The Small-x Message from HERA

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- Introduction
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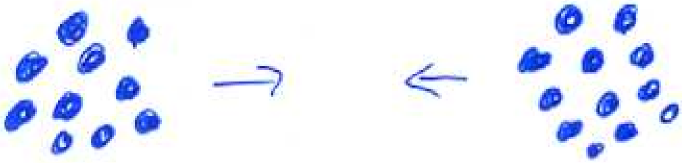
Introduction

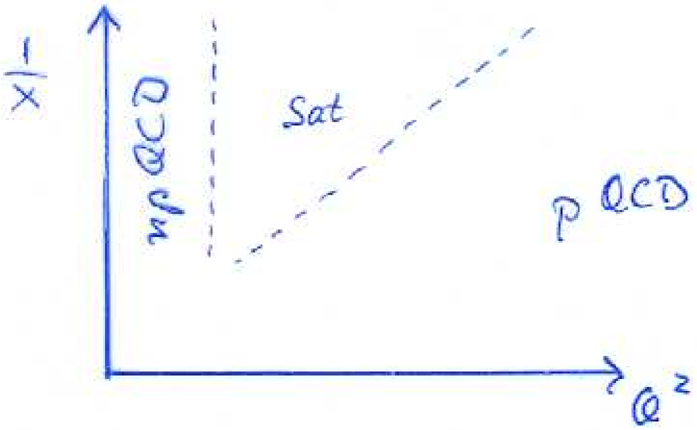
Analysis of heavy ion collisions: interest in parton (gluon) content of nuclei.



DIS at HERA: provides information on partons inside the proton,
as seen by the 'small-size' photon:

By varying Q^2 (size of the $q\bar{q}$ pair): continous transition.

What defines small-x physics:



- new kinematic limit: large Q^2 and large $1/x$
- pQCD offers DGLAP and BFKL (CCFM)
- close to nonperturbative region:
  Regge limit in hadron-hadron scattering

This talk: two 'messages'

- Structure Functions
- Diffraction (more work to be done)

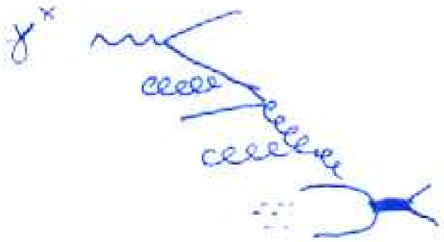
Has stimulated (ongoing) theoretical work, e.g.

- QCD dipole picture
- concept of saturation
- concept of unintegrated parton densitites
- BFKL field theory

Structure Functions

Seen: strong rise of F_2 at small x and low Q^2
(new kinematic regime).
Question: what does it mean?

(i) How far down in Q^2 and/or x can we describe F_2 by (leading twist) DGLAP evolution:
the picture of quasi-free (dilute) partons inside the proton?



Fits seems to work well even at rather low Q^2 and small x, but there are several warnings and doubts:

- small (even negative) gluon at low Q^2
- sensitivity to higher order corrections
- positive curvature of DGLAP not seen in the data
- higher twist studies indicate potential cancellation of twist four in $F_2 = F_T + F_L$, but not in F_L.
  Needs more theoretical work.

JB, Golec-Biernat, Peters
Levin et al

Need for a measurement of F_L:

- interesting per se
- potentially very useful for testing validity of leading twist DGLAP: large variations even within DGLAP
- theoretical argument from above: F_L could have large twist four correction

Illustration: a model study

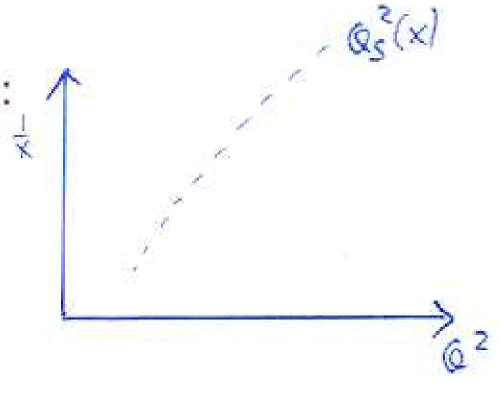
JB, Golec-Biernat, Peters

(ii) What is going on at low Q^2 and/or small x?
Saturation: gluons become dense

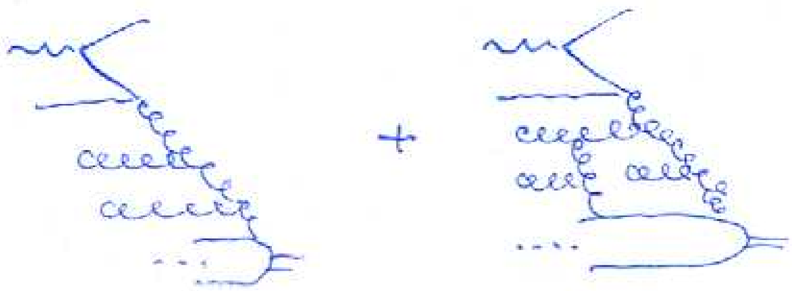
Gribov, Levin, Ryskin
Mueller, Qiu
...

- DGLAP predicts rise of $xg(x, Q^2)$ at small x (seen in data)
- must reach a value in x where gluons inside the proton overlap and start to interact: stops the growth.
- new saturation scale $Q_s^2(x)$
- strong (classical) field: new 'state' in QCD, Color Glass Condensate

