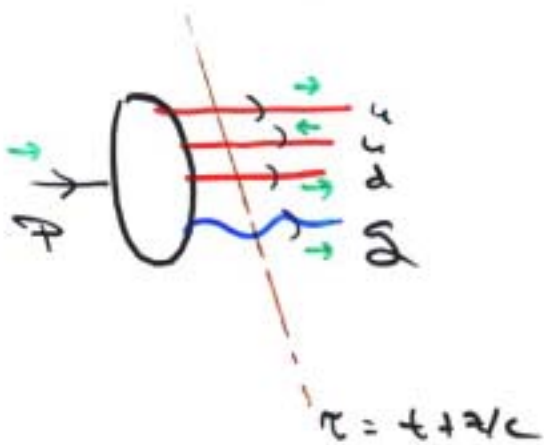
Riken BNL Research Center Workshop

High p_T Physics at RHIC

Novel Spin Effects in QCD

S. Brodsky
SLAC



$$J_z^p = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z$$

$$\left\{ \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right\}$$

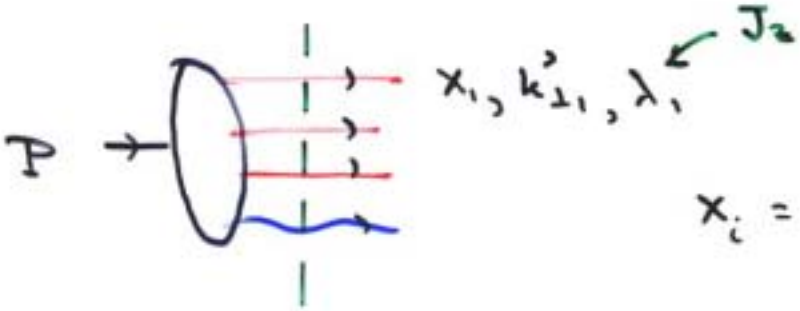
$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{p^0 + p^z} \qquad \sum_{i=1}^{n} x_i = 1$$

$$\sum \vec{k}_{\perp i} = 0$$

Light-Front Wavefunctions in QCD

Essential quantities for QCD calculations

at amplitude level



$$x_i = \frac{k^+_i}{P^+} = \frac{k^0_i + k^z_i}{P^0 + P^z}$$

Fixed $\tau = t + z/c$ "Light-Front Time"
Dirac

$$|\Psi_P\rangle = \sum_n |n\rangle\langle n|\Psi_P\rangle$$

$$\boxed{\langle n|\Psi_P\rangle = \Psi_{n/P}(x_i, \vec{k}_{\perp i}, \lambda_i)}$$

$$\sum_{i=1}^{n} x_i = 1 \quad , \quad \sum_{i=1}^{n} \vec{k}_{\perp i} = 0$$

Relativistic representation of hadrons
in terms of
quark and gluon degrees of freedom

J^z conservation for each LC Fock comp.

X. Ji

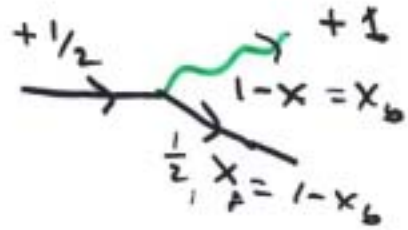
$$J^z_{(n)} = \sum_{j=1}^{n} S_j^z + \sum_{j=1}^{n-1} \ell_j^z$$

Hwang
Ma
Schmidt
SJB

Only $(n-1)$ relative ℓ_j^z !

$$J^z \left| \Psi^{\uparrow}_{\lambda_f, \lambda_b} \right\rangle = \tfrac{1}{2} \left| \Psi^{\uparrow}_{\lambda_f, \lambda_b} \right\rangle$$

Helicity retention at $x_b \to 1$



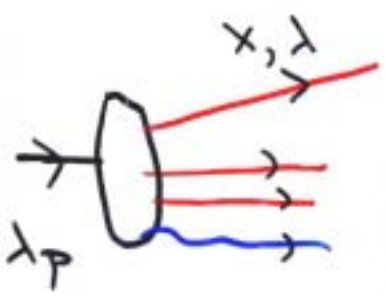
$$\left| \Psi^{\uparrow}_{1/2,1} \right|^2 \sim \frac{1}{(1-x_b)^2} \mathcal{C}^2$$

SJB + GPL
Burkardt, Schmidt
SJB

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$$q_{\lambda/\lambda_p} = \sum \Big|$$



$$\Big|^2$$

$$q_{\lambda/\lambda_p}(x, \Lambda)$$

transversity: density matrix

light-cone helicity

$$= \sum_{n,\ell} \int \left| \psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1}^{n} dx_j \prod_{j=1}^{n} d^2 k_{\perp j}$$

$$\delta\left(\sum_i x_i - 1\right) \delta\left(\sum_i \vec{k}_{\perp i}\right)$$

$$\delta(x - x_\ell)\, \delta_{\lambda,\lambda_\ell}$$

$$\Theta(\Lambda^2 - m_n^2)$$

Factorization

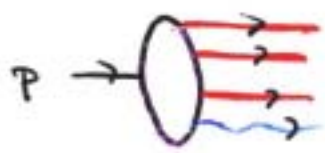
Light-Cone Scheme

* Quark, gluon structure of hadrons:

central focus of QCD

* Light-cone Fock expansion:

rigorous repn of quantum field theory



$$\{\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$$

gauge-invariant measures

$$q(x, Q^2) = \sum_n \int^{m_n^2 < Q^2} |\Psi_n(x_i, k_{\perp i}, \lambda_i)|^2 \pi d^2k_{\perp i}\, dx_i\, \delta(x_i - x_q)$$

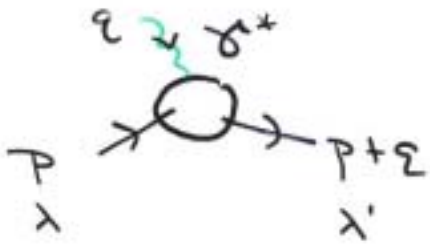
$\Rightarrow$ parton distributions, inclusive process

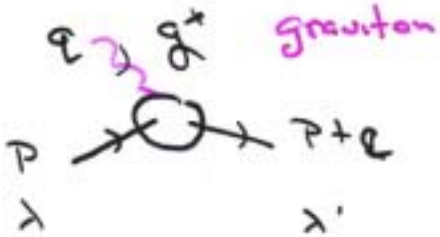
$$\phi(x_i, Q^2) = \int^{m_n^2 < Q^2} \pi d^2k_{\perp i}\; \Psi_{val}(x_i, k_{\perp i}, \lambda_i)$$

$\Rightarrow$ distribution amplitude, exclusive proc.

