PERTURBATIVE QCD
FOR HIGH PT REACTIONS

George Sterman
Workshop on high pT
physics at RHIC
BNL, Dec. 3, 2003

I. Concepts

II. Fixed Orders and PDFs

III. Resummations

IV. Power Corrections

V. Particle Specta

⋆ pQCD Concepts

- **Infrared safety & asymptotic freedom:**

$$Q^2\, \hat\sigma_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) \;=\; \sum_n c_n(Q^2/\mu^2)\,\alpha_s{}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$
$$= \sum_n c_n(1)\,\alpha_s{}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

– e^+e^- total; jets

- Generalization: **factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

μ = **factorization scale**; m= **IR scale**
(m *may* be perturbative)

– New physics in $\omega_{\rm SD}$; $f_{\rm LD}$ "universal"
– Deep-inelastic, $p\bar{p} \to Q\bar{Q}$ ...
– Exclusive decays: $B \to \pi\pi$
– Exclusive limits: $e^+e^- \to$ JJ as $m_J \to 0$
– Exclusive scattering (large t): $\pi\pi \to \pi\pi$

- Factorization proofs:
- (1) $\omega_{\rm SD}$ **incoherent with LD dynamics**
- **(2) mutual incoherence when** $v_{\rm rel} = c$
- All orders in perturbation theory; all powers $\ln \mu/Q$

  $t \ll s$ **elastic:** Sen 1980... Kucs 2002
- An aside:
  - Whenever there is factorization, there is **evolution**

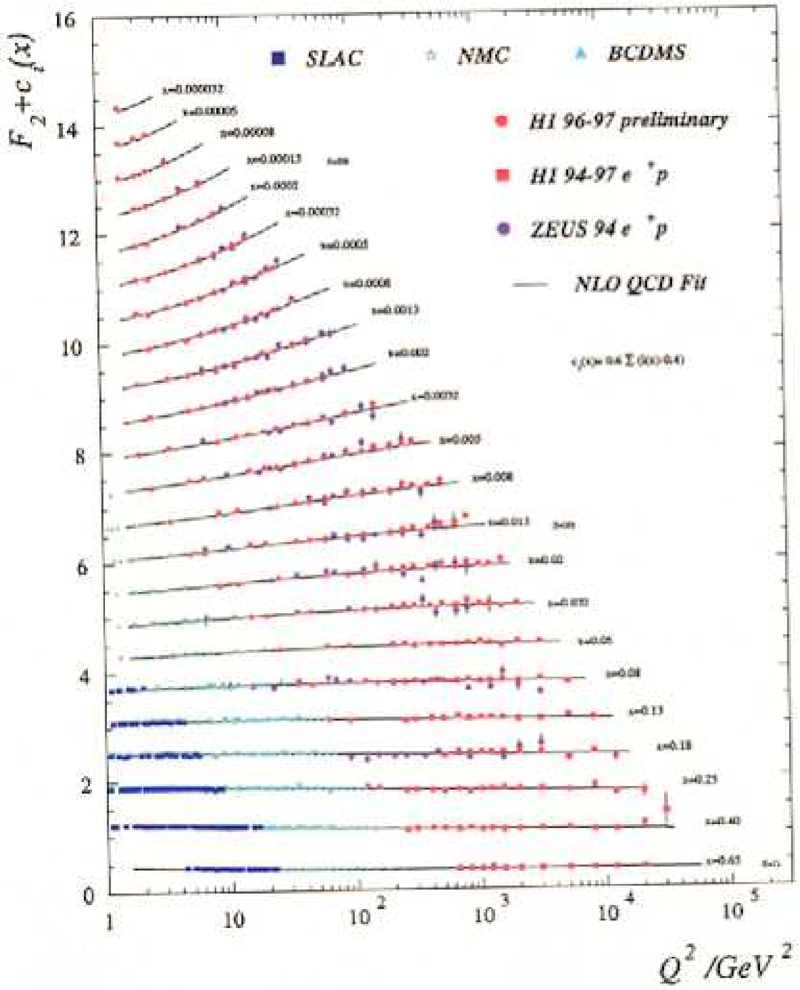
$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

  - Wherever there is evolution, there is **resummation**

$$\ln \sigma_{\rm phys}(Q, m) = \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

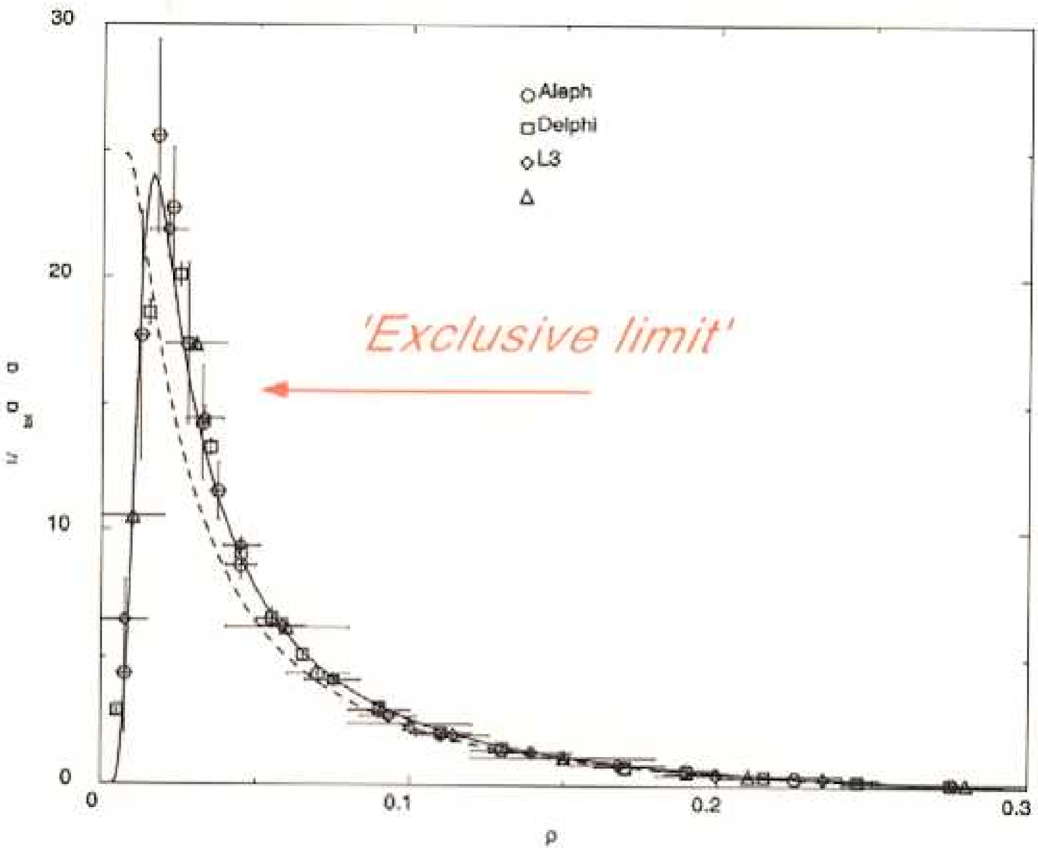
What it gives: inclusive DIS



F_2 data

Single dimensional scale: Q

What it gives: semi-inclusive e^+e^- annihilation



Heavy jet distribution at the Z pole. From Korchemsky and Tafat (2000)

- Two dimensional scales: $Q = \sqrt{s}$ and m_J $(\rho = m_J^2/Q^2)$
- Resummed PT (NLL in m_J/Q) alone not enough: power corrections in the lighter scale: $1/m_J^p$
- How to organize NP corrections starting with PT
- Can we make statements "to all orders"?

