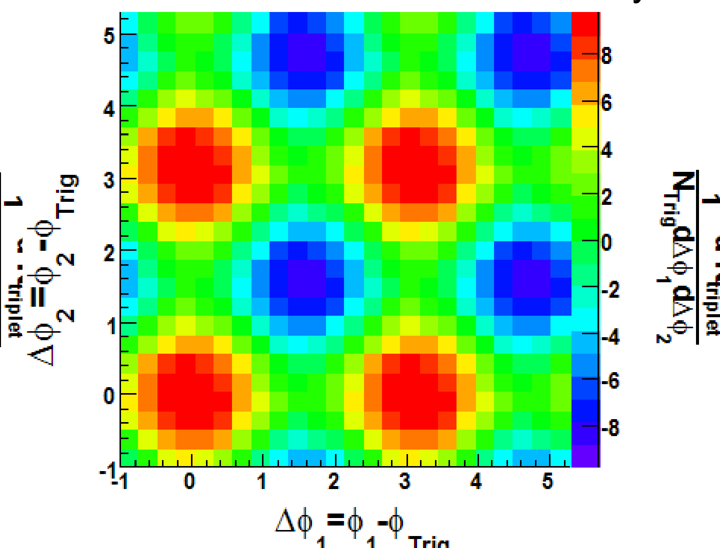
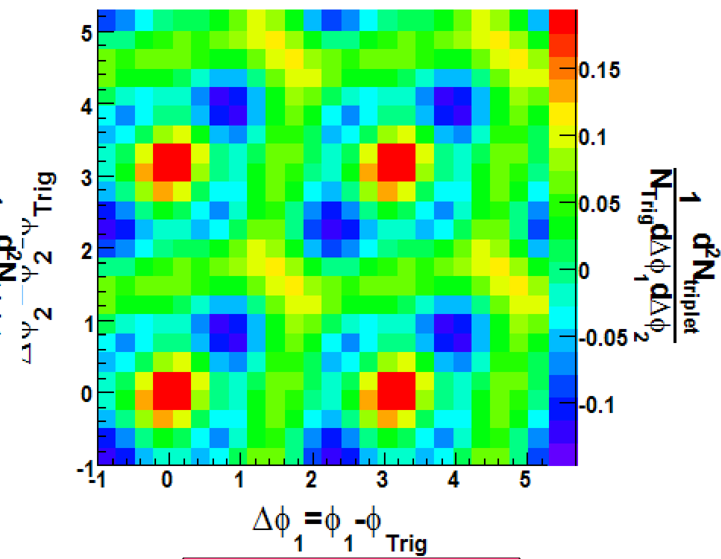


Soft-Soft, J_2^{Inc}



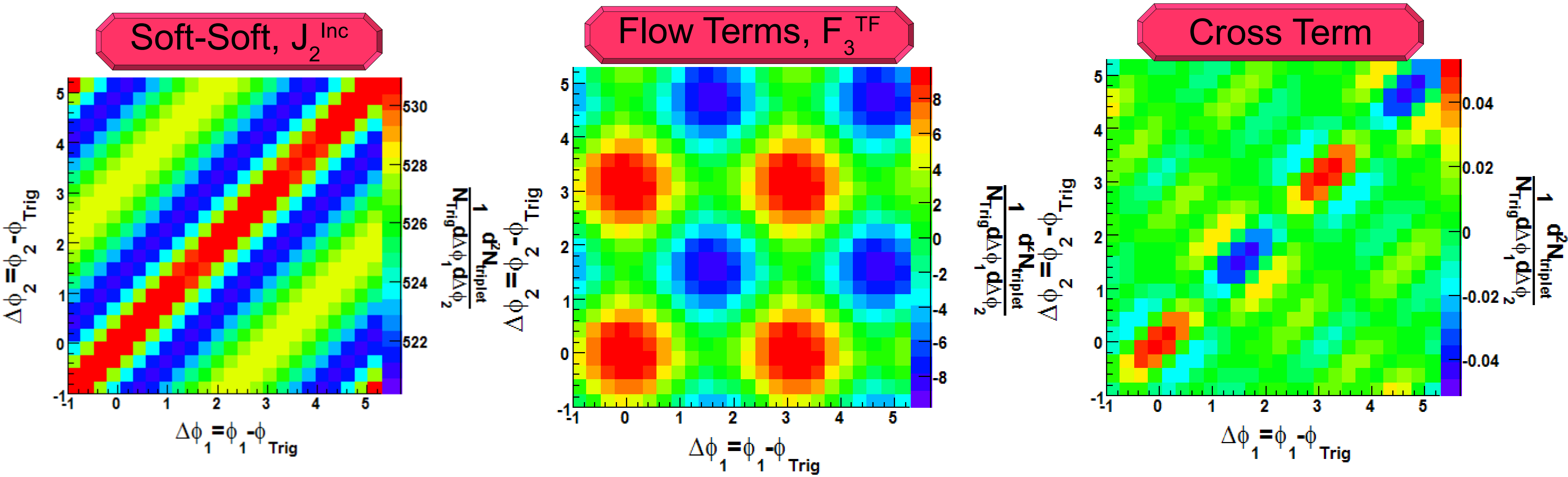
v_2v_2



v_4v_4 & $v_2v_2v_4$

- Soft-soft contains correlations between the associated particles independent of the trigger.
- Flow terms contain v_2 and v_4 related correlations.
- normalized by a^2b

Soft-Soft Cross Term



- Higher order correction for correlation of the non-flow component of the soft-soft with the reaction plane.