Ψ_n: fluctuations in particle number, size, helicity, flavor, momentum correlations

Electromagnetic and Gravitational Form Factors





$$\langle p+q\uparrow | \frac{J^+(0)}{2p^+} | p\uparrow\rangle = F_1(q^2)$$

electromagnetic

$$\langle p+q\uparrow | \frac{J^+(0)}{2p^+} | p\downarrow\rangle = -(q_1 - iq_2)\frac{F_2(q^2)}{2M}$$

$$\langle p+q\uparrow | \frac{T^{++}(0)}{2p^{+2}} | p\uparrow\rangle = A(q^2)$$

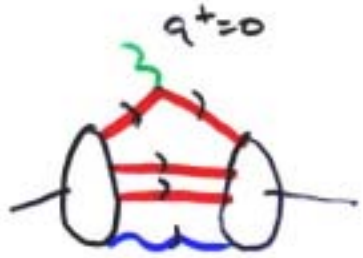
gravity

$$\langle p+q\uparrow | \frac{T^{++}(0)}{2p^{+2}} | p\downarrow\rangle = -(q_1 - iq_2)\frac{B(q^2)}{2M}$$

* Choose $q = (q^+, q^-, \vec{q}_\perp) = (0, \frac{2p\cdot q}{p^+}, \vec{q}_\perp)$

$$p = (p^+, p^-, \vec{p}_\perp) = (p^+, \frac{M^2}{p^+}, \vec{0}_\perp)$$

$$2p\cdot q = -q^2 = Q^2 = \vec{q}_\perp^{\,2}$$

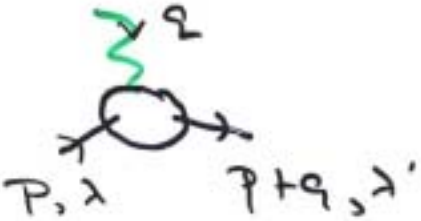


diagonal $n = n'$ only!

Light-Front Wavefunctions

* Space-like form factors computed from diagonal $n = n'$ overlap

$$q^+ = 0, \quad q^2 = -q_\perp^2 = -Q^2$$



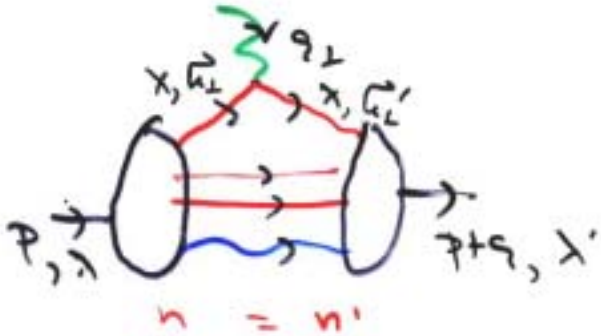
$$= \langle p+q | \frac{J^+(0)}{p^+} | p \rangle$$

Drell, Yan, West
Drell, SSA

$$F_{\lambda\lambda'}(q^2) = \sum_q e_q \sum_{n=n'} \int [d^2k_\perp] \int [dx]$$

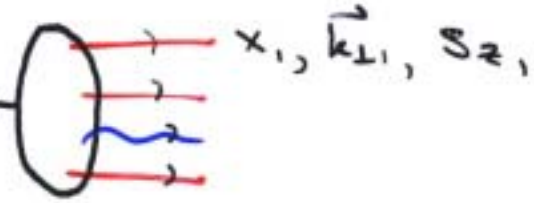
$$\psi_{n'}(x, \vec{k}_\perp', \lambda') \, \psi_{n,\lambda}(x, \vec{k}_\perp, \lambda)$$

$$\vec{k}_\perp' = \begin{cases} \vec{k}_\perp + (1-x)\vec{q}_\perp & \text{struck parton} \\ \vec{k}_\perp - x\,\vec{q}_\perp & \text{spectators} \end{cases}$$



no ghosts (l.c.g.)
no vacuum graphs
no infinite sum of irreducible kernels

Light-Front Spin :

$1, 0, J_z \rightarrow$ 

$\Psi_n(x_i, \vec{k}_{\perp i}, S_{zi})$

$$\sum_{i=1}^{n} x_i = 1 \quad , \quad \sum_{i=1}^{n} \vec{k}_{\perp i} = \vec{0}_\perp$$

$$\boxed{J_z = \sum_{i=1}^{n} S_z^{(i)} + \sum_{i=1}^{n-1} L_z^{(i)}}$$

$$\rightarrow \quad L_z^{(i)} = -i k_x^{(i)} \frac{\partial}{\partial k_y^{(i)}} - k_y^{(i)} \frac{\partial}{\partial k_x^{(i)}}$$

- J_z conserved for each Fock State
- $2J+1$ degeneracy $J_z = 0, \pm 1, \cdots \pm J$

- LF spinors
- LF polarization vectors $(A^+ = 0)$

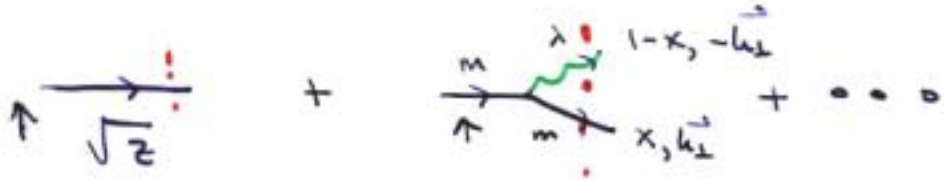
$\frac{\alpha}{2\pi}$

Example of Light-cone wavefunctions

B+Drell
B. Ma Huang
Schmidt

quantum fluctuation of $|e^-\rangle$ in QED

$A^+ = 0$



dom at $x_g \to 1$

large $\Delta g(x)$

$$\psi^{\uparrow}_{+1/2,+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(-k^1 + i k^2)}{x(1-x)}\,\varphi(x,\vec{k}_\perp)$$

$l^z = -1$

$$\psi^{\uparrow}_{+1/2,-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(k^1 + i k^2)}{1-x}\,\varphi(x,\vec{k}_\perp)$$

$l_z = +1$

$$\psi^{\uparrow}_{-\frac{1}{2},+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M - \frac{m}{x}\right)\varphi(x,\vec{k}_\perp)$$

$l_z = 0$

$$\varphi(x,k_\perp) = \frac{e}{\sqrt{1-x}}\,\frac{1}{M^2 - \frac{k_\perp^2+m^2}{x} - \frac{k_\perp^2+\lambda^2}{1-x}}$$

Cons. of J^z : $J^z = \sum_{i=1}^{n} S_i^z + \sum_{j=1}^{n-1} l_j^z = \frac{1}{2}$

$$l_j^z = -i\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

$n-1$ only!

Some advantages of Light-Front-Quantized QCD

* No Fermion doubling
* Multiple fermion flavors
* Minkowski space
* Gauge-Fixed : Physical Degrees of Freedom
* Manifest Frame-Independence : $P^+, \vec{P}^\perperp$ arbitrary
  J_z kinematical
* L.F. Vacuum Trivial + Zero modes
* Vanishing anomalous gravitomagnetic moment

$$B(0) \equiv 0 \quad ; \quad \lim_{M_R \to 0} M_A = 0$$

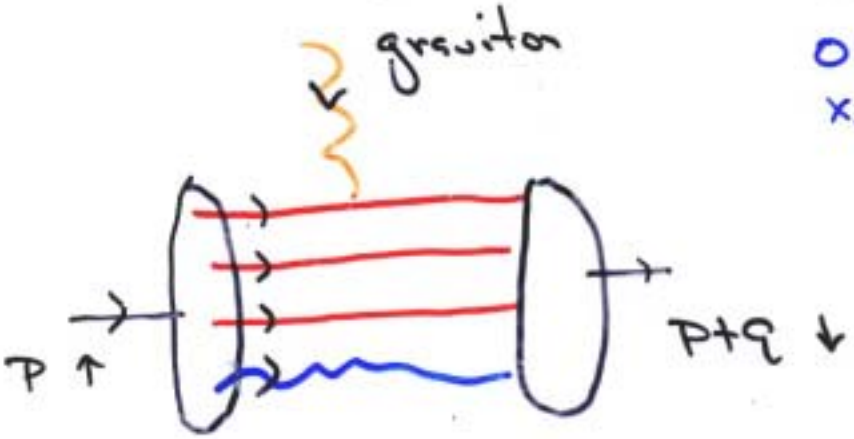
* DLCQ discretization retains symmetries
  continuum limit : $K \to \infty$
* LF Wavefunctions, continuum solns.,
  amplitudes, phases as well as spectra
* Solns for QCD(1+1), SUSY(1+1)
  QCD(3+1) : Challenging!