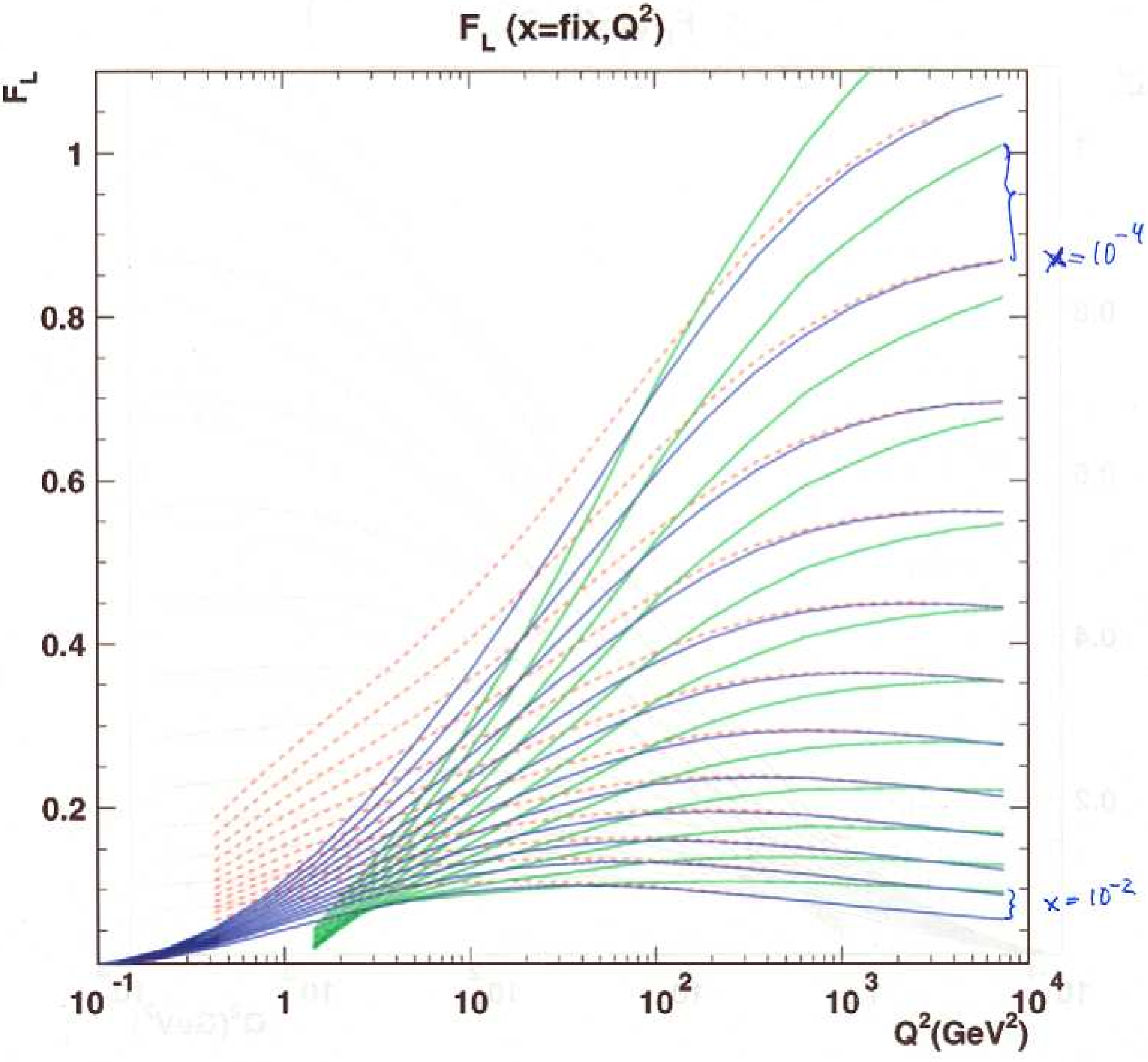
Kinematic plane:



Physical picture:



Golec Biernat



McLerran, Venugopalan

Have we seen Saturation in DIS? Positive evidence from:

Golec-Biernat, Wuesthoff
...

- success of saturation models

JB, Golec-Biernat, Kowalski
...

- scaling laws

Golec-Biernat, Kwiecinski, Stasto
...

One of the models (3 parameters):

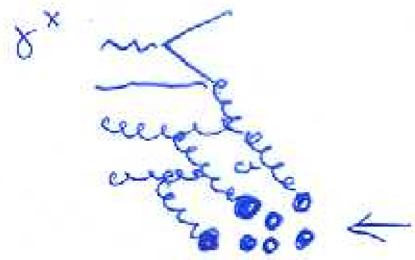
Itakura et al.

$$\sigma(x,r^2) = \sigma_0 \left(1 - e^{-\frac{\pi^2 r^2 \alpha_s(\mu^2) x g(x,\mu^2)}{3\sigma_0}} \right), \quad \mu^2 = \frac{C}{r^2} + \mu_0^2$$

Is based upon the idea of saturation. Decribes F_2 well (even at larger Q^2); is also in agreement with diffraction (see below). Quantitative estimate of saturation scale $Q_s^2(x)$:

$$1 = \frac{4\pi^2 \alpha_s(\mu^2) x g(x,\mu^2)}{3Q^2\sigma_0}, \quad \mu^2 = \frac{CQ^2}{4} + \mu_0^2$$

(iii) stronger evidence for saturation expected from DIS on nuclei:



Saturation scale should scale as $A^{1/3}$, i.e. saturation sets in at larger x already.