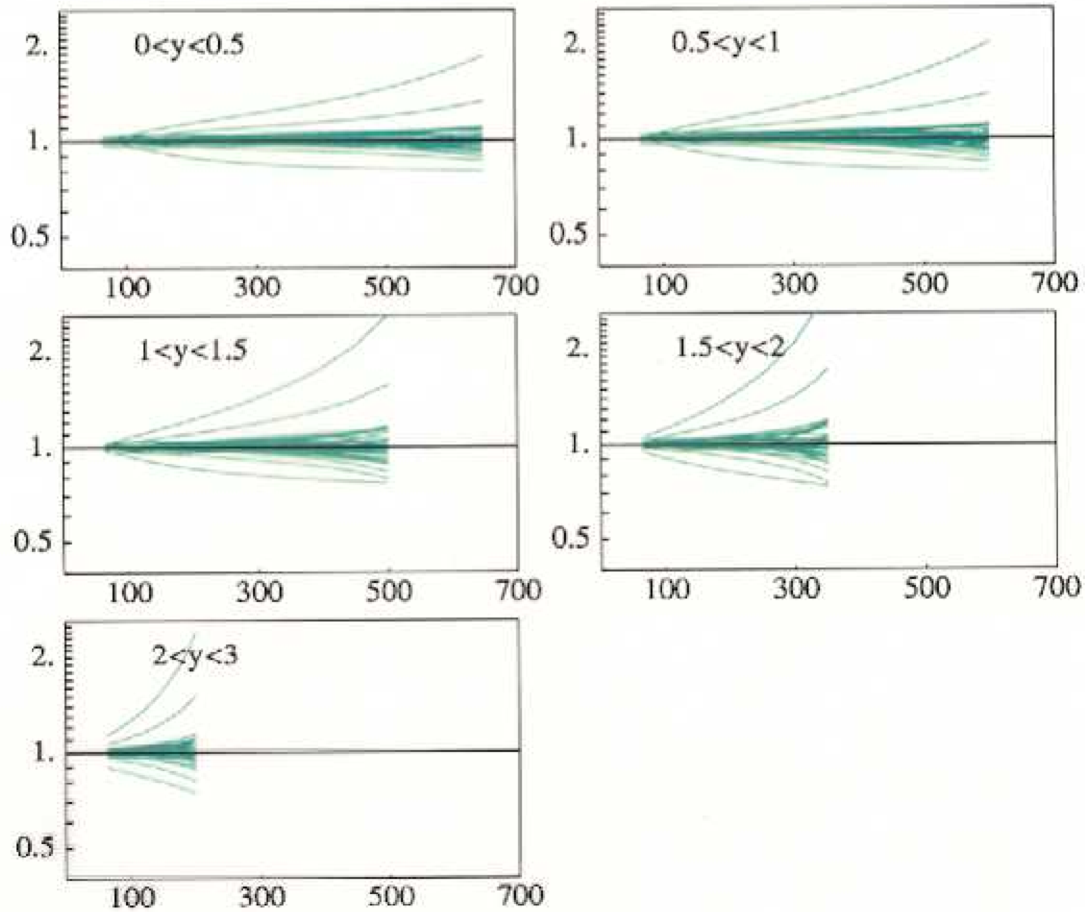
$\star$ Hadron Structure

Parton Distributions

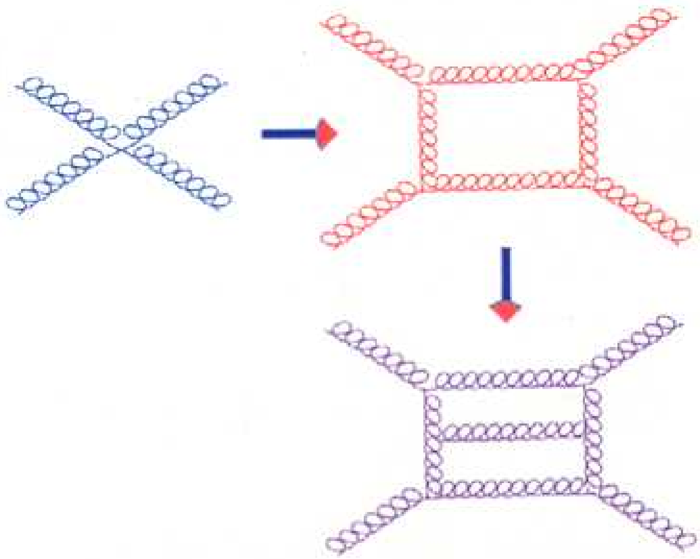
- Examples of '98-03 "refinements" (Martin 99, Lai 00)
  - d/u: CDF W asymmetry and Higher twist reanalysis of $F_2^{(D)}$, F_s^p
  - $\bar{d}/\bar{u}$ from DY
  - Dimuons from ν scattering (*c.f.* NuTeV)
- Lesson:
  indirect constraints on PDF's are tentative
- Issues for global fits
  (CTEQ(N= dots 6), MRST2001,2)
  - Example of direct photons: potentially non-perturbative corrections at moderate k_T (E706; controversy)
  - Role replaced by Tevatron high-p_T jet data
  - Still needed: systematic understanding of *differences* in higher-order corrections between processes
  - parton distribution uncertainties: statistical analysis *vs* role of higher orders & NP corrections? (DIS: Alekhin, Botje, Barone *et al.*, Giele & Keller, Global: Brock *et al*,Stump *et al.*,Pumplin *et al.*)

– CTEQ approach: eigenvectors of error matrix in N-dimensional PDF parameter space generate "best" set + "cloud" of $2N$ admissible variant basis sets

From Stump *et al.* hep-ph/0303013 (JHEP)
Each line a "basis set"



⋆ Toward a Two-Loop Phenomenology



- $\alpha_s^2 \sim 1\%$; scale of NP and "new physics"
- NLO up to four jets e^+e^-; full NNLO only for one-scale problems: e^+e^- total, DIS.
- 2002: $\mathcal{O}({\alpha_s}^3)$ for 3-jet amplitudes (Garland et al.)
- Progress toward $P(z)$ to α_s^3 (Larin, Moch et al.,C.F.Berger, 2002)
- 2000-2003: Two-loop parton-parton scattering (Tausk, Smirnov, Bern et al., Anastasiou,Glover ... Tejeda-Yeomans)
- A number of years yet to true NNLO jet cross sections ...