Theorem

Anomalous gravitomagnetic moment $B(0) = 0$

manifestation of Equiv. Princ.

Okun & Kobzarev (62)
X. Ji, Terayev

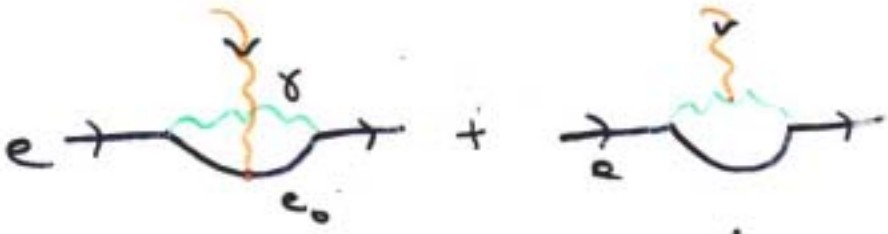


$$\sum_{i=1}^{n} B_i(0) = 0.$$

$$\langle p+q \downarrow | \frac{T^{++}(0)}{2p^{+2}} | p \uparrow \rangle = (q_1 - i q_2)\, B(q^2)$$

e.g. quantum fluctuations of electron

Hwang, Ma, Schmidt 98



$$\frac{\alpha}{3\pi} \qquad -\frac{\alpha}{3\pi}$$

$B_e + B_\gamma = 0$

at $q^2 = 0$.

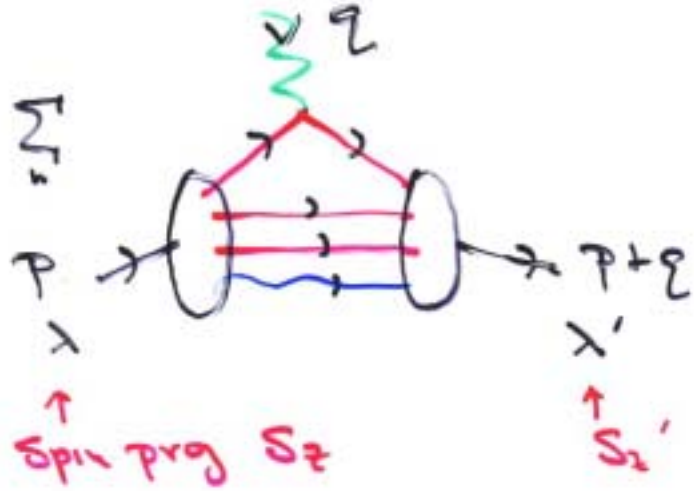
$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$:

Exact repn. of local current m.e's

Spacelike F.F.

$q^+ = 0$ frame

$q^2 = -q_\perp^2$



$\langle p+q_\lambda | J^+ | p_{\lambda'} \rangle \Rightarrow F_i(q^2)$ DYW Drell + SSB

$\langle p+q | T^{++} | p \rangle \Rightarrow A(q^2), B(q^2)$

grav.t. f.f. Huang Ma Schmidt SSB Okun

Prove: $B(0) = 0$ vanishing anom. grav. mom.

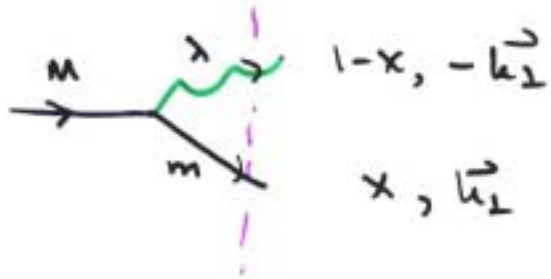
$\mu_A = F_2(0) \Rightarrow 0$ for $MR \to 0$

Use LCG $(A^+ = 0)$: no ghosts; unitary

SM, SSB Srivastava, SPR

Testing ground for light-cone wavefunctions

Quantum fluctuations of electron in QED



$+ \cdots$

$L_z = -1$ $$\Psi^{\uparrow}_{+1/2,+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(-k^1+ik^2)}{x(1-x)}\,\varphi(x,\vec{k}_\perp)$$

$L_z = 1$ $$\Psi^{\uparrow}_{+1/2,-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{(k^1+ik^2)}{1-x}\,\varphi(x,\vec{k}_\perp)$$

$L_z = 0$ $$\Psi^{\uparrow}_{-1/2,+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M-\frac{m}{x}\right)\varphi(x,\vec{k}_\perp)$$

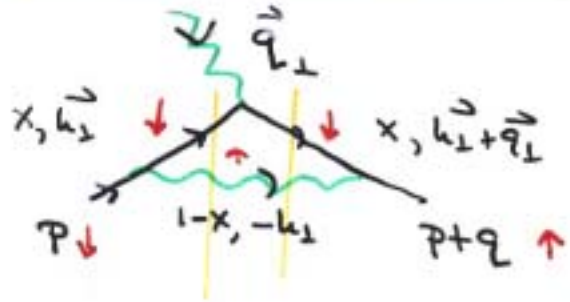
$$\varphi(x,\vec{k}_\perp) = \frac{e}{\sqrt{1-x}}\,\frac{1}{M^2-\frac{k_\perp^2+m^2}{x}-\frac{k_\perp^2+\lambda^2}{1-x}}$$

S. Drell + SJB

Huang, Ma, Schmidt, SJB

LC Calculation of $(g-2)_e$

Drell + SJB



$q^+ = 0$ frame

only one LC time-order

$$-(q_1 - i q_2)\,\frac{F_2(q^2)}{2m} = \langle p+q\uparrow \,|\, \frac{J^+(0)}{2p^+} \,|\, p\downarrow \rangle$$

$$= \int \frac{d^2k_\perp}{2(2\pi)^3} \int_0^1 dx \; \psi^{\uparrow}_{e\gamma\,\uparrow\uparrow}(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)\, \psi^{\downarrow}_{e\gamma\,\uparrow\uparrow}(x, \vec{k}_\perp)$$