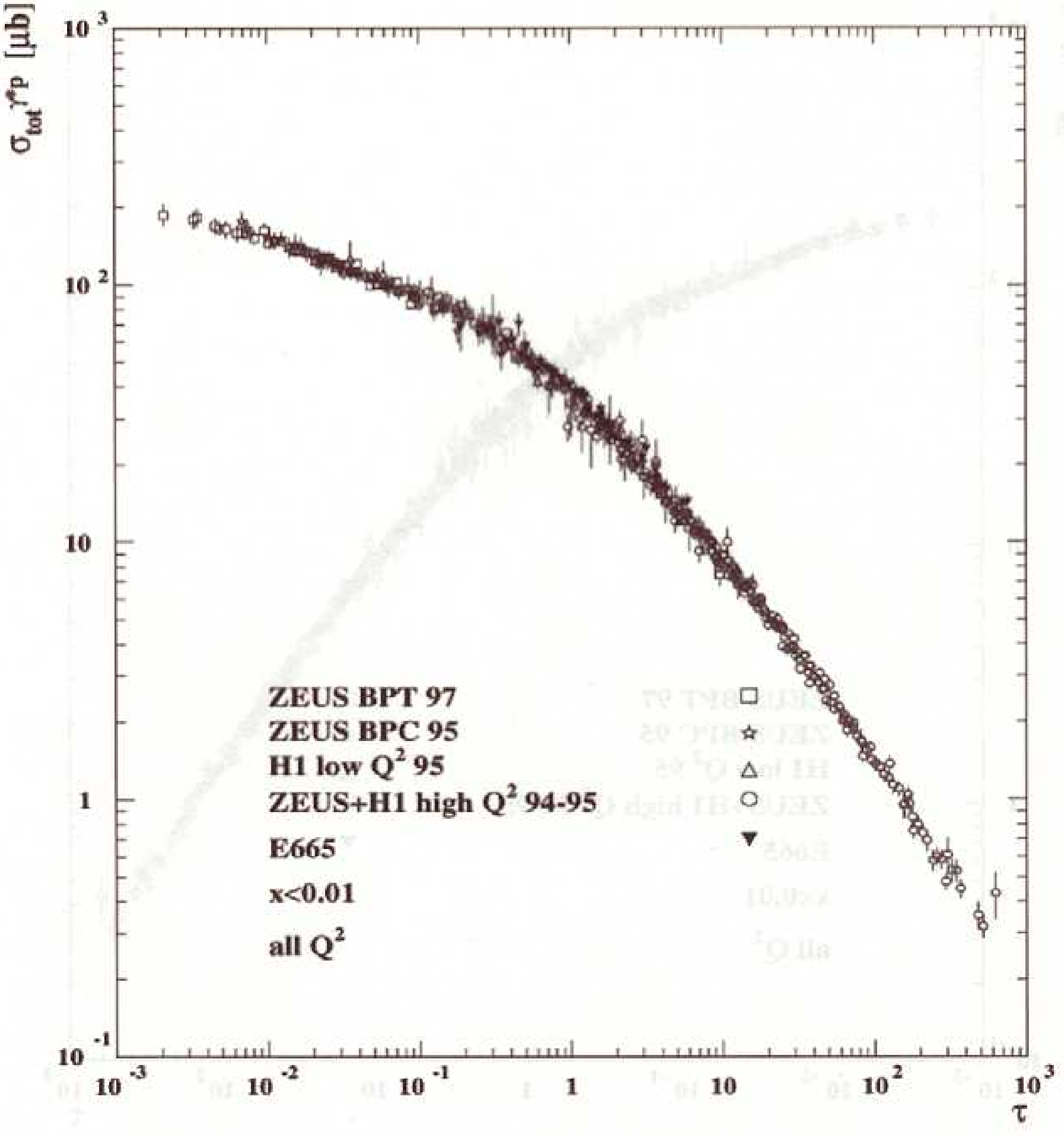


Figure 1: Experimental data on $\sigma_{\gamma^* p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

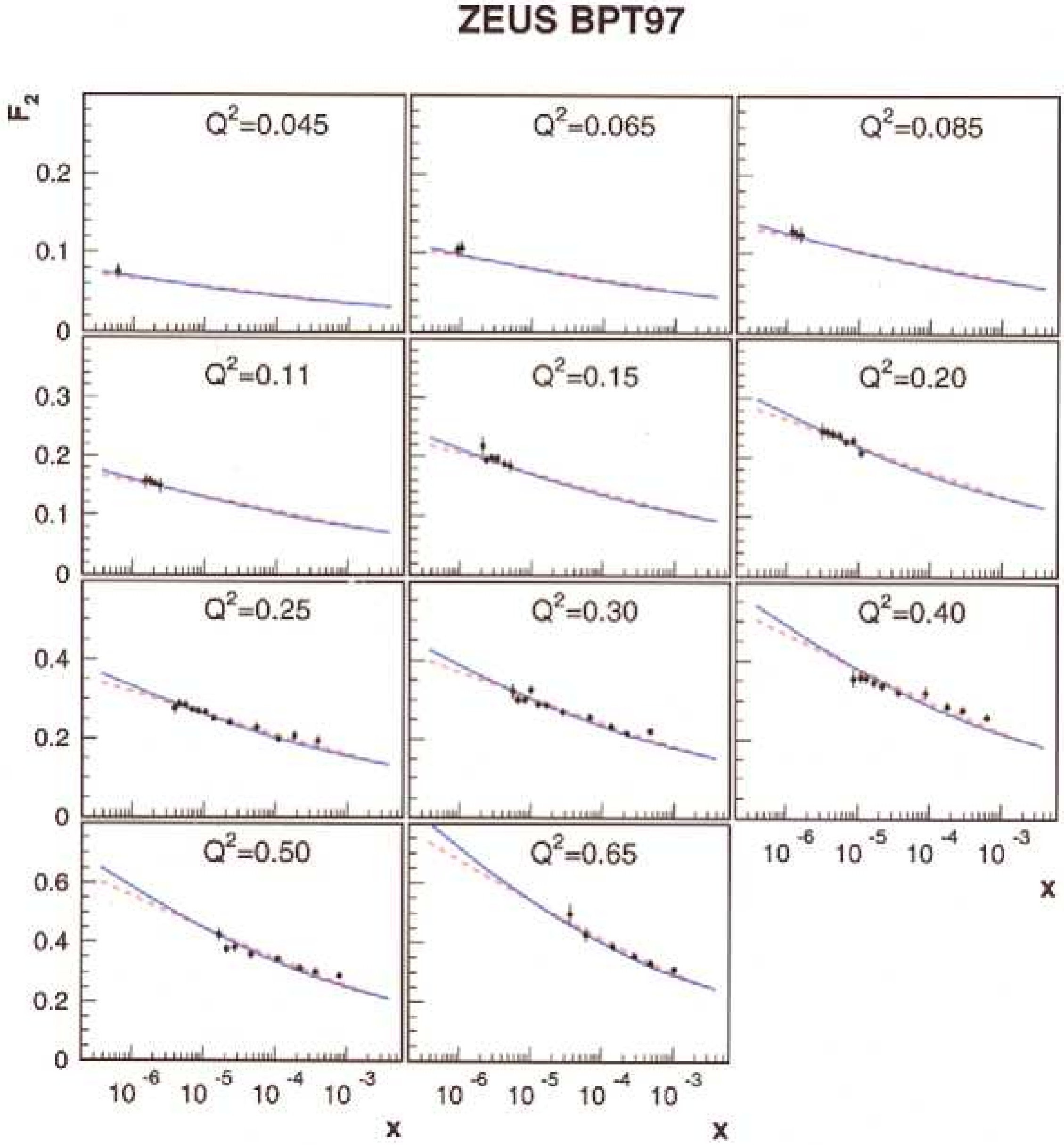


Figure 1: F_2 *as a function of* x *for fixed low* Q^2 *values. The comparison with the low* Q^2 *data from ZEUS. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted lines: the saturation model (2).*