⋆ Threshold Resummation for high-p_T physics:

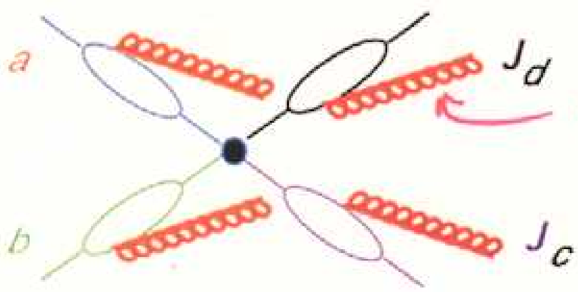
- Factorization

$$Q^2\sigma_{AB\to F}(Q) = f_{a/A}(x_a)\otimes f_{b/B}(x_b)\otimes\omega_{ab\to F}\left(z=\frac{Q^2}{x_a x_b S}\right)$$

- **Partonic threshold:** $z\to 1$
- Singular corrections

$$\omega^{(r)}_{ab\to F}(z)\ \sim\ C^{(r)}\left(\frac{\alpha_s}{\pi}\right)^r\frac{1}{r!}\left[\frac{\ln^{2r-1}(1-z)}{1-z}\right]_+$$

- Elastic kinematics at $z=1$: $m^2[J_i]\sim(1-z)s$



collinear effective theory C. Bauer et al

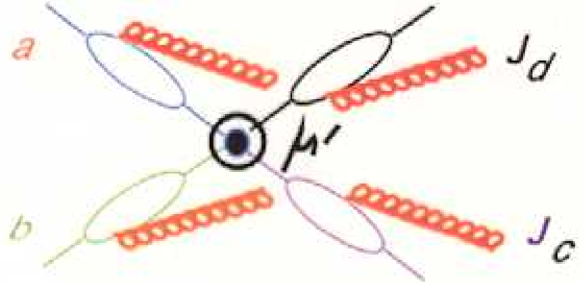
- **Refactorization** → evolution
→ resummation ... z^{N-1} 'real'; -1 'virtual':

$$\int_0^1 dz z^{N-1}\omega_{a\bar a\to F}(z,\mu)$$
$$=\exp\left[-\int_0^1 dz\frac{z^{N-1}-1}{1-z}\int_{(1-z)^2Q^2}^{\mu^2}\frac{dm^2}{m^2}A_a(\alpha_s(m))+\ldots\right]$$
$$\sim\exp\left[\int_{Q^2/N^2}^{\mu^2}\frac{dm^2}{m^2}A_a(\alpha_s(m))\ln\left(\frac{Nm}{Q}\right)+\ldots\right]$$

Enhancement, but under control

⋆ Color Mixing

- Refactorization scale μ' between $m[J_i]$ and Q



- Changes in $\mu' \to$ resummation. NLL factors

$$\exp \int_{Q/N}^{Q} \frac{dm}{m} \left[\lambda^{(\mathrm{f})}(\alpha_s(m))\right]$$

- (f) labels color exchange basis λs: eigenvalues of color exchange (anom. dim.) matrix Γ_S

- Example: For $g+g \to g+g$

$$\mathrm{Tr}\,\left[\,T_{a_1}T_{a_2}T_{a_3}T_{a_4}\,\right] \text{ and 5 perms}$$
$$\left[T_{a_1}T_{a_2}\right]\ \mathrm{Tr}\,\left[T_{a_3}T_{a_4}\right] \text{ and 2 perms}$$

- Color mixing governed by a 9×9 matrix

- **Example:** $g+g \to g+g$ (Kidonakis 98)

$$\Gamma_{S'} = \frac{\alpha_s C_A}{\pi}\begin{pmatrix} T & 0 & 0 & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & U & 0 & 0 & 0 & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & T & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & 0 & 0 & (T+U) & 0 & 0 & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ 0 & 0 & 0 & 0 & U & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & 0 & 0 & 0 & (T+U) & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ \frac{T-U}{N_c} & 0 & \frac{T-U}{N_c} & \frac{T}{N_c} & 0 & \frac{T}{N_c} & 2T & 0 & 0 \\ -\frac{U}{N_c} & -\frac{T}{N_c} & -\frac{U}{N_c} & 0 & -\frac{T}{N_c} & 0 & 0 & 0 & 0 \\ 0 & \frac{U-T}{N_c} & 0 & \frac{U}{N_c} & \frac{U-T}{N_c} & \frac{U}{N_c} & 0 & 0 & 2U \end{pmatrix}$$

Tejeda-Yeomans
GS 02

- **Same matrix generates** $4-D$ **poles in**

$\mathcal{O}(\alpha_s^2)$ $gg \to gg$ **scattering** (Glover et al., catani): $\exp(I^{(1)})$

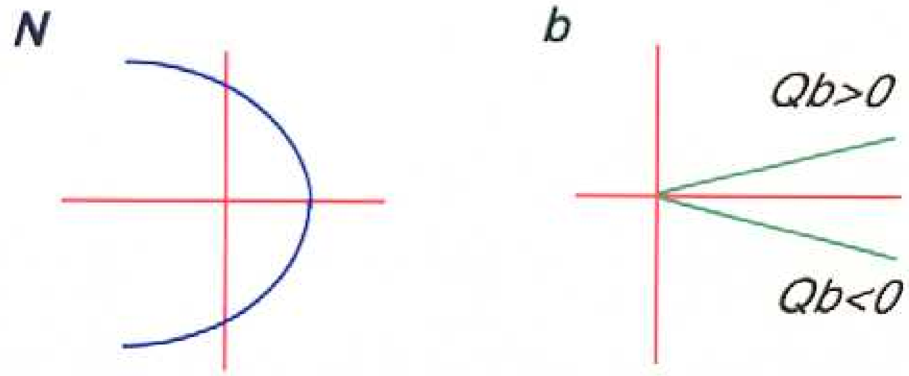
$$I^{(1)}(\epsilon)(s=\mu_R^2) \sim \left(\frac{1}{\epsilon^2}+\frac{\beta_0}{N_c\epsilon}\right)\Gamma_S$$

$\star$ k_T-Resummation

- Example: Joint Resummation for **Z** Q_T
- Double inverse transform

$$\frac{d\sigma^{\mathrm{res}}}{dQ^2 dQ_T^2} \sim \sigma_0^{a\bar{a}\to Z} \int_C dN\, \tau^{-N} \int_{C_b} d^2b\, \mathrm{e}^{i\vec{Q}_T\cdot\vec{b}} \times \mathcal{C}_a(Q,b,N)\, \mathrm{e}^{E_{a\bar{a}}(N,b,Q)}\, \mathcal{C}_{\bar{a}}(Q,b,N)$$

$$C_a(Q,b,N,\mu) = \sum_j C_{a/j}(N,\alpha_s(\mu))\, f_j(N, Q/(N+bQ))$$



- Perturbative exponent

$$E_{a\bar{a}}(N,b,Q) \sim \int_{[Q/(N+bQ)]^2}^{\mu^2} \frac{dm^2}{m^2} A_a(\alpha_s(m))\, \ln\left(\frac{Nm}{Q}\right)$$

- Incorporates energy and p_T-conservation $\rightarrow$ Average p_T^2 function of $\ln(1-x)Q$.