Mathematics

3-Particle Cumulant:

$$C_3(\phi_1,\phi_2,\phi_3) = \frac{\rho_3(\phi_1,\phi_2,\phi_3) - \rho_2(\phi_1,\phi_2)\rho_1(\phi_3) - \rho_2(\phi_1,\phi_3)\rho_1(\phi_2) - \rho_2(\phi_2,\phi_3)\rho_1(\phi_1)}{\rho_1(\phi_1)\rho_1(\phi_2)\rho_1(\phi_3)} + 2$$

3-Particle Correlation

$$J_3^{Final} = J_3(\Delta\phi_1,\Delta\phi_2)$$

$$- \{[J_2(\Delta\phi_1) - aB_{Inc}(1+F(\Delta\phi_1))] \otimes [aB_{inc}(1+F(\Delta\phi_2)]$$

$$+ [J_2(\Delta\phi_2) - aB_{Inc}(1+F(\Delta\phi_2))] \otimes [aB_{inc}(1+F(\Delta\phi_1)]\}$$

$$-a^2 b\, J_2^{Inc}(\Delta\phi_1,\Delta\phi_2)\, [1 + F_3^{TF}(\Delta\phi_1,\Delta\phi_2)\, / J_2^{Inc,flow}(\Delta\phi_1,\Delta\phi_2)]$$

Normalization
3-Particle Correlation

- a determied such that the background subtracted 3-particle correlation is ZYAM
- b $= \dfrac{\langle N_{Trig}(N_{Trig}-1)\rangle / \langle N_{Trig}\rangle^2}{\langle N_{Inc}(N_{Inc}-1)\rangle / \langle N_{Inc}\rangle^2}$
  - accounts for $\langle a\rangle^2 \neq \langle a^2\rangle$
  - assumes the deviation from Poisson statistics for underlying events is the same as for entire triggered event
- For 0-5% most central:
  - a=0.994 + 0.005 – 0.004
  - b=1.00021 + 0.0003 - 0.0005

Normalization

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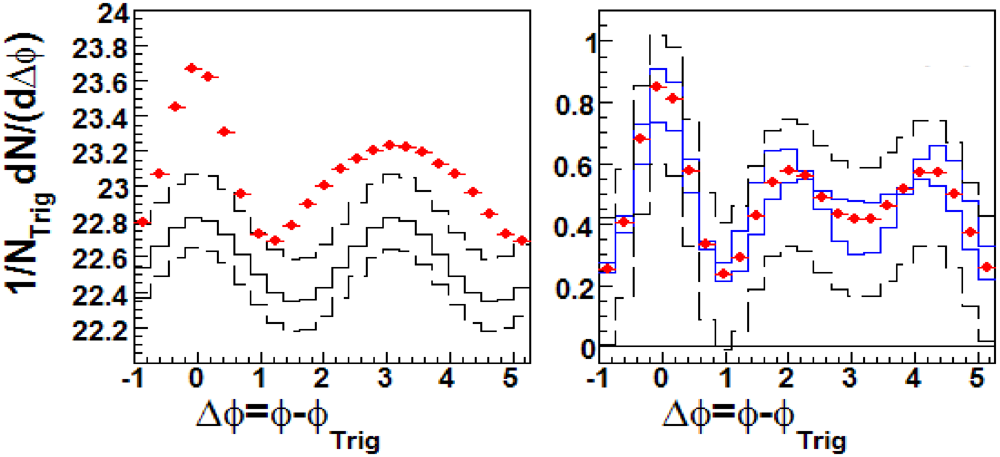

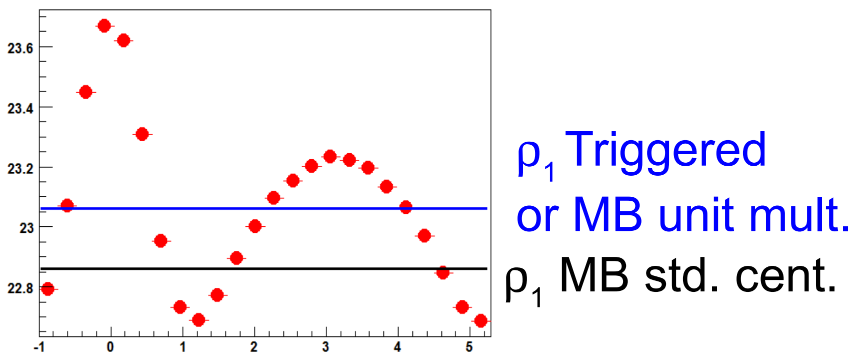

- 3-Particle Correlation:
  - background level always fixed by ZYAM
- 3-Particle Cumulant:
  - level dependent mixed event selection (minimum bias events or triggered events, multiplicity bin width etc...)