* $q_\perp$-linear term gives $F_2(0) = \frac{\alpha}{2\pi}$

* Higher-order : alternating denominator renormalization

Roskies
Suaya
SJB

* Non-perturbative : DLCQ

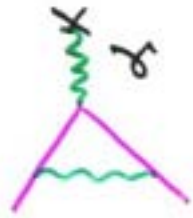
truncation in particle number not e

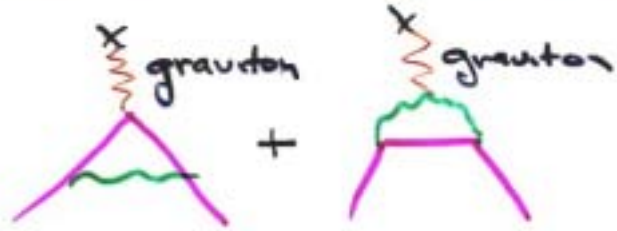
McCartor

J. Hiller
SJB

Anomalous Gravitomagnetic Moment $B(0)$

QED





$$F_2(0) = \frac{\alpha}{2\pi}$$

$$B(0) = \frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0.$$

$$B(q^2) \sim \frac{\alpha}{\pi}\sqrt{q^2/m^2}$$

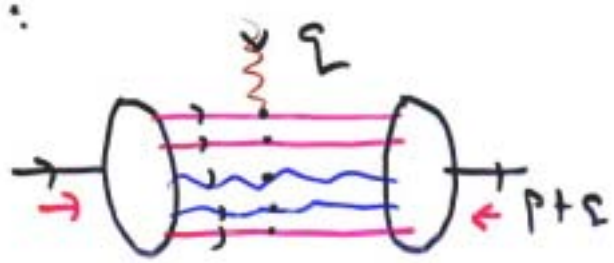
Equivalence Principle : $B(0) = 0$

Okun + Kobzarev (62)
X. J., Tergaev

any spin 1/2 system

L.F. :

$$\lim_{q^+ \to 0} \sum_{i=1}^{n}$$



$(\Delta L_z = 1)$

$$\sum_{i=1}^{n} B_i(0) = 0$$

Fock state by Fock state.

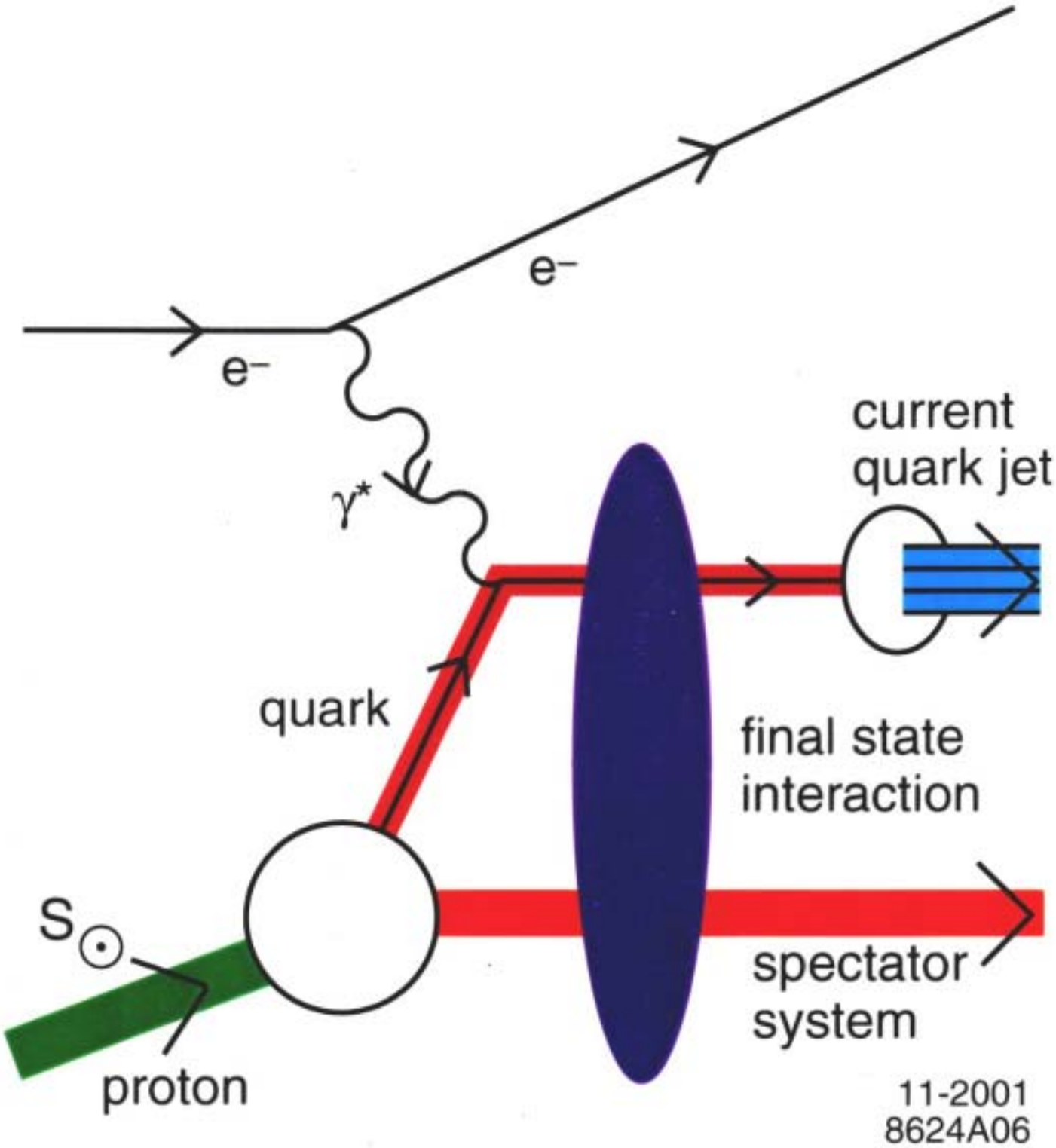
Result of Lorentz prop of LF wavefunctions.

Hwang, Ma
Schmidt, SJB

key Question for LGTh

$$B_p(0) = 0 \ ?$$

Important indicator of lattice errors.


e⁻
e⁻
γ*
current
quark jet
quark
final state
interaction
S
spectator
system
proton
11-2001
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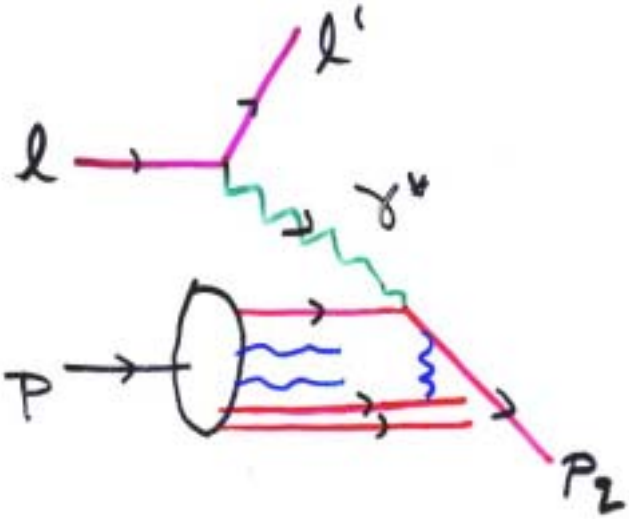
Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange after photon acts

not in LF WF

* Single-spin asymmetry $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Bjorken-scaling

* Diffraction at Leading Twist
* Nuclear Shadowing (interference of diff channels)
* Energy Loss, p_T Broadening

Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

* $\Delta L_{Ioffe} = \frac{2\nu}{Q^2} = \frac{1}{M x_{Bj}}$

References

"Structure functions are not parton probabilities"

S.J.B., P. Hoyer, N. Marchal, S. Peigne + F. Sannino

BHMPS

PRD 65 114025 (2002) hep-ph/0104291

"Final-state interactions and Single-spin Asymmetries in Semi-inclusive Deep Inelastic Scatt."