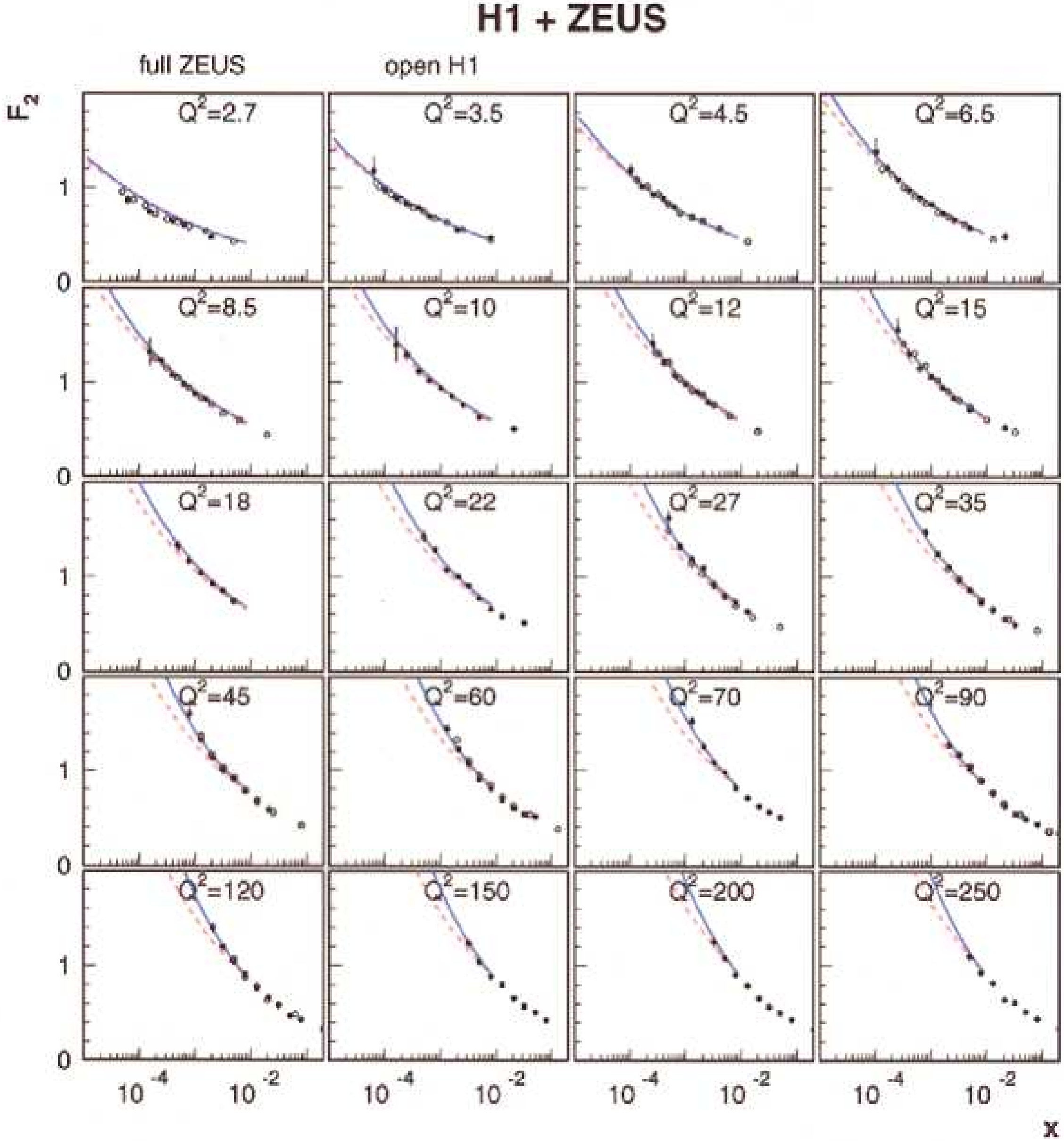


Figure 2: *H1 and ZEUS data on F_2 as a function of x for fixed values of $Q^2 > 1$ GeV^2 and the saturatiom model curves. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted lines: the saturation model (2).*