Comparison Raw

$\rho_3(\phi_1,\phi_2,\phi_3)/[\rho_1\rho_1\rho_1]$

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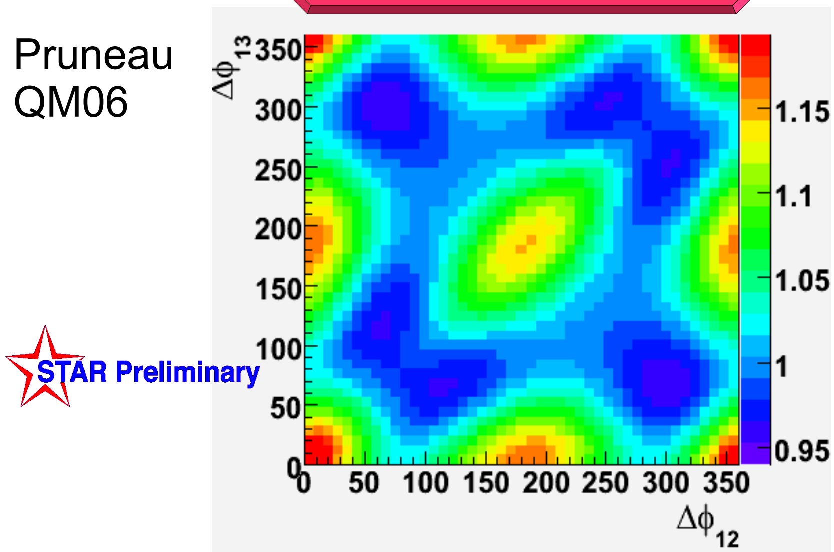

$J_3(\Delta\phi_1,\Delta\phi_2)$

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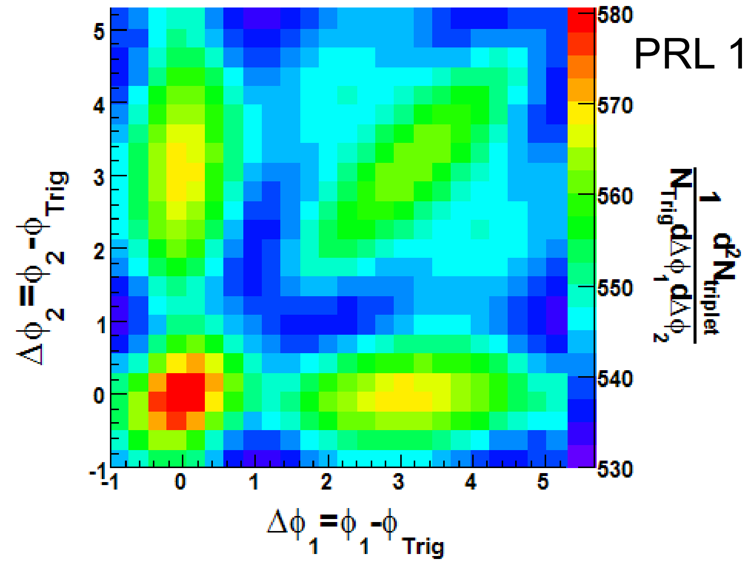

- Only difference in the unsubtracted signals should be a scaling factor that persists through all terms.
  - I'll refer to as k