S.J.B., Dae Sung Hwang, I. Schmidt

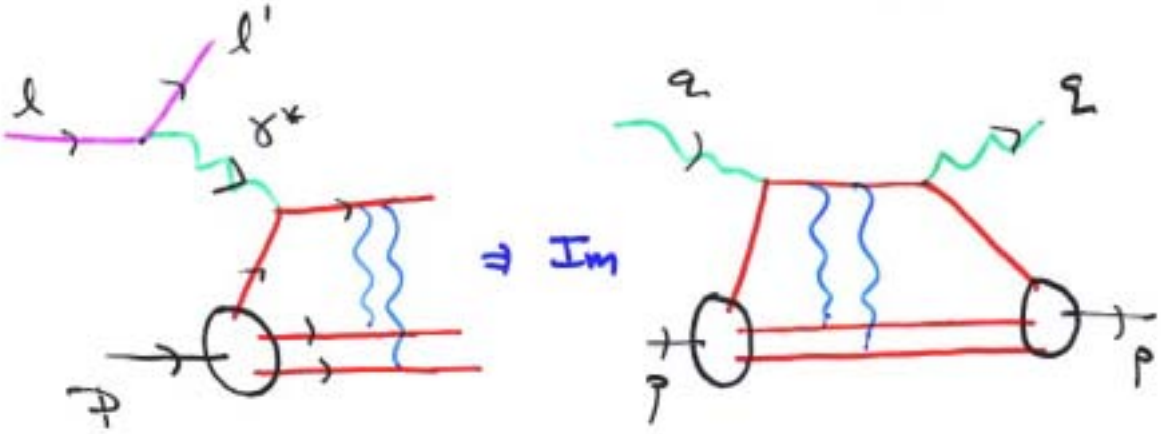
PLB 530 99 (2002) hep-ph/0201296

* J.C. Collins hep-ph/0204004

* X. Ji + F. Yuan hep-ph/0206057

* S.J.B, A. Hebecker + E. Quack PRD 55, 2584 (1997)

New perspectives on Final-State-Interactions



$$f_{q/N}(x_B, Q^2) = \frac{1}{8\pi}\int dy^- \, e^{-i x_B p^+ y^-}$$

$$\langle N(p)|\bar{q}(y^-)\gamma^+ \mathcal{P}\, e^{ig\int_0^{y^-} d\omega^- A^+(\omega^-)} q(0)|N(p)\rangle$$

Usual argument $A^+ = 0$ gauge

no effect from FSI !

phase irrelevant

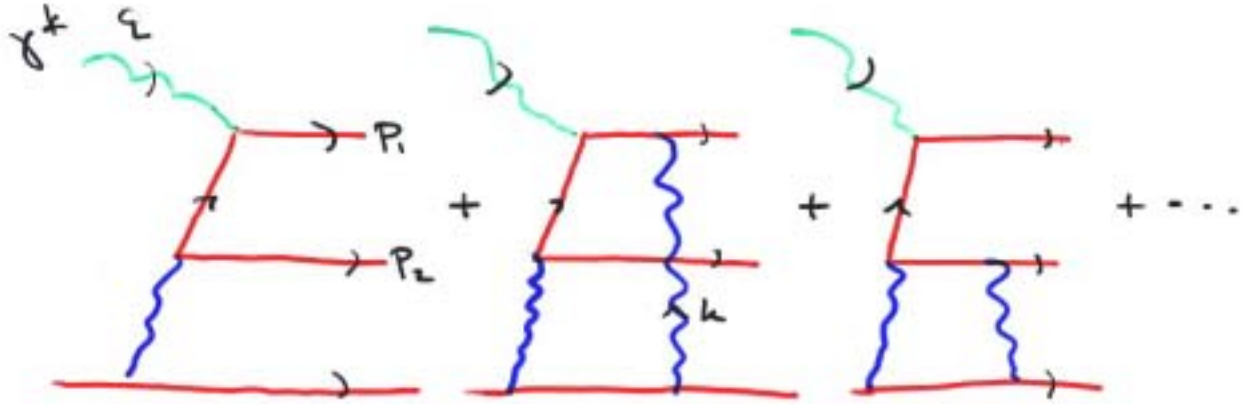
$\therefore$ Identify $f_{q/N}$ with LC Prob. Dist

$$P_{q/N}(x_B, Q^2) = \sum_n \int^{k_{\perp i}^2 < Q^2} \left[\prod dx\, d^2k_\perp\right] |\Psi_n(x, \vec{k}_{\perp i})|^2 \sum_q \delta(x_B - x_j)$$

Explicit calculation of FSI

BHMPS

$q^+ \leq 0$ frame



LFTOPT : gluon exchanged after photon acts

Find non-zero leading-twist FSI effect

Leading twist diffraction , Eikonal form

Shadowing correction , not Coulomb phase

Gauge - independent :

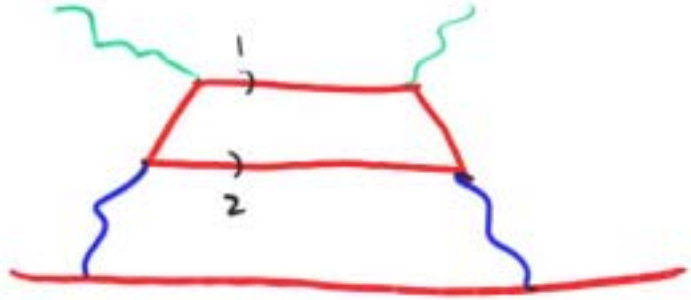
Checked: Feynman , LCG $[A^+=0]$ $\begin{cases} \text{ML} \\ \text{PV} \\ k \end{cases}$ prescriptions

Lesson for LCG:

$$g^{\mu\nu} - \frac{n^\mu k^\nu + k^\nu n^\mu}{n \cdot k}$$

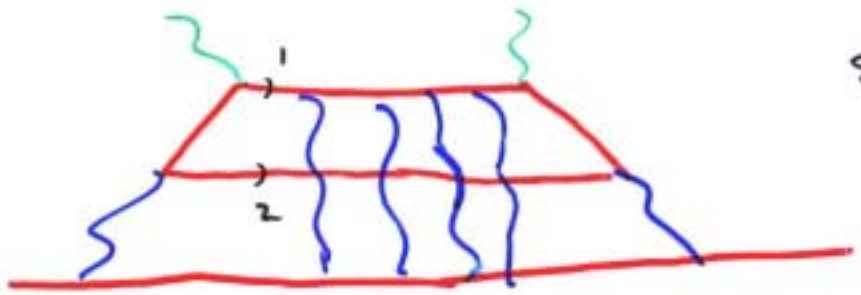
At pole: $n \cdot k = k^+ = O(\frac{1}{v})$.