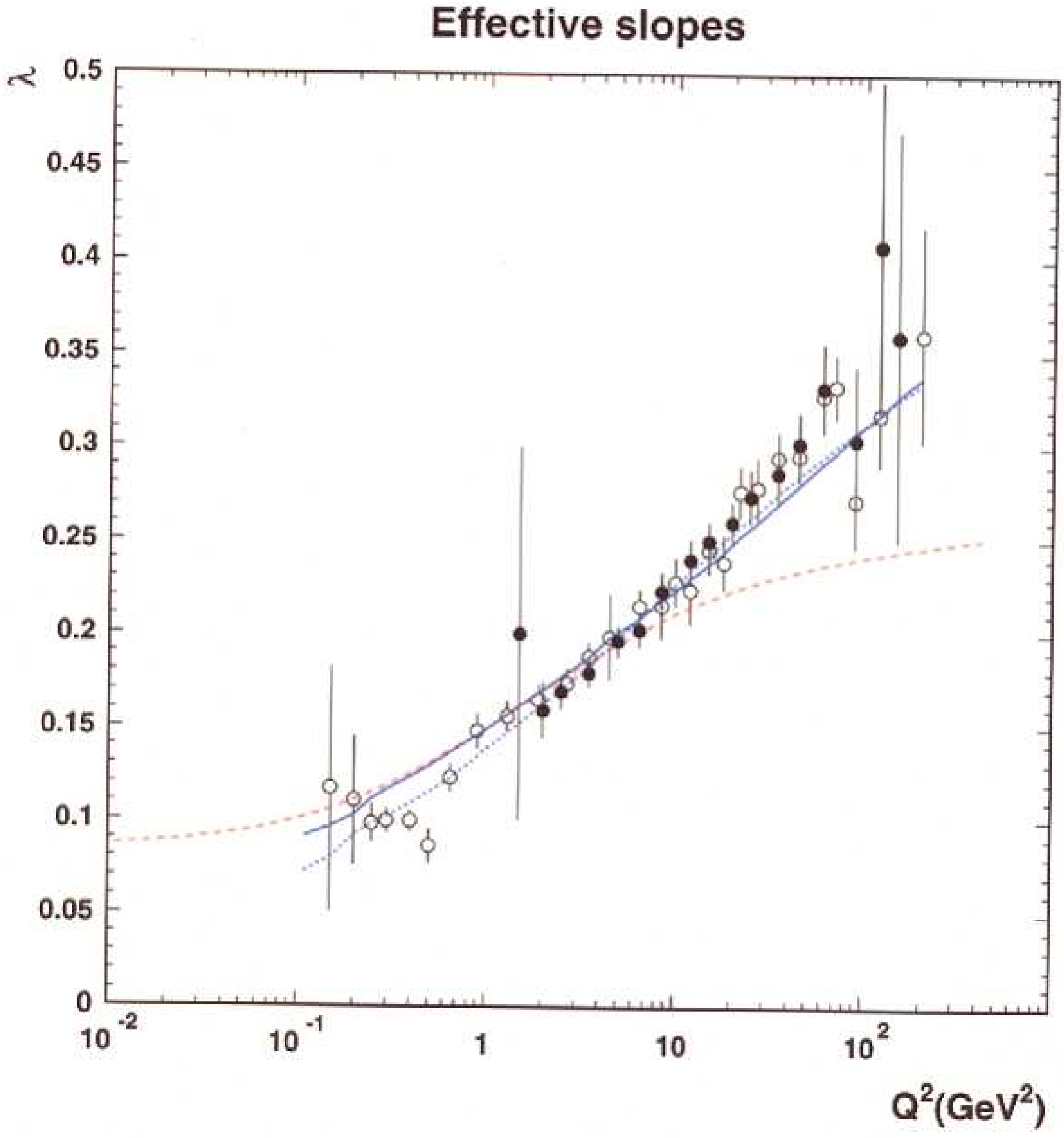
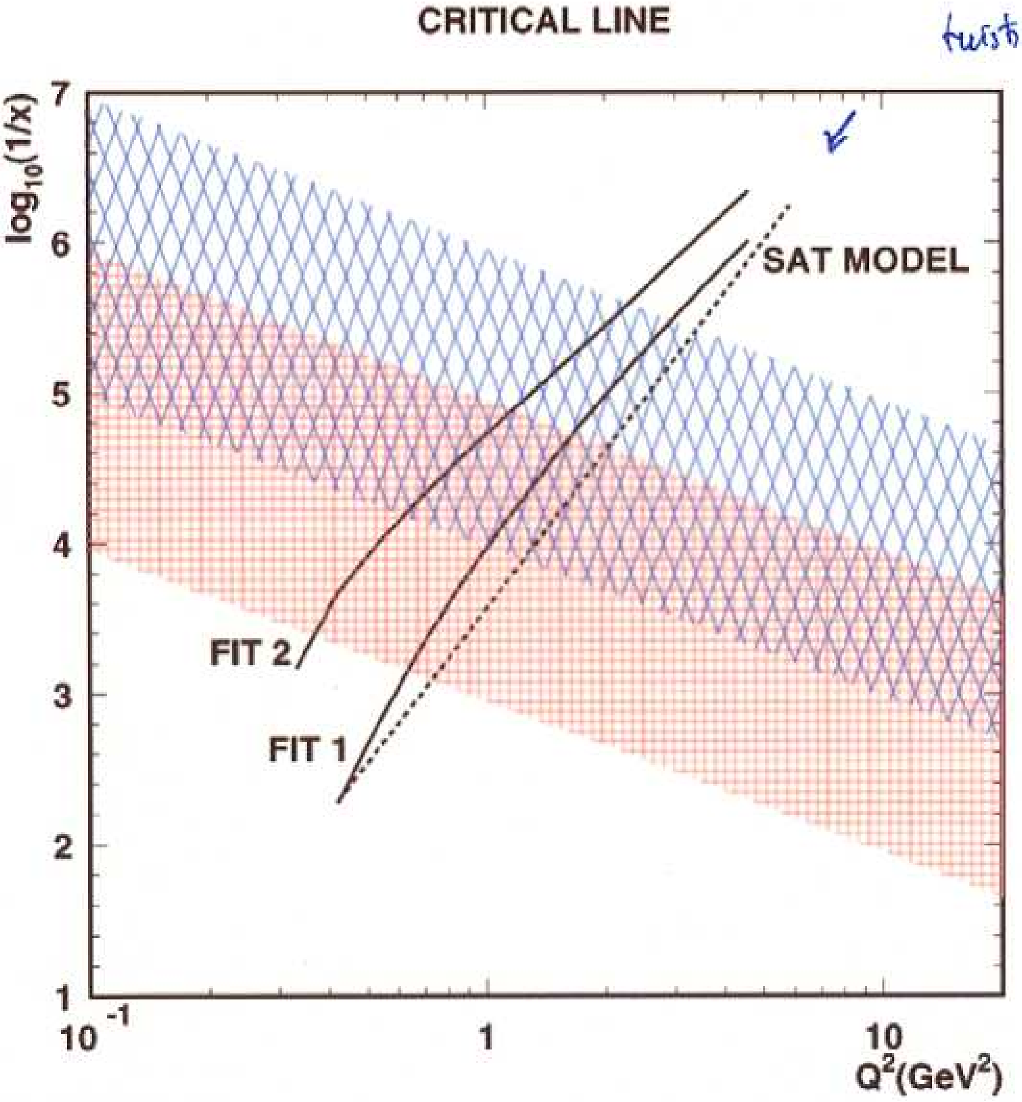


Figure 3: *The effective slope $\lambda(Q^2)$ from the parameterization $F_2 \sim x^{-\lambda(Q^2)}$ as a function of Q^2. The model with the DGLAP evolution (8): the solid line (FIT 1) and the dotted line (FIT 2). The saturation model (2): the dashed line. The open circles: ZEUS analysis and the full circles: H1 data [20].*



twists accumulate

GBW: $Q_s^2 \sim \left(\frac{1}{x}\right)^\lambda$

$\lambda = 0.3$

Figure 8: *The position of the critical line in the (x, Q^2) plane in the DGLAP improved model (solid lines) and the original saturation model (dshed line). The bands indicate acceptance regions for the colliders HERA (lower) and future THERA (upper).*