* EXPONENT IN JOINT RESUM.
(closer look)

$E_{ab}(N, b, Q)$

$$= \int \frac{d^2k_T}{(2\pi)^2} \frac{1}{k_T^2} \sum_{i=a,b} A(\alpha_s(k_T^2)) \times$$

(Laenen 01)

$$\times \left[e^{-i\underline{b}\cdot\underline{k}_T} K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{Nk_T}{Q}\right)\right]$$

$$A_i(\alpha_s(k_T^2)) = C_i \frac{\alpha_s(k_T^2)}{\pi} + \cdots$$

* POWER CORRECTIONS

FROM $k_T^2 \to 0$

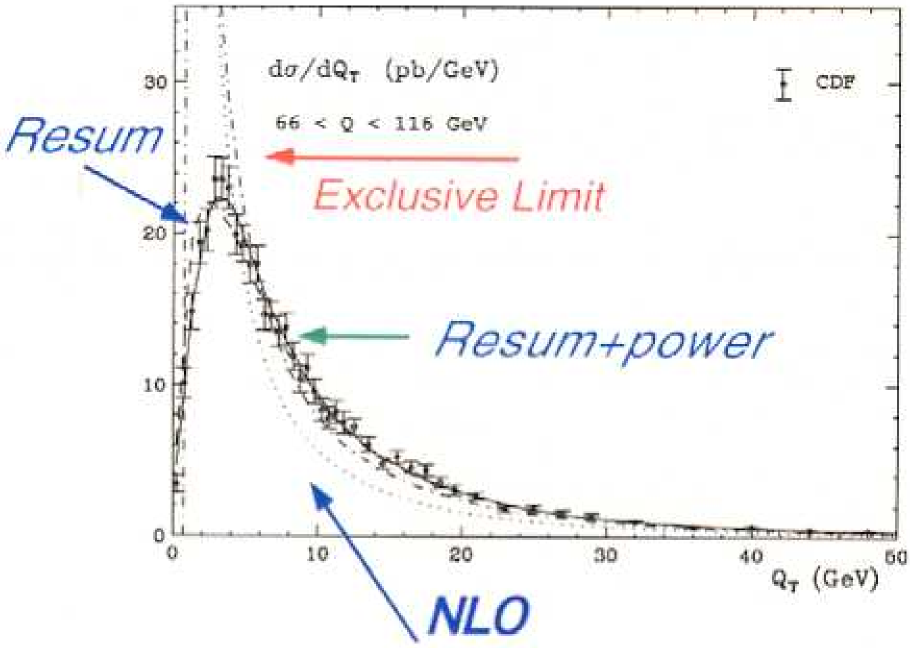
EVEN POWERS of $b, \frac{N}{Q}$

Identify NP parameters

$$g_b \leftrightarrow \int_0^{\mu_{IR}} dk_T\, k_T\, A(\alpha_s(k_T^2)) \ln^b \frac{k_T}{\mu_{IR}}$$

$$\Rightarrow e^{-(b^2 + \frac{2N^2}{Q^2}) g_1 \ln Q + \cdots}$$

"shape function"

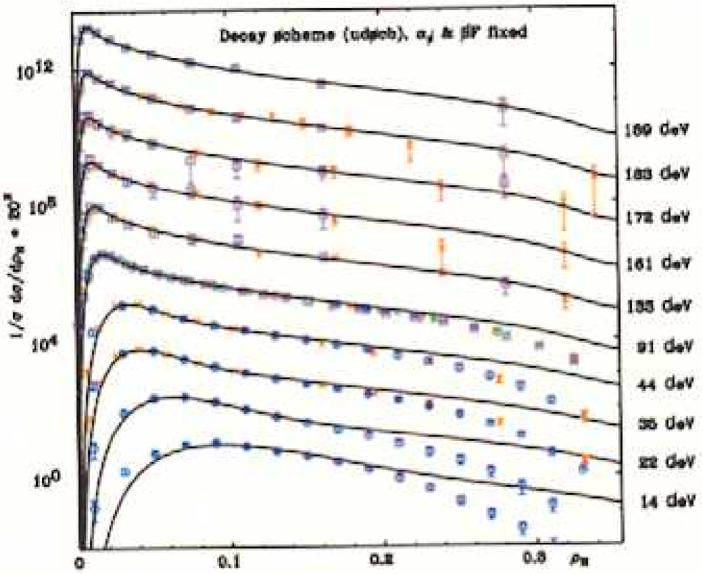


- Power corrections: $\exp[-gb^2]$, $g \sim 0.8$ GeV^2
  Kulesza et al,
  [redacted]; Qiu and Zhang

- PT without cutoff lowers power correction

Shape function phenomenology

$(e^+e^- \to Z)$



Strategy: $f_e(\epsilon)$ **at Z pole; predict other** Q

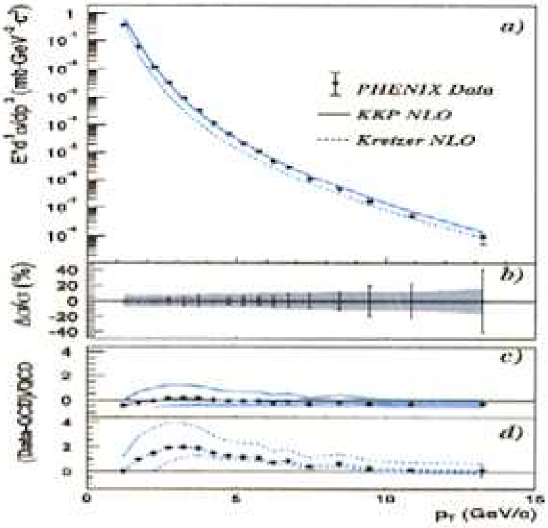
Korchemsky,GS; Gardi Rathsman

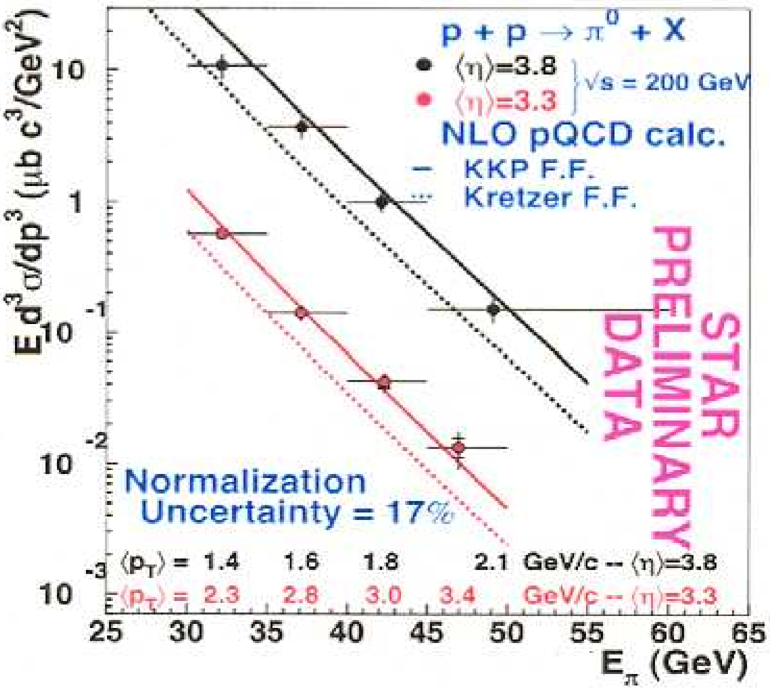
FIRST PASS:
(Belitsky 01)