Consider



$q^+ \leq 0$

In Feynman gauge, must keep



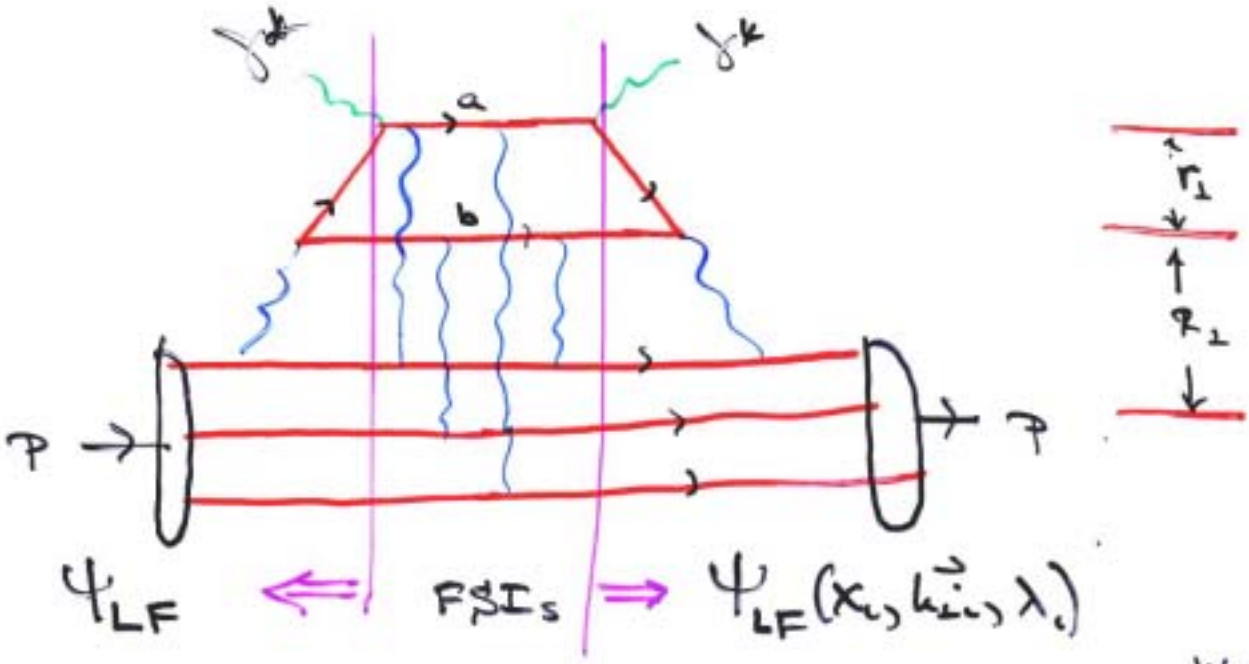
Shadowing effect from on-shell states

$$|\mathcal{M}|^2 \Rightarrow \left| \frac{\sin[g^2 W(r_\perp, R_\perp)/2]}{g^2 W(r_\perp, R_\perp)/2} \mathcal{M}_0 \right|^2$$

$$W(r_\perp, R_\perp) = \int d^2k_\perp \frac{1 - e^{+i r_\perp \cdot \vec{k}_\perp}}{k_\perp^2} e^{i \vec{R}_\perp \cdot \vec{k}_\perp}$$

$$= \frac{1}{2\pi} \log\left(\frac{|\vec{R}_\perp + \vec{r}_\perp|}{|R_\perp|} \right)$$

Hoyer, Marchal
Peigne, Sannino, SB



$$W = \frac{g^2}{2\pi} \log \frac{|\vec{R}_\perp - \vec{r}_\perp|}{|\vec{r}_\perp|}$$

phase $e^{iW_a} e^{iW_b} = e^{iW}$

subtract tree gluon

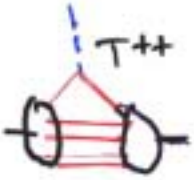
Augment LFWF

$$\Psi_{LF} \Rightarrow \Psi_{LF} \frac{e^{iW} - 1}{iW}$$

$$F_2 \Rightarrow F_2 \left| \frac{\sin W/2}{W} \right|^2 < F_2$$

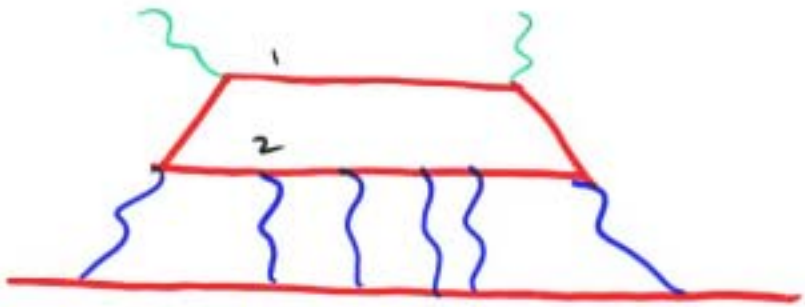
Two Wilson lines !: like external field. (BJY)

* Shadowing not contained in Ψ_{LFWF}
  - not in LGTL or Ψ_{BS}
  - not in local operators



In Light-Cone gauge ("Kovchegov" prescription)

must keep



Final state rescattering

p_z line

These graphs are suppressed in Feynman gauge

but in l.c.g

$$d_{LC}^{\mu\nu} = \frac{i}{k^2+i\epsilon}\left[-g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k}\right]$$

and $n\cdot k = k^+ = O\left(\frac{1}{\nu}\right)$ for on-shell states!

* Result: identical answer as Feynman gauge

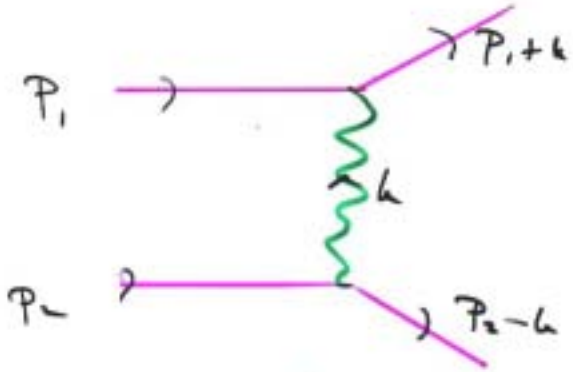
* Not included in l.c.g wavefunctions!

M-L, PV prescriptions differ by resid. singularities

Light-cone gauge evaluation

$n^\mu = (0, 2, 0_\perp)$

$n^2 = 0$



$2p_1 \cdot k + k^2 = 0$

$$\mathcal{M} = e^2 \underset{\perp}{(2p_1 + k)_\mu} \frac{\left[-g^{\mu\nu} + \frac{n^\mu \overset{\perp}{k^\nu} + \overset{\perp}{k^\mu} n^\nu}{n \cdot k} \right]}{k^2 + i\epsilon} \underset{+}{(2p_2 - k)_\nu}$$

where: $n \cdot k = k^+ = \dfrac{-k_\perp^2}{p_1^-} \to 0$ for $s \gg -t$

Dominant term in l.c.g. from non-eikonal current

$$\mathcal{M} = e^2 \frac{-k_\perp^2 \, \frac{2p_2^+}{k^+}}{k^2 + i\epsilon} \Rightarrow -2e^2 \frac{s}{t}$$

$\vec{A}_\perp$ plays crucial role in lcg: $k^+ = O\left(\frac{1}{p_1^-}\right)$

Similarly: $\int d\omega^\mu A_\mu \neq \int d\omega^- A^+$ in l.c.g.