Comparison Soft-Soft

$\rho_2(\phi_2,\phi_3)\rho_1(\phi_1)/[\rho_1\rho_1\rho_1]$

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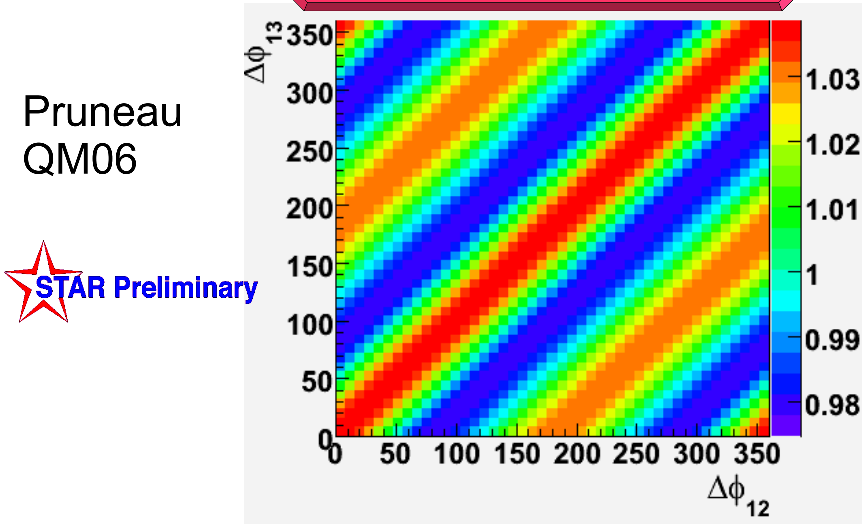
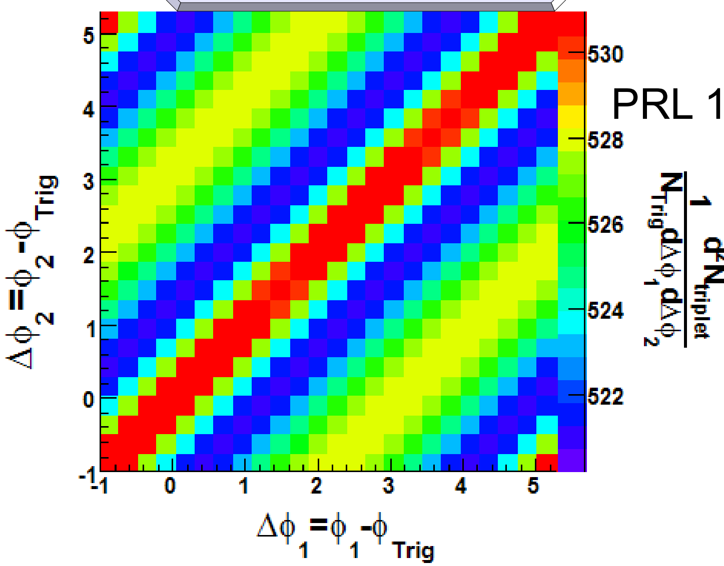
QM06



$a^2b\ J_2^{Inc}(\Delta\phi_1,\Delta\phi_2)$



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- Additional difference due to normalization.
- Different by factor of ka^2b.
  - This is largely a pedestal shift as this term has a small signal on a large background.

Comparison Hard-Soft and Flow

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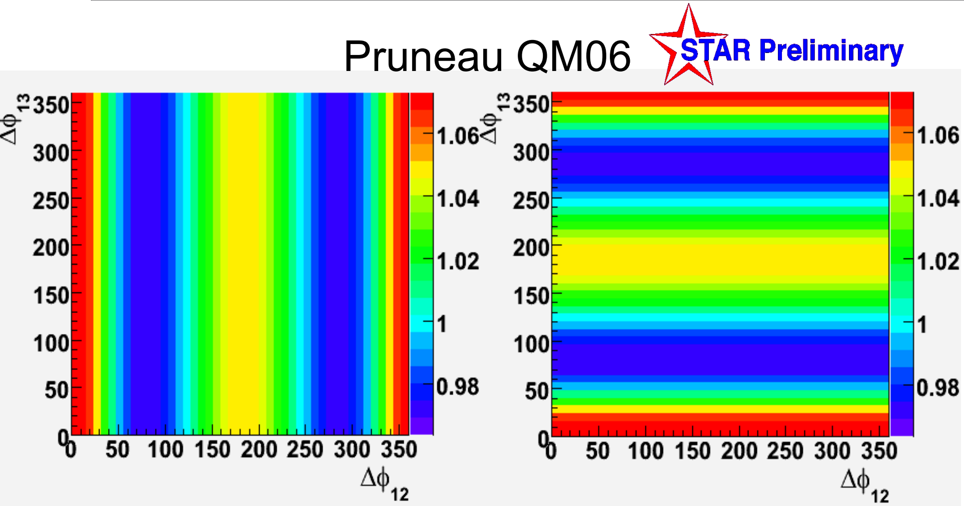

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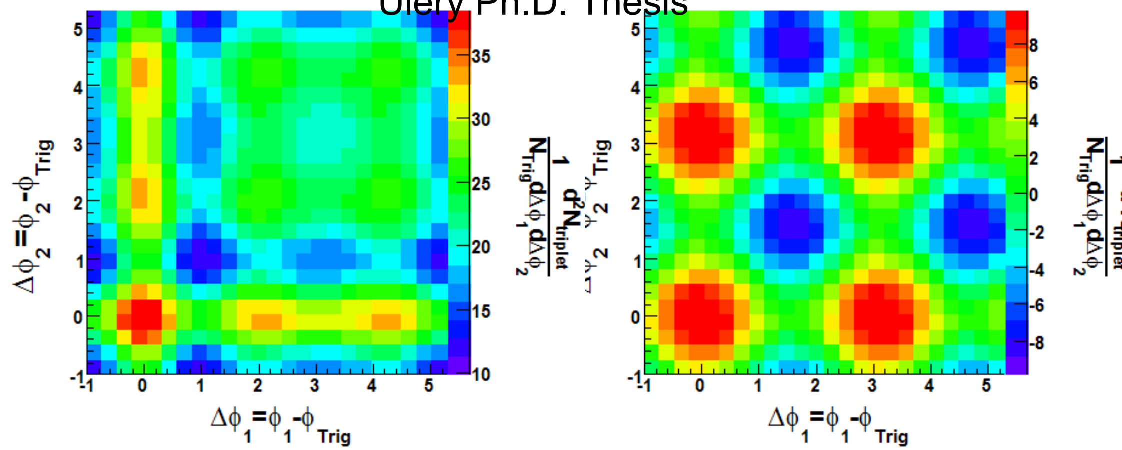
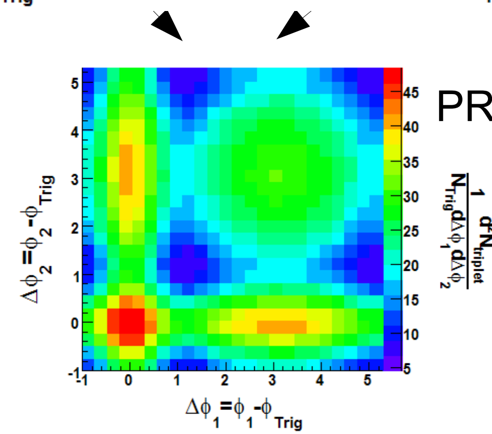