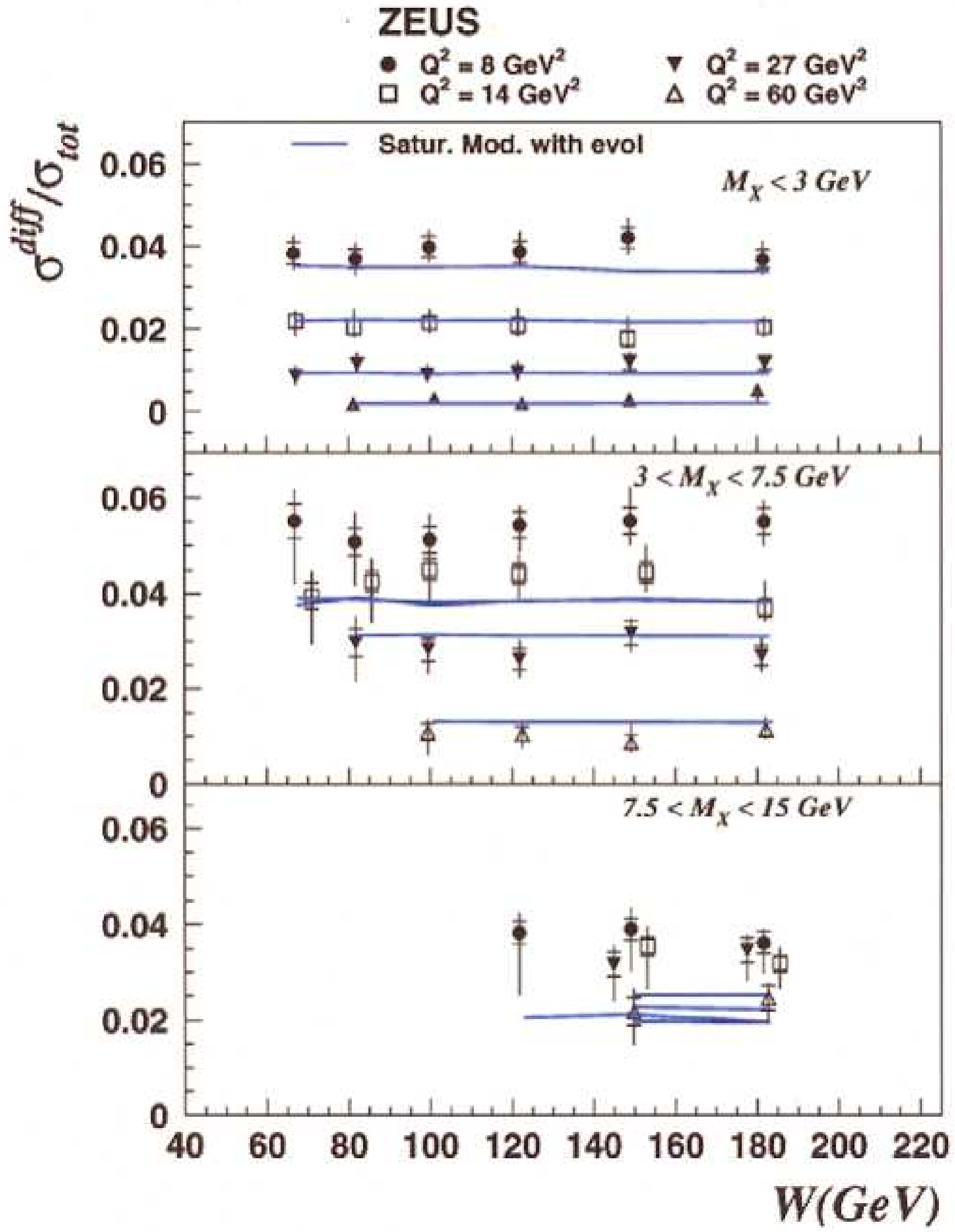


Figure 9: *The ratio of $\sigma_{diff}/\sigma_{tot}$ versus the $\gamma^* p$ energy W. The data is from ZEUS and the solid lines correspond to the results of the DGLAP improved model with massless quarks (FIT 2).*

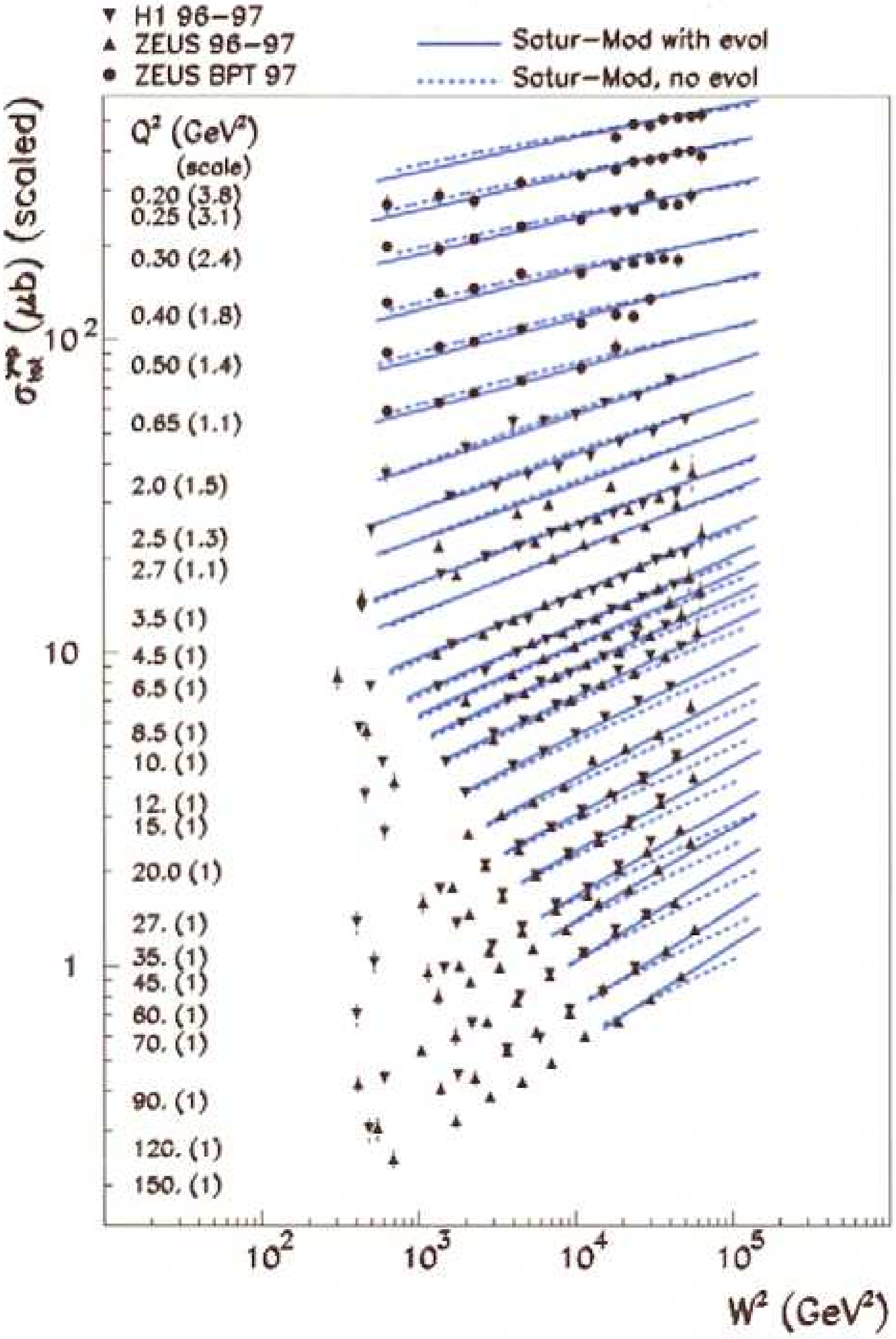


Figure 4: *The $\gamma^* p$ cross section as a function of energy W^2 at various Q^2. The solid lines: the model with the DGLAP evolution (8) (FIT 1) and the dotted line: the saturation model (2), shown for $x < 0.01$.*

Further insight:
look into final states, e.g. collinear vs.k_t factorization:

- photo-, electro production of D^*
- photo-, electro production of J/ψ
- Di-jets
- forward jets, forward pions
- ...