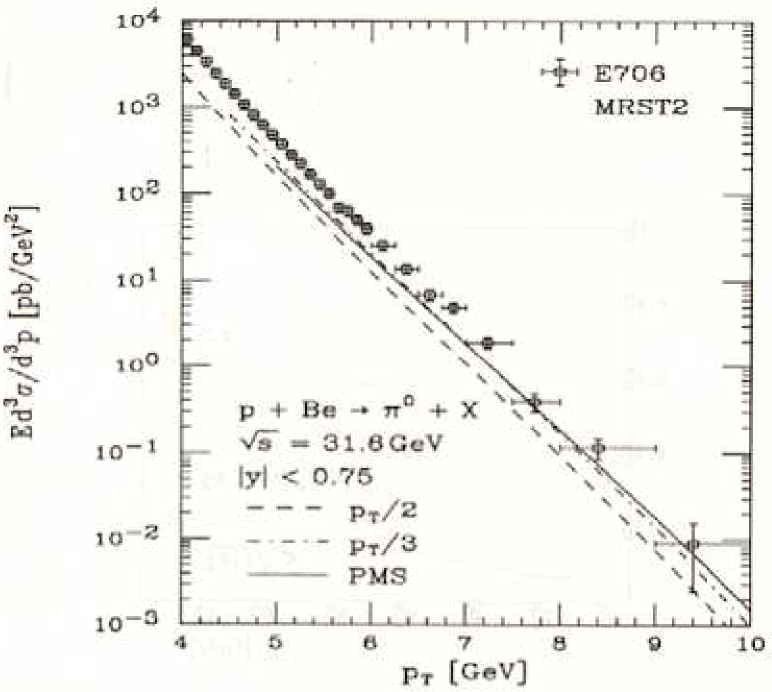
$$f_e(\rho) \sim (\text{const.})\, \rho^{a-1} e^{-b\rho^2}$$