$[\rho_2(\phi_1,\phi_2)\rho_1(\phi_3)+\rho_2(\phi_1,\phi_3)\rho_1(\phi_2)]/[\rho_3(\phi_1)\rho_3(\phi_2)\rho_3(\phi_3)]$



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- Here the comparison gets a bit more complicated. Terms have to summed from both analyses for comparison.

Difference Hard-Soft and Flow

$[\rho_2(\phi_1,\phi_2)\rho_1(\phi_3)+\rho_2(\phi_1,\phi_3)\rho_1(\phi_2)]/[\rho_3(\phi_1)\rho_3(\phi_2)\rho_3(\phi_3)]$ Cumulant Terms

$\{[J_2(\Delta\phi_1)-aB_{Inc}(1+F(\Delta\phi_1))]\otimes[aB_{inc}(1+F(\Delta\phi_2))] + [J_2(\Delta\phi_2)-aB_{Inc}(1+F(\Delta\phi_2))]\otimes[aB_{inc}(1+F(\Delta\phi_1))]\}$

$+a^2b\ J_2^{Inc}(\Delta\phi_1,\Delta\phi_2)\ [\ F_3^{TF}(\Delta\phi_1,\Delta\phi_2)\ /J_2^{Inc,flow}(\Delta\phi_1,\Delta\phi_2)]$ Correlation Terms

let $J'_2=J_2-aB_{Inc}(1+F)$

- Difference is:
  - $k[aB_{inc}\{J'_2(\Delta\phi_1)\ F(\Delta\phi_2)+J'_2(\Delta\phi_2)F(\Delta\phi_1)\}$ Jet-Flow Cross Term
  - $+(a-1)B_{inc}[J'_2(\Delta\phi_1)+J'_2(\Delta\phi_2)]$ Residual 2-Particle Correlation
  - $+a^2(b\langle J_2^{Inc}\rangle-B_{Inc}^{\ 2})(v_2v_2+v_4v_4)$ Residual 2-Particle Flow
  - $+a^2b\langle J_2^{Inc}\rangle v_2v_2v_4$ 3-Particle Flow
  - $+a^2b\ F_3^{TF}(\Delta\phi_1,\Delta\phi_2)\ [\ J_2^{Inc}(\Delta\phi_1,\Delta\phi_2)\ /J_2^{Inc,flow}(\Delta\phi_1,\Delta\phi_2)-1]]$ Soft-Soft-Flow Cross Term –very small

Difference Terms

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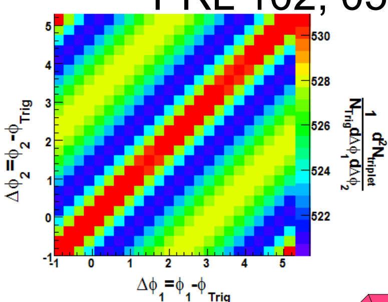
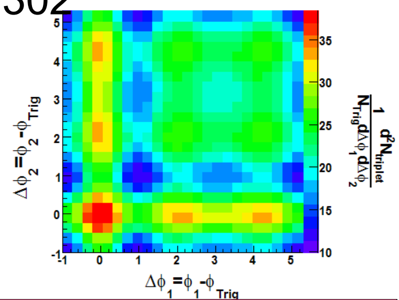
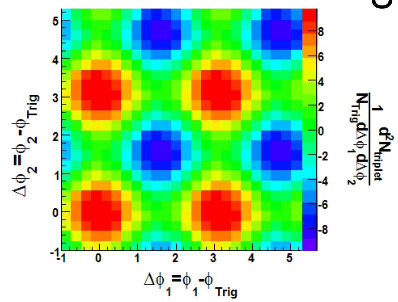


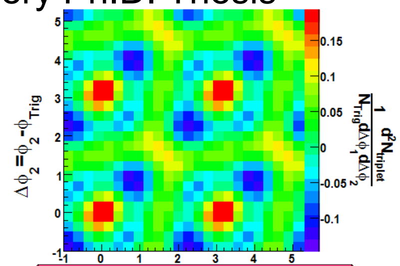
a^2b



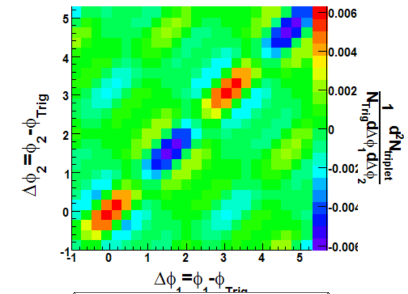
Jet ⊗ Flow
Resid. 2-Part. Corr.