* Alternative approach

Belitsky, Ji, Yuan

⇒ Augment LFWF (lcg (PV)) with transverse gauge link

$$\Psi \Rightarrow \Psi L$$

$$L = P \exp\left[i g \int_0^\infty d\vec{\xi}_\perp \cdot \vec{A}_\perp (\xi^- = \infty, \vec{\xi}_\perp) \right]$$

where $\vec{A}_\perp = -\frac{g}{2\pi} \Theta(\xi^-) \vec{\nabla}_\perp \ln \mu \vec{r}_\perp$

corresponds to solving LFWF in Coulomb field of charged particle!

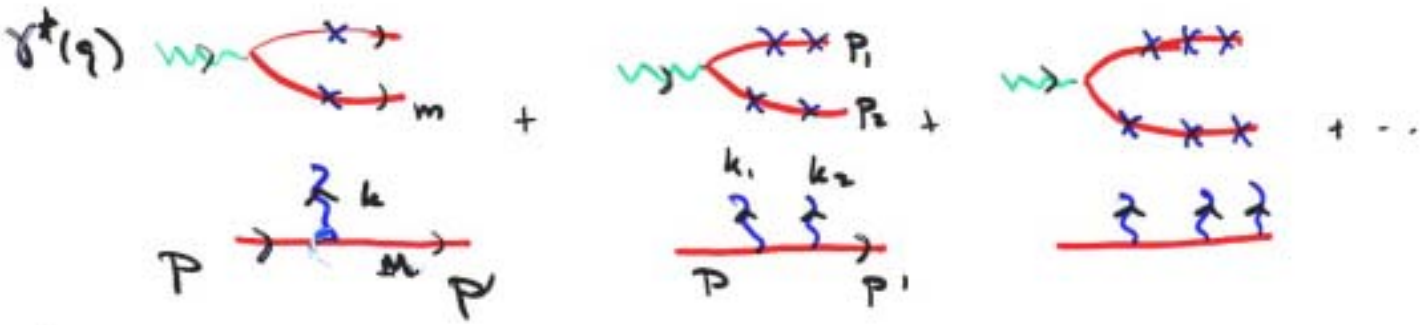
Equivalent to

$$d^{\mu\nu}(k) = \frac{-i}{k^2}\left(g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n + i\epsilon} \right)$$

non-causal b.c. (advanced boundary)

No Wilson line but external field present.

Model Calculation



Scalar quarks, crossed + uncrossed graphs, large M
+ seagulls

Eikonal factorization in $\vec{r}_\perp$, $\vec{R}_\perp$
Verified to 3-loops in Feynman, l.e.g.

$$* \quad \mathcal{M} = \mathcal{M}_{Born}\left[1 - e^{-ig^2 W}\right]$$

$$\mathcal{M}_{Born} = -2ieMQ\,p_2^-\,V(m_{\parallel}, r_\perp)$$

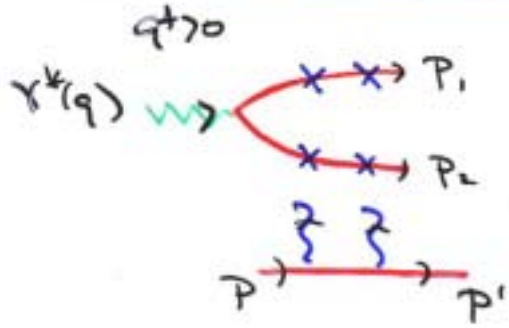
$$V(m_{\parallel}, r_\perp) = \int \frac{d^2p_\perp}{(2\pi)^2}\,\frac{e^{i\vec{r}_\perp\cdot\vec{p}_\perp}}{p_\perp^2 + m_{\parallel}^2} = \frac{1}{2\pi}K_0(m_{\parallel} r_\perp)$$

$$m_{\parallel}^2 = p_2^- M x_B + m^2$$

$$W(\vec{r}_\perp, \vec{R}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\,\frac{1 - e^{i\vec{r}_\perp\cdot\vec{k}_\perp}}{k_\perp^2}\,e^{i\vec{R}_\perp\cdot\vec{k}_\perp} = \frac{1}{2\pi}\ln\frac{|\vec{R}_\perp + \vec{r}_\perp|}{R_\perp}$$

$$Q^4\frac{d\sigma}{dQ^2\,dx_B} = \frac{\alpha}{16\pi^2}\,\frac{1-y}{y^2}\,\frac{1}{2MW}\int \frac{dp_2^-}{p_2^-}\,d^2r_\perp\,d^2R_\perp\,|\mathcal{M}|^2$$

Effect of Rescattering on the DIS Cross Section



$+ \cdots$

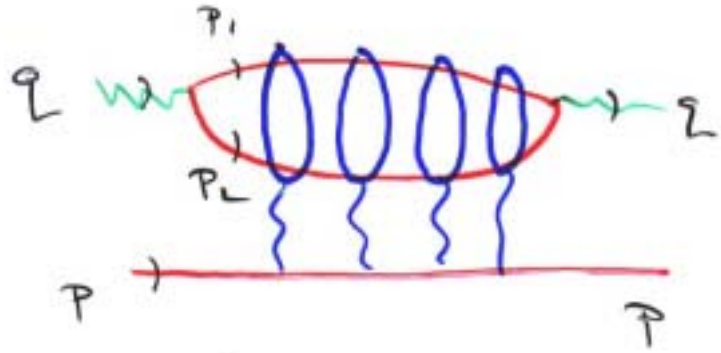
p_1: aligned jet
p_2: spectator jet

$$Q^4 \frac{d\sigma}{dQ^2\, dx_B} = \frac{\alpha}{16\pi^2} \frac{1-y}{y^2} \frac{1}{2M\nu} \int \frac{dp_2^-}{p_2^-}\, d^2r_\perp\, d^2R_\perp\, |M|^2$$

$$|M| = \left| \frac{\sin\left[g^2 W(\vec{r}_\perp, \vec{R}_\perp)/2\right]}{g^2 W(\vec{r}_\perp, \vec{R}_\perp)/2}\, M_{Born}(p_2^-, \vec{r}_\perp, \vec{R}_\perp) \right|$$

< 1 for all $\vec{r}_\perp, \vec{R}_\perp$

Equiv. to sum of cuts of forward virt. Compt. ampl.



Find shadowing only arises from diagrams involving attachments to p_1 in F.G.!