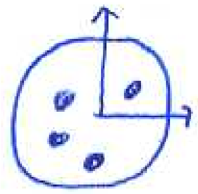
DGLAP

Many theoretical issues:

B. Anderson et al.

- DGLAP vs.BFKL
- k_t factorization: NLO?
- unintegrated parton densities; NLO?
- dipole picture: NLO?

Diffraction



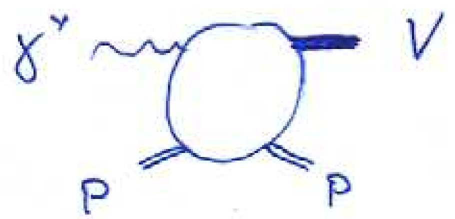
An additional source of information comes from diffraction: distribution in the transverse plane.

So far: all information on partons inside the proton integrated over all positions in transverse plane:

$$\sigma_{tot}^{\gamma^* p}(W^2) = \frac{4\pi^2\alpha}{Q^2} F_2(x, W^2) = \frac{1}{W^2}\, Im\; T_{el}^{\gamma^* p}(W^2, t=0)$$

$$T_{el}^{\gamma^* p}(W^2, t) = is \int d^2 b e^{i\vec{q}\vec{b}} T(s,b), \;\; t = -\vec{q}^{\,2}$$

Closest to $T_{el}^{\gamma^* p}(W^2 0, t)$:
DIS diffractive scattering, e.g. $T^{\gamma^* p \to Vp}(W^2, t)$



Measurement of t-dependence investigates b-dependence.
What can be deduced from measurements:

- interaction radii (t-slopes)
- profile function
- nonforward parton densities

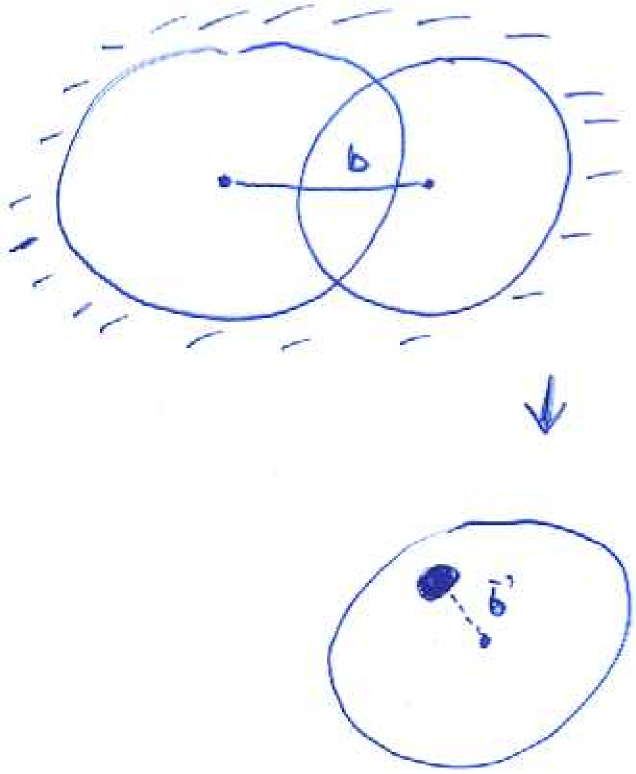
Measurements of t-slopes:

$$\frac{d\sigma}{dt} \sim e^{-B(W^2)\,|t|}$$

$$B(W^2) \sim < b^2 > \sim R_{int}^2$$

- Q^2-dependence in ρ production:
  radius shrinks with size of projectile
- energy dependence in J/ψ-production:
  less shrinkage (diffusion) than in hadron-hadron scattering

Qualitative picture:



$R^2 = 2R_0^2 + 2\alpha' \ln s$

J/ψ in between

$R^2 = R_0^2 = \text{const}$

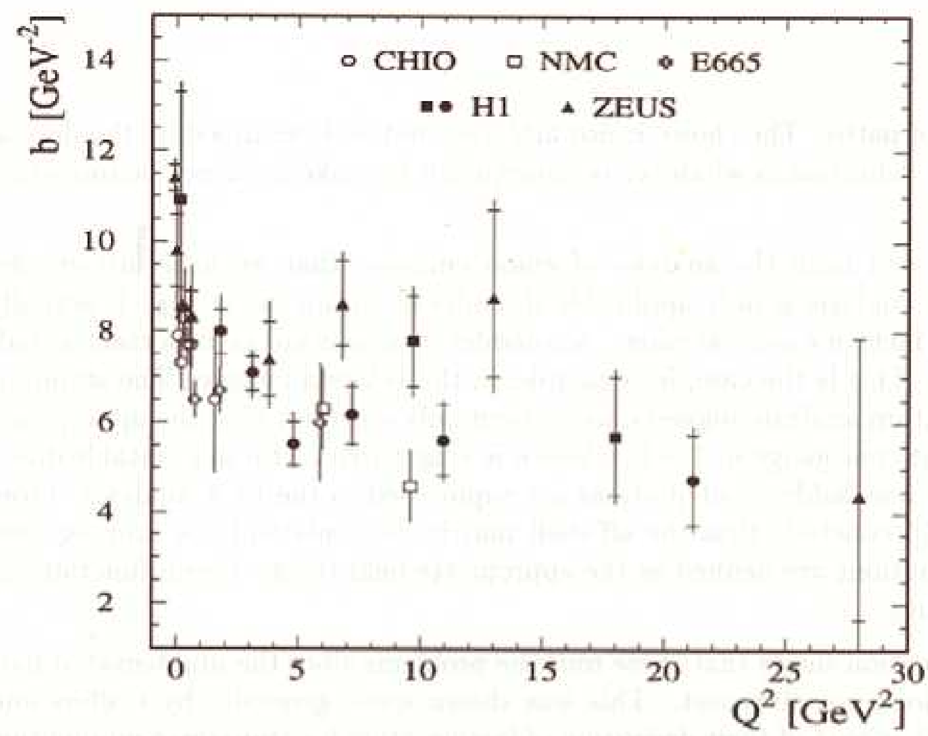


Figure 19: Measurement of the slope parameter b of the exponential t dependence for elastic ρ production. For the present measurements (full circles), the inner error bars are statistical and the full error bars include the systematic errors added in quadrature. The other measurements are from H1 [26] and ZEUS [28, 39] in photoproduction, and from CHIO [3], NMC [4], E665 [5], H1 [1] and ZEUS [2] in electroproduction. It should be noted that the definition of the parameter b is not unique (see text).