Resid. v_2v_2



Residual v_4v_4
$v_2v_2v_4$ Terms

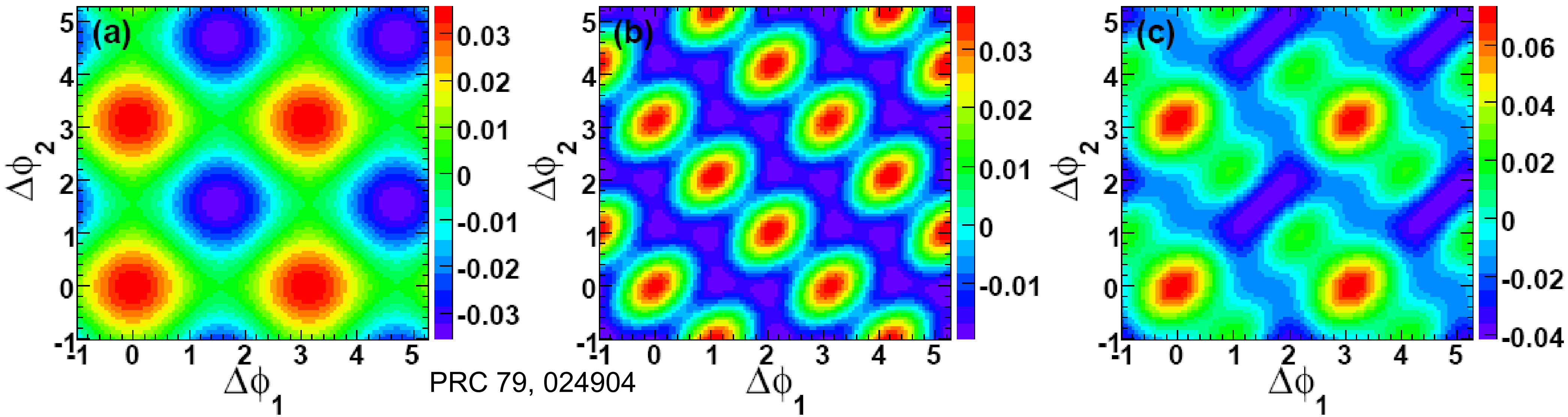


Soft-Soft ⊗ Flow

- Visual Summary.
- (note the analyses are equivalent iff a=1, b=1, v_2=0, and v_4=0, other then the constant factor)

3-Particle Cumulant From Only Flow

From calculation with $v^{(t)}_2 = 7.5\%, v^{(1)}_2 = v^{(2)}_2 = 5\%$, $v_4 = v_2^2$. and $\langle B_1^2\rangle - \langle B_1\rangle^2 = 0.1\langle B_1\rangle$.



Residual $v_2 v_2$ and $v_4 v_4$ if $\langle B^2\rangle \neq \langle B\rangle^2$

Irreducible Flow from $v_2 v_2 v_4$

(c)=(a)+(b)

- If only flow is present 3-particle cumulant will still give a signal.

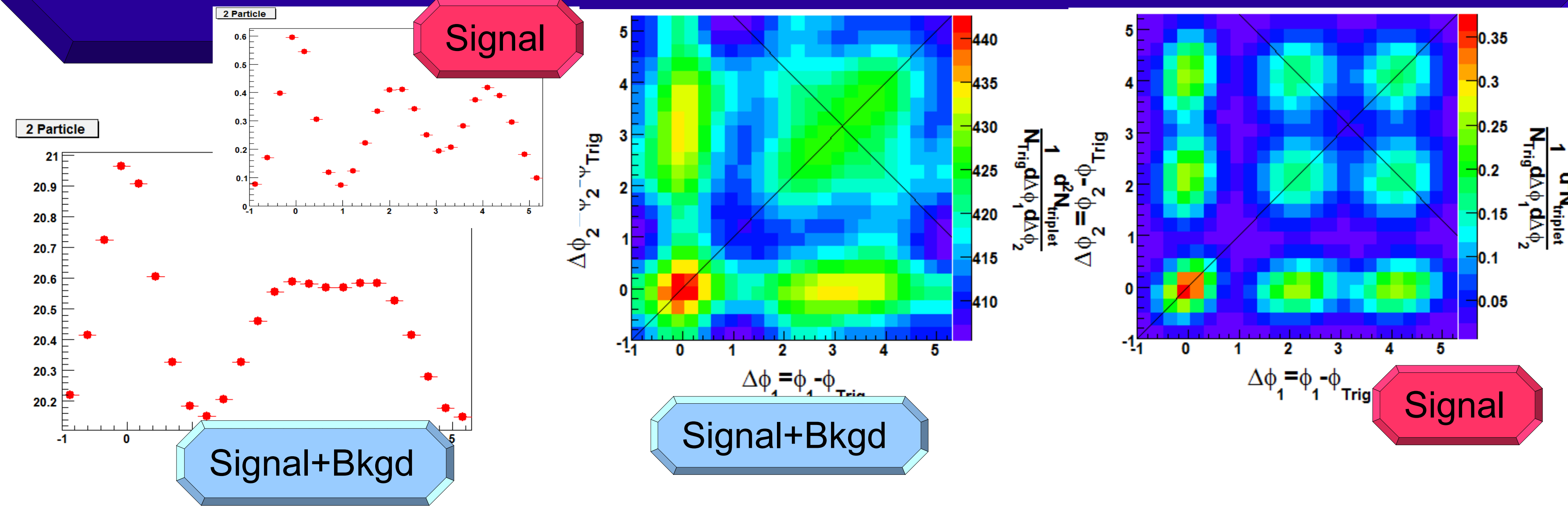
Reaction Plane Frame 3-Particle Cumulant