cuts give Glauber-Gribov shadowing

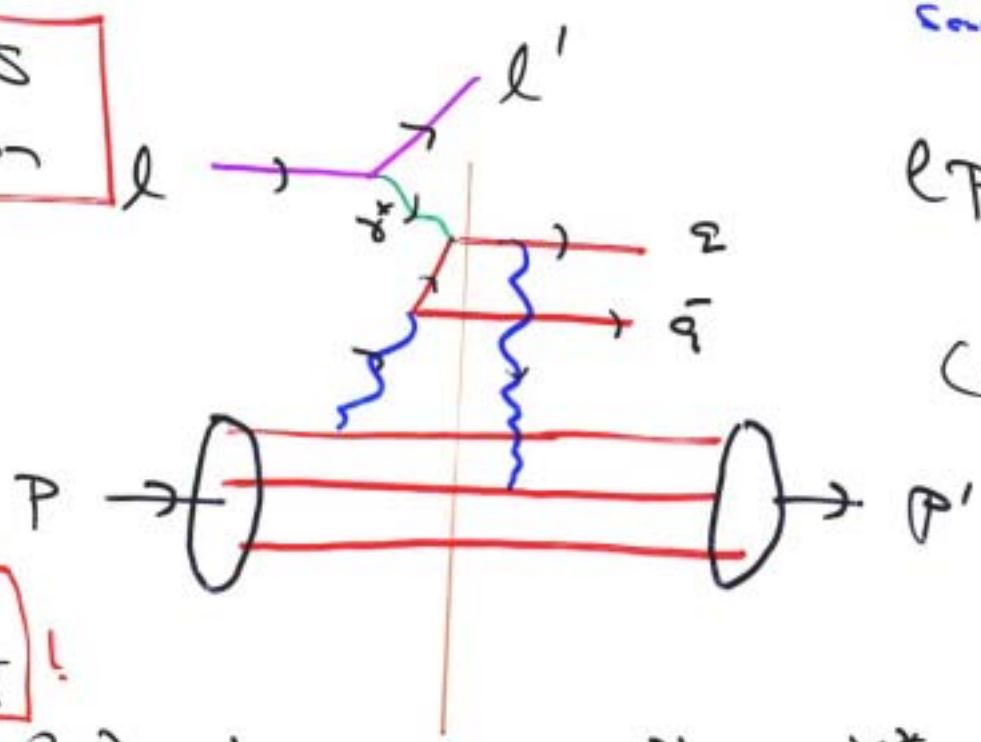
Same result in Feynman, l.c.g. (ML)

from $\frac{2\vec{p}_{\perp 1} - \vec{k}_\perp}{k^+}$ term

Non-universal Pomeron Coupling

Hoyer, Peigne, Marchal, Sannino, SJB

DIS Diffraction



$lp \to l'p'X$

(rap gap)

β-dep like $g \to q\bar{q}$!

- 2nd gluon occurs after γ^* interaction exchange

$\Delta r = O(\frac{1}{Q})$

- Cut gives imaginary phase !

- Coupling depends on color dipole moment of $q\bar{q}$

Hebecker, Quack, SJB Kopeliovich

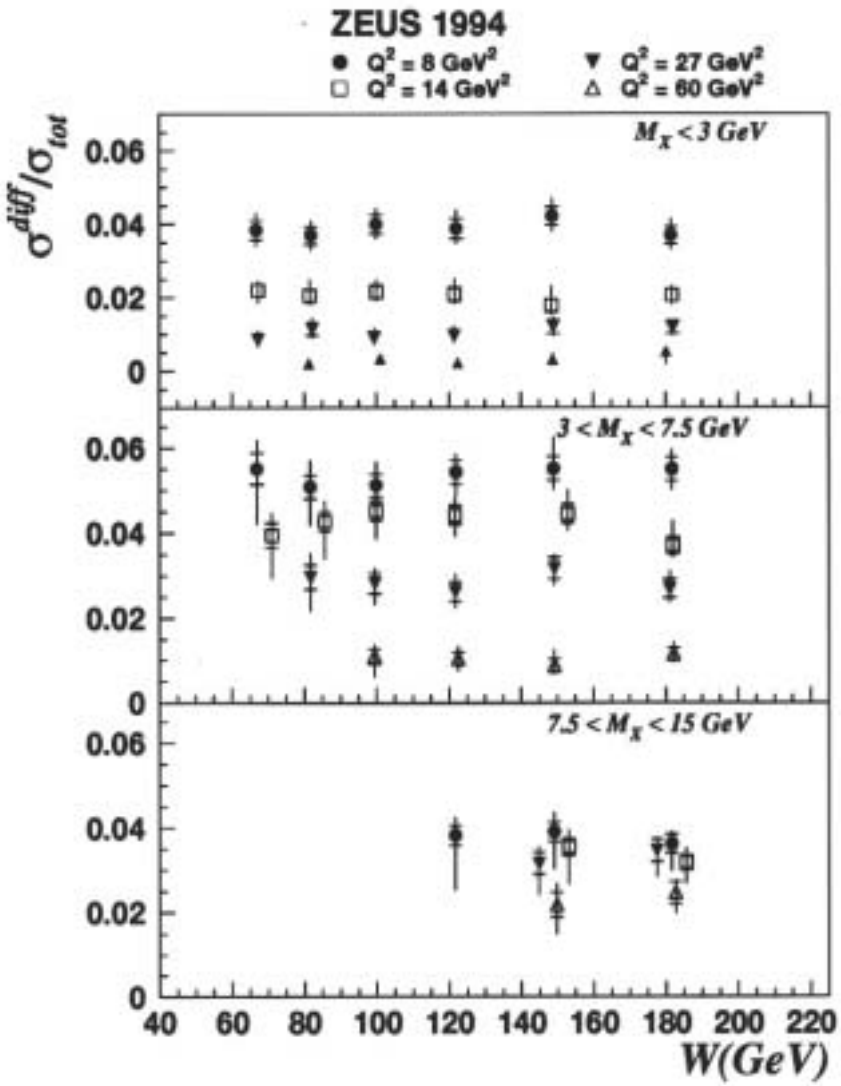
∴ Pomeron not part of proton w.f.

Not universal !

No !

$P_{\gamma/p}(x)$, $P_{q/p}(x)$, $P_{g/p}(x)$, $\cancel{P_{pom/p}(x)}$

Comparison of diffractive and DIS cross sections



Diffractive cross section has same energy dependence as inclusive cross section !

pQCD

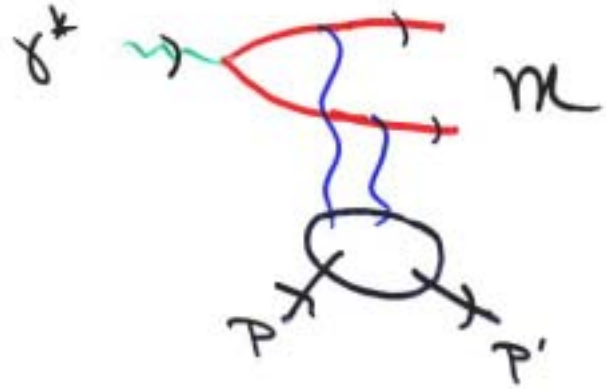
$$\frac{\sigma^{diff}}{\sigma^{tot}} \sim \frac{(xg)^2}{(xg)} \sim (W^2)^{\lambda_{eff}}$$

optical theorem $\left.\frac{d\sigma^{diff}}{dt}\right|_{t=0} \sim \sigma^{diff} \sim (\sigma^{tot})^2 \sim (W^2)^{\lambda_{eff}}$

Regge $\sigma^{diff} \simeq (W^2)^{0.08}$

Diffractive Dissociation (large rapidity gaps)

is leading twist in QCD



$$\frac{d\sigma}{dm^2}(\gamma^* p \to \mathcal{M} p')$$

Review by Hebecker

Kopeliovich, Nicolaev

$$\frac{d\sigma_T}{dm^2} \sim \left(\frac{1}{m^2+Q^2}\right)^2, \quad \sigma_T \sim \frac{1}{Q^2}$$

aligned jet regime Bj + Kogut

large color dipole Hoyer, Magnea, SJB

- Coherence for $L_{Iorpe} = \frac{2\nu}{Q^2} > R_N$
- Shadowing in nuclei for

$$L_{Iorpe} = \frac{2\nu}{Q^2} > R_A$$