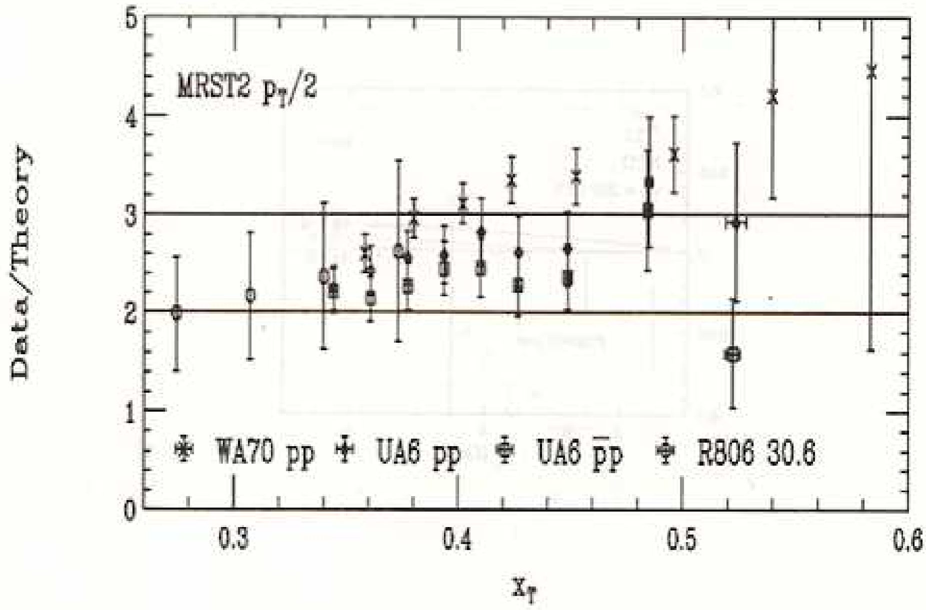
a: ⟨# particles/unit rapidity⟩

⁎ Particle Spectra: Fixed Target vs. Collider

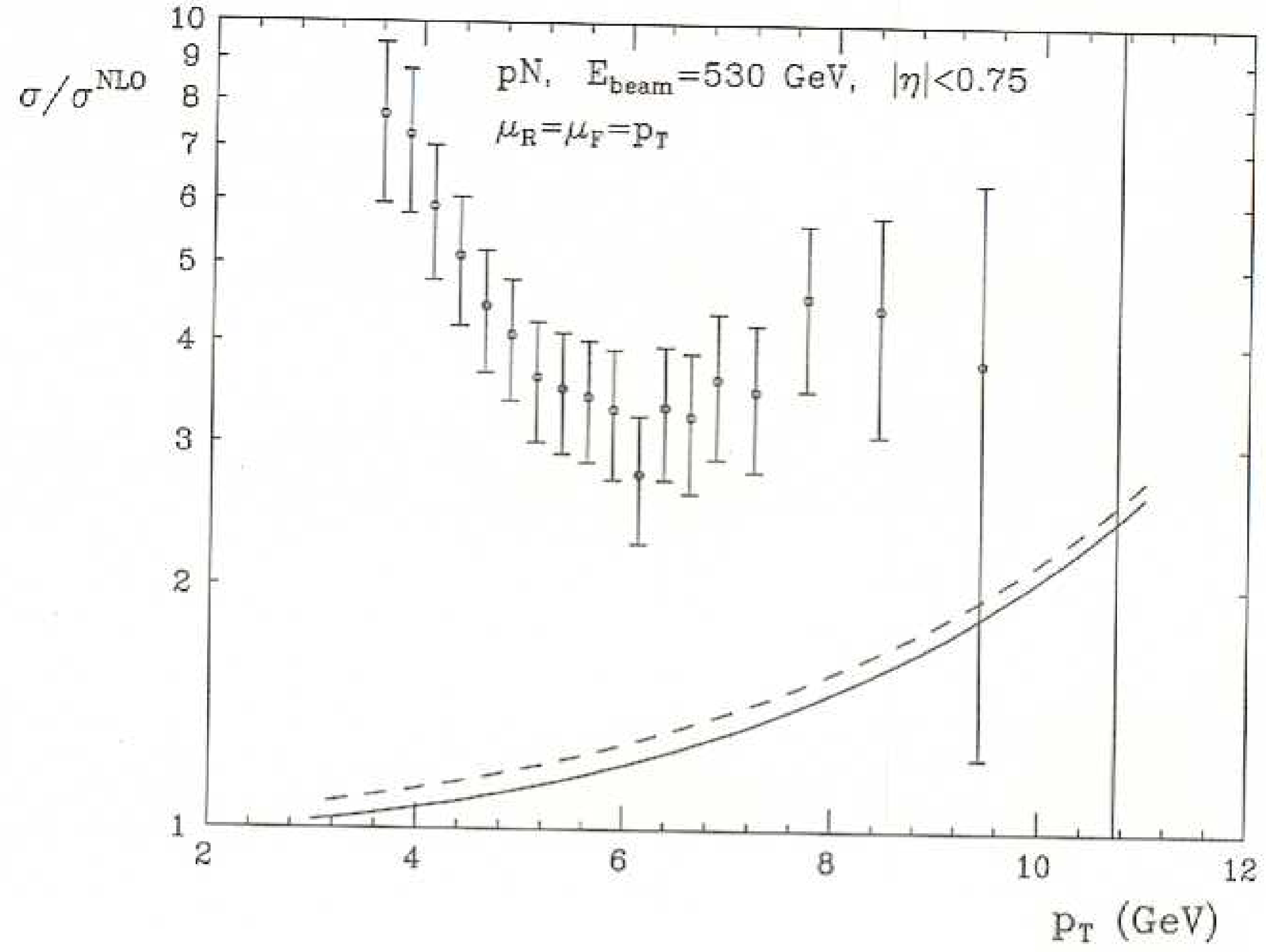








Aurenche et al hep-ph/9910252 Eur. J. Phys.



Catani...
Vogelsang 99

- **Self-consistent recoil in Joint Resummation**

Laenen, GS, Vogelsang

- **Double inverse transform and approximation:**

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \int_{-\infty}^{\infty} d^2b\, d^2Q_T\, \mathrm{e}^{i\vec{Q}_T\cdot\vec{b}}$$

$$\times\ \tilde{\sigma}_{ab}^{(0)}(N)\, \mathrm{e}^{E(N,b,p_T)}$$

$$\times \qquad \underbrace{\left(\frac{S}{4(\vec{p}_T-\frac{1}{2}\vec{Q}_T)^2}\right)^{N+1}}_{=\left(x_T^2\right)^{-N-1}\, \mathrm{e}^{N\vec{Q}_T\cdot\vec{p}_T/p_T^2}\left(1+\mathcal{O}(1/N,Q_T^2/p_T^2)\right)}$$

- Q_T, b **integrals** (N **imaginary**) $\Rightarrow$

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN\, \tilde{\sigma}_{ab}^{(0)}(N)\ \left(x_T^2\right)^{-N-1}$$

$$\mathrm{e}^{E\mathrm{thr}(N,p_T)}\ \mathrm{e}^{\delta\, E\mathrm{recoil}(N,p_T)}$$

$$\delta\, E_{\mathrm{recoil(N,p_T)}} = \pi \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i\left(\alpha_s(k_T^2)\right)$$

$$\left[\left(I_0\left(\frac{Nk_T}{p_T}\right)-1\right)K_0\left(\frac{Nk_T}{p_T}\right)\right]$$

• Isolate perturbative recoil; NLL in N:

$$\delta\, E_{\rm recoil}(N,p_T) = \delta E_{\rm PT} + \delta\, E_{\rm np}$$
$$\delta E_{\rm PT} \propto \frac{\alpha_s(p_T^2/N^2)}{\pi}\,\frac{\zeta(2)}{2}$$

• isolate low scales $\leftrightarrow$ strong coupling

$$\delta\, E_{\rm recoil} \;=\; {\rm PT} \;+\; \lambda_{\rm ab}\,\frac{{\rm N}^2}{{\rm p_T^2}}\;\ln\frac{\rm p_T}{\rm N}$$

$$\lambda_{ab} \sim\; 2g_{\rm EW} \;\propto\; \int dk_T^2\,\alpha_s(k_T^2)$$

$$N \;\leftrightarrow\; \frac{1}{\ln x_T^2}$$

$$\delta\, E_{\rm recoil} \;=\; {\rm PT} \;+\; \lambda_{\rm ab}\,\frac{1}{{\rm p_T^2}\ln^2\left(\frac{4{\rm p_T^2}}{\rm S}\right)}\;\ln\left({\rm p_T}\ln\left(\frac{4{\rm p_T^2}}{\rm S}\right)\right)$$