$$C_3(\phi_1,\phi_2,\phi_3,\psi) = \frac{\rho_3(\phi_1,\phi_2,\phi_3,\psi) - \rho_2(\phi_1,\phi_2,\psi)\rho_1(\phi_3,\psi) - \rho_2(\phi_1,\phi_3,\psi)\rho_1(\phi_2,\psi) - \rho_2(\phi_2,\phi_3,\psi)\rho_1(\phi_1,\psi)}{\rho_1(\phi_1,\psi)\rho_1(\phi_2,\psi)\rho_1(\phi_3,\psi)} + 2$$

$$C_3(\phi_1,\phi_2,\phi_3) = \int C_3(\phi_1,\phi_2,\phi_3,\psi)\,d\psi$$

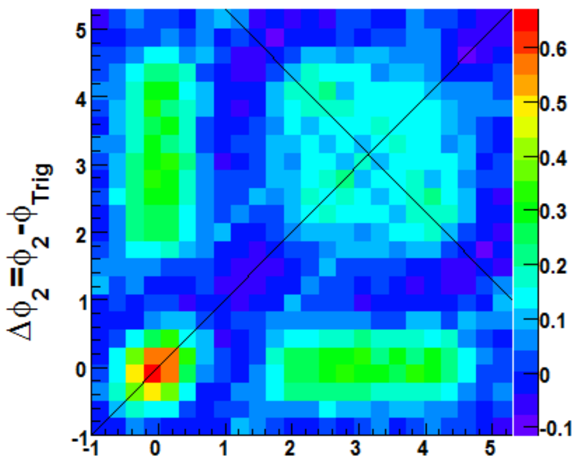
- The cumulant can handle to flow if the reaction plane angle is taken into account.
- In practice could be done using measured v_2 and v_4.
- This method should be consistent with the 3-Particle Correlation Method except for the normalization.

Toy Model Simulation

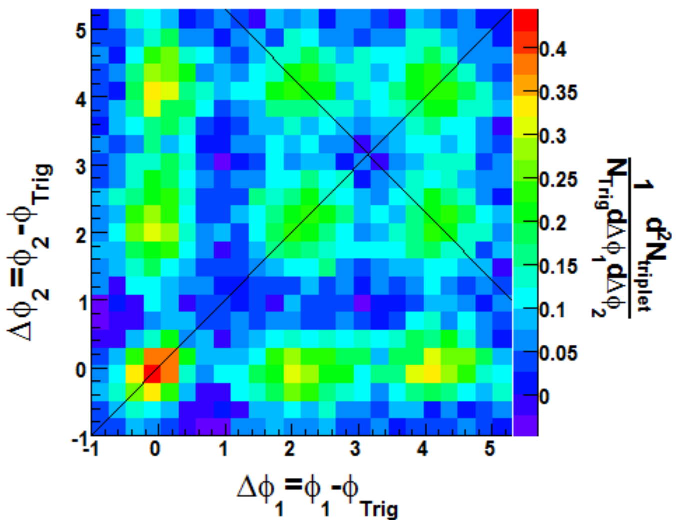


- Jet axis correlated with reaction plane.
- Poisson # of triggers about jet axis.
- Poisson # of associated jet particles in 3 Gaussians about jet axis.
- Poisson # of background particles with v_2.
- Background level, v_2, and jet correlation similar to real data.