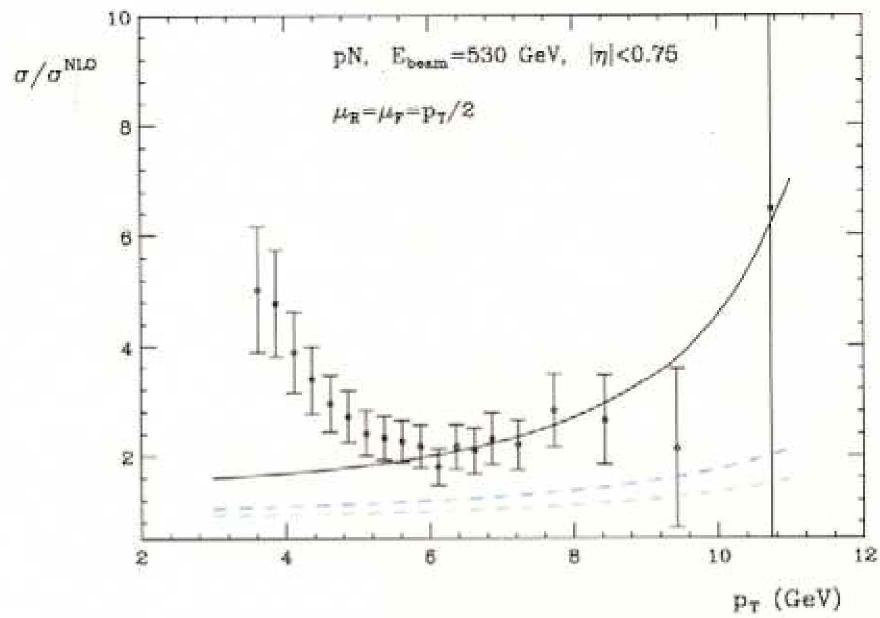
• **power suppressed in p_T**

• **decreases with S at fixed p_T**

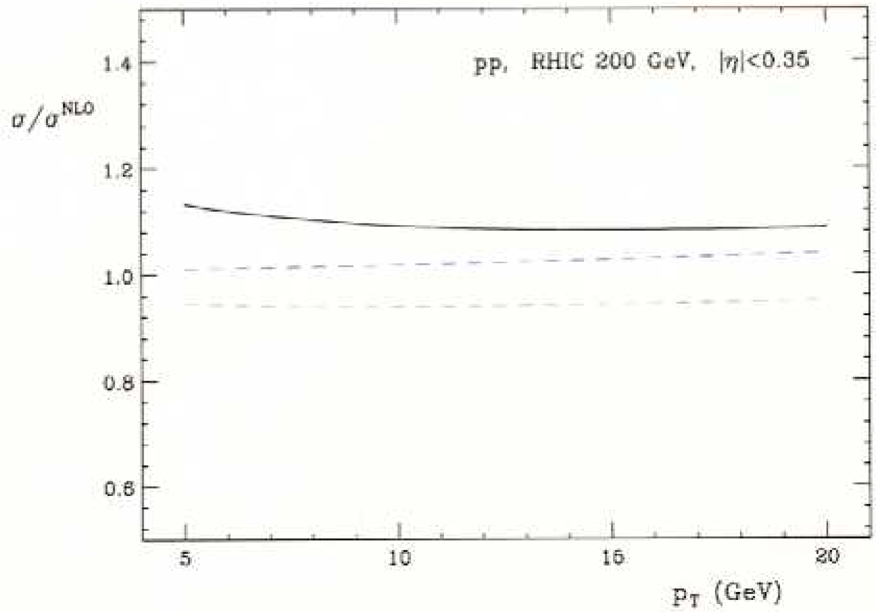
• **match to large- and small-N behavior of Bessel functions $\rightarrow$**
sample 'shape function' of N/p_T only:

$$\delta E_{\rm np} = \frac{N^2}{p_T^2}\;\frac{ln\left(1+\frac{2p_T}{N}\right)}{\left(1+\frac{p_T}{N}\right)^2}$$

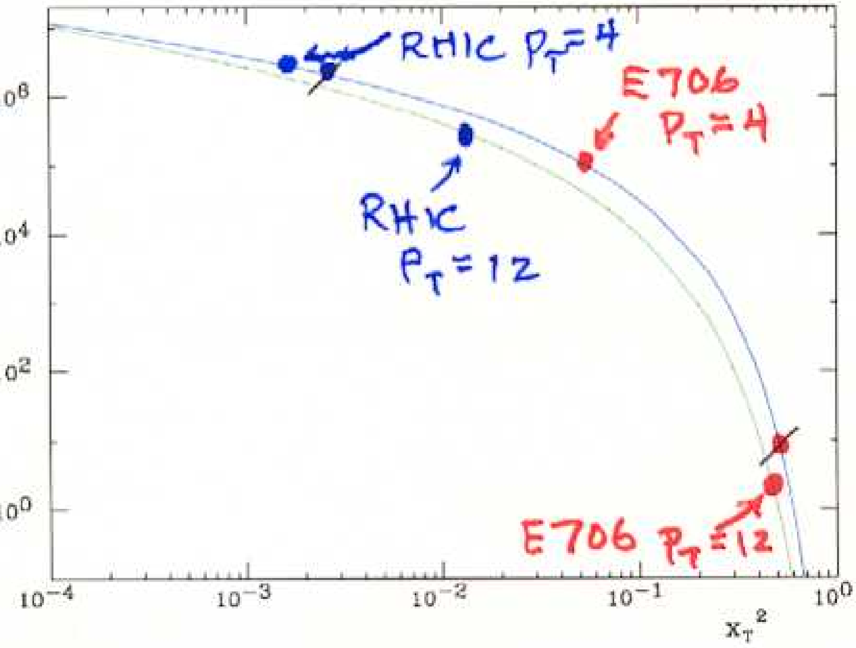
(p_T in GeV)



Including $E_{\rm recoil}$ for direct photons at E706



Including $E_{\rm recoil}$ for RHIC



The NLO cross section $p_T^3 d\sigma_\gamma/dp_T$ *vs.* x_T^2.

Origin of high-p_T enhancement:

$\exp CN^2/p_T^2$ **on** $p_T^3 d\sigma_\gamma/dp_T \sim (x_T^2)^{-\lambda} \Rightarrow (x_T^2)^{-\lambda} \times \mathrm{e}^{C\lambda^2/p_T^2}$

OUTLOOK

Higher fixed order

Phenomenology of PT $\leftrightarrow$ NP transition

Nail down pp 'baseline' to pA, AA

Explore quantum field theory

SELF-CONSISTENT RECOIL IN JOINT RESUMMATION

(Vogelsang, GS)

$$p_T^3 \frac{d\sigma_{ab\pi}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \int_{-\infty}^{\infty} d^2 b \, d^2 Q_T \, e^{i \underline{Q}_T \cdot \underline{b}}$$

$$\cdot \, \tilde{\sigma}^{(0)}_{ab\pi}(N) \cdot e^{E(N, b, p_T)}$$

$$\cdot \left(\frac{S}{4(\underline{p}_T - \frac{1}{2}\underline{Q}_T)^2} \right)^{N+1}$$

$\leftarrow$ singular

$N \to \infty$

$$(x_T^2)^{-N-1} \, e^{-N \underline{Q}_T \cdot \underline{p}_T / p_T^2}$$

$\leftarrow$ nonsingular

Q_T, b integrals $\Rightarrow$

$$p_T^3 \frac{d\sigma_{ab\pi}}{dp_T} \sim \int dN \, \tilde{\sigma}^{(0)}(N) \, (x_T^2)^{-N-1} \cdot$$

$$\cdot \, e^{E_{\mathrm{Thr.}}(N, p_T)}$$

$$\cdot \, e^{\delta E_{\mathrm{recoil}}(N, p_T)}$$

$$\delta E_{\mathrm{recoil}} = \pi \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A(\alpha_s(\underline{\underline{k_T^2}})) \cdot$$

$$\cdot \left[\left(I_0\left(\frac{N k_T}{p_T}\right) - 1 \right) K_0\left(\frac{N k_T}{p_T}\right) \right]$$