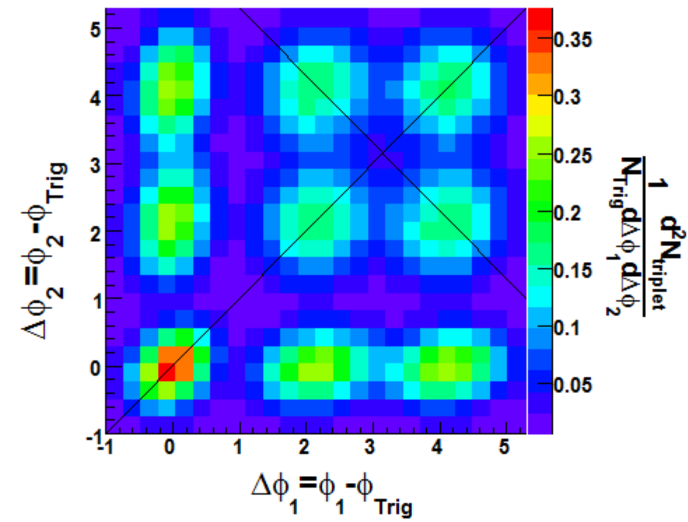
Toy Model



a=b=1
No Flow Subtraction
Cumulant Equivalent



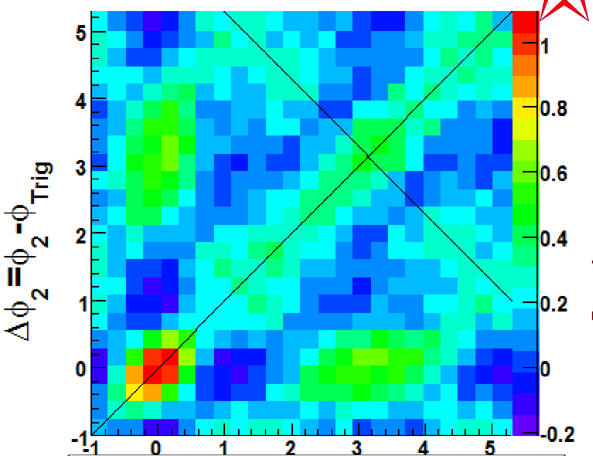
3-Particle Correlation



True Signal

- 3-Particle Correlation reproduces the signal.
- 3-Particle Cumulant does not, (Jet⊗Flow).
- Poisson generation, signal is not ZYAM, true level of background is a=b=1.

Real Data Comparison



a=b=1
No Flow Subtraction
Cumulant Equivalent

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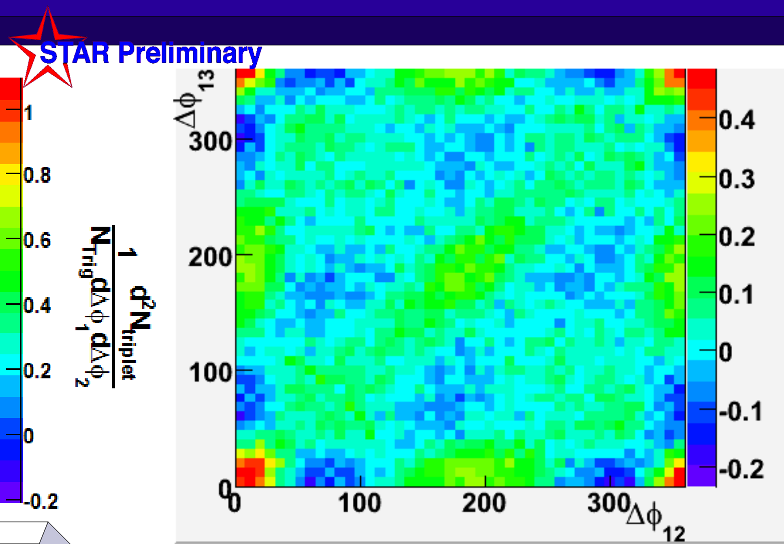

3-Particle Cumulant

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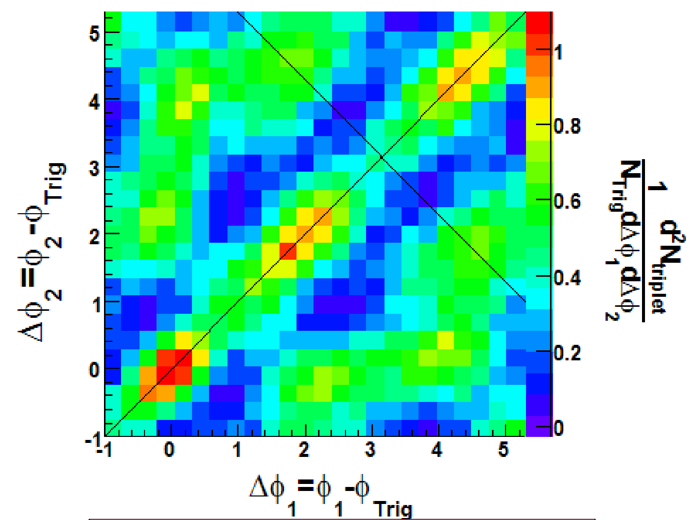

3-Particle Correlation

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- Cumulant equivalent analyses consistent with the Cumulant
- Cumulant displays no distinct off-diagonal peaks.
- 3-Particle Correlation displays significant peaks.

Summary

- Two analyses are consistent iff a=b=1 and no flow is subtracted.
- 3-Particle Correlation:
  - Removes all known backgrounds
  - Significant signal
    - interpretable within the model assumptions.
- 3-Particle Cumulant:
  - Model independent, uninterpretable without a model.
  - Residual 2-particle correlations and flow in addition to 3-particle correlations.