Isolate low scales $\leftrightarrow$ strong coupling

$$\delta E_{recoil} = PT + \lambda_{ab} \frac{N^2}{p_T^2} \ln \frac{p_T}{N}$$

$$\lambda_{ab} \propto \int dk_T^2 \, \alpha_s(k_T^2)$$
$$\sim 2 g_{1,EW}$$

$$N \leftrightarrow \frac{1}{\ln x_T}$$

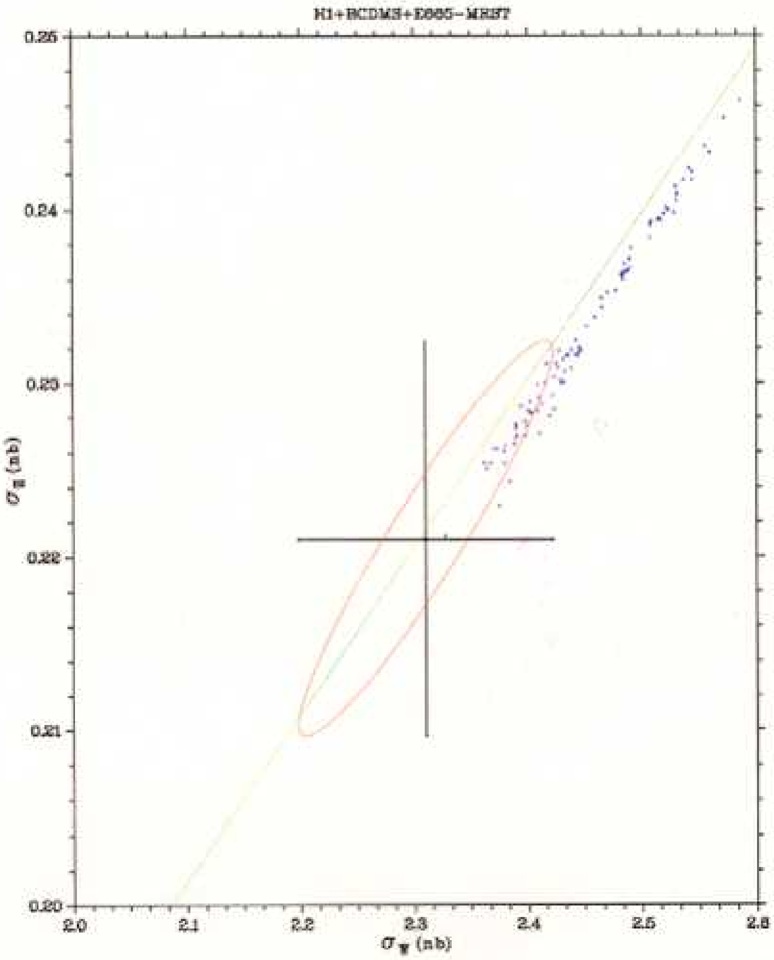
$$\delta E_{recoil} = PT + \lambda_{ab} \frac{1}{p_T^2 \ln^2 \frac{4p_T^2}{s}} \ln\left(\frac{p_T}{\ln \frac{p_T}{\sqrt{s}}}\right)$$

- Power suppressed in p_T
- Decreases with s at fixed p_T

⋆ Parton Distribution Uncertainties

The problem in precision QCD at the Tevatron & LHC

Giele (2001) Pumplin (2001)



- **Inclusive W vs Z cross sections**
- **The D0 measurement is represented by the one-standard deviation error ellipse**
- **The theory prediction is represented by a random sampling PDF set (using H1, ZEUS and E665)**

PERTURBATIVE QCD
OVERVIEW

George Sterman
Workshop on QCD
Confinement and
Heavy Ion Collisions
Tuscon, Oct. 29, 2003

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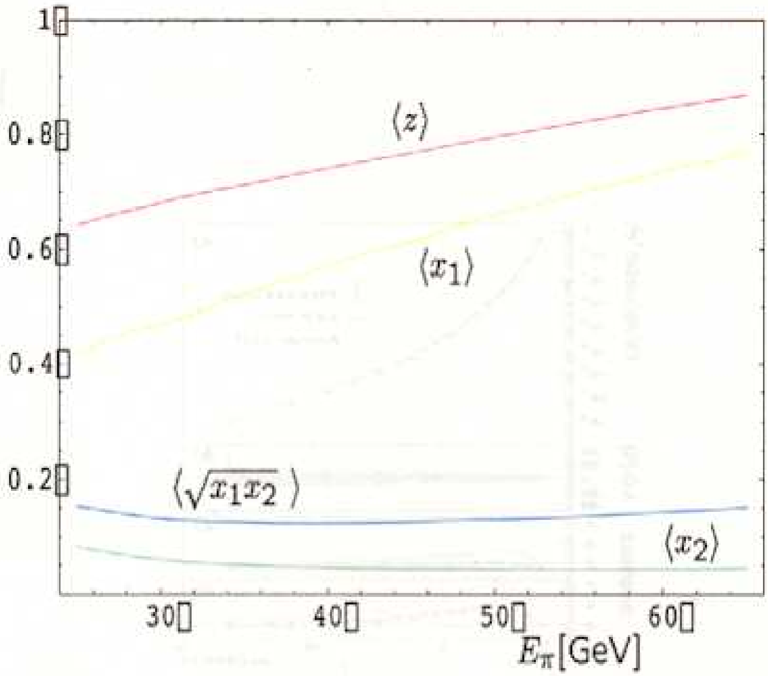
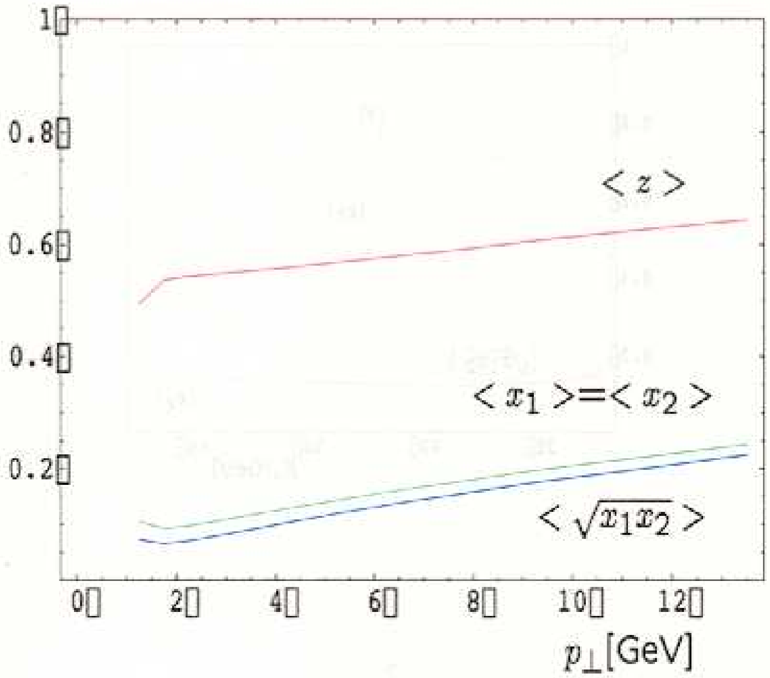
Parton Distributions

- **Examples of '98-01 refinements** (Martin 99, Lai 00)
  - d/u: CDF W asymmetry and Higher twist reanalysis of $F_2^{(D)}$, F_s^p
  - $\bar{d}/\bar{u}$ from DY
- Lesson:
  indirect constraints on PDF's are tentative
- Issues for global fits
  (CTEQ(N= dots 6), MRST2001,2)
  - Example of direct photons: potentially non-perturbative corrections at moderate k_T (E706; controversy)
  - Role replaced by Tevatron high-p_T jet data
  - Needed: systematic understanding of *differences* in higher-order corrections between processes
  - parton distribution uncertainties: statistical analysis *vs* role of higher orders & NP corrections? (Giele 98, Brock 00)



1
0.8
0.6
0.4
0.2
30
40
50
60
E_π[GeV]
$\langle z\rangle$
$\langle x_1\rangle$
$\langle\sqrt{x_1x_2}\,\rangle$
$\langle x_2\rangle$



1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
$p_\perp$[GeV]
$<z>$
$<x_1>=<x_2>$
$<\sqrt{x_1x_2}>$