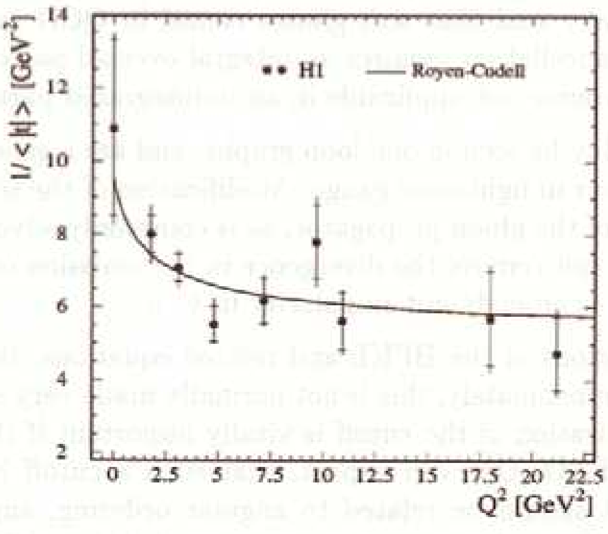
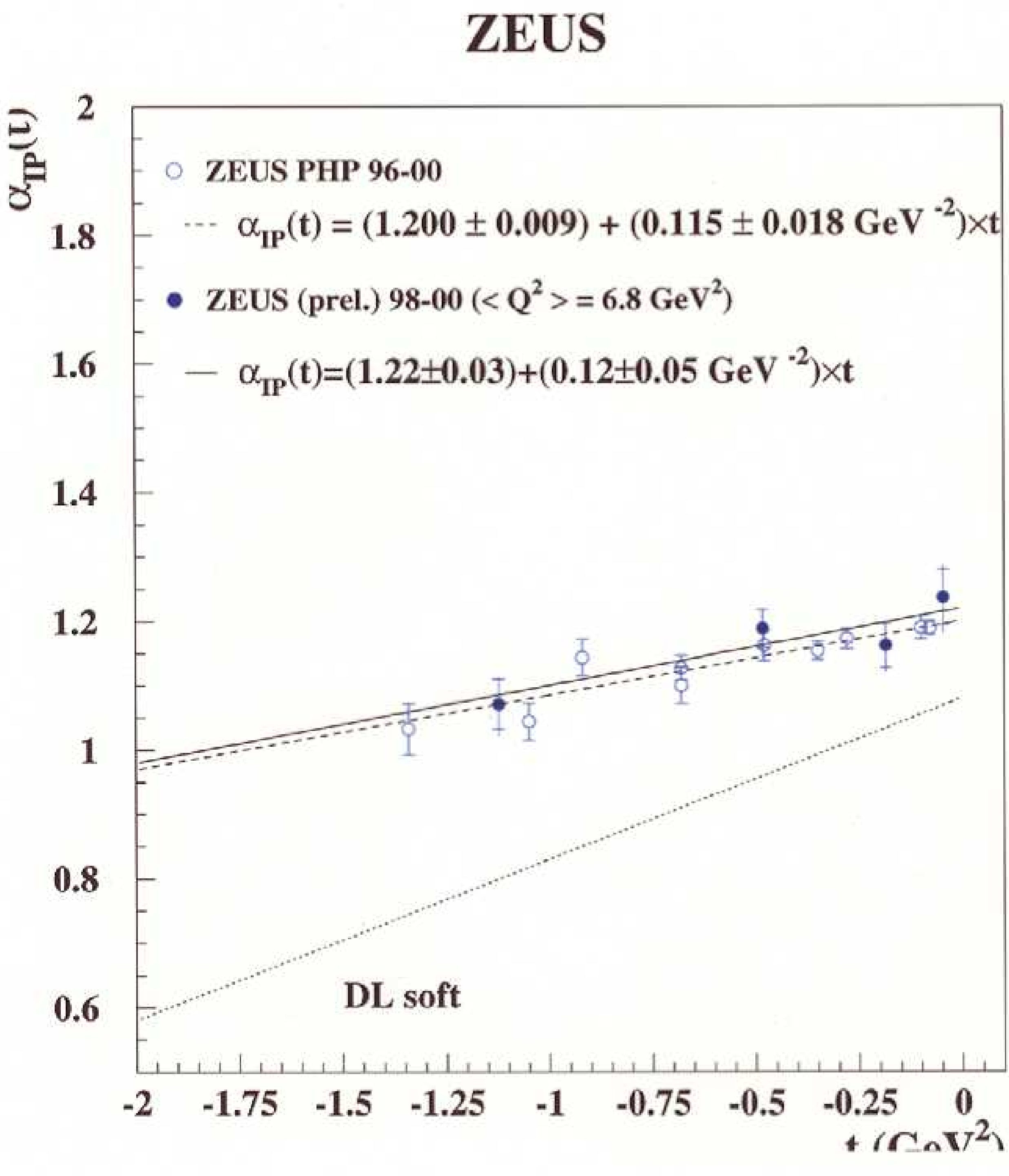


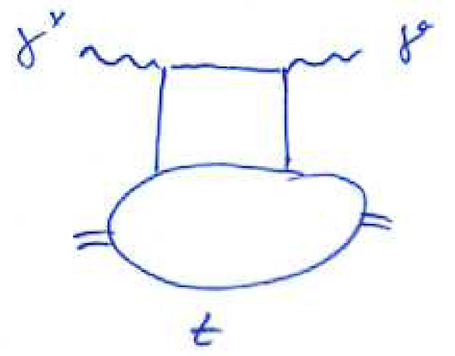
Figure 20: Q^2 dependence of the slope parameter b for ρ elastic production by H1 (these and previous measurements [1, 26]), compared to the predictions of the model of Royen and Cudell [35] for the HERA energy range, presented in the form of the variable $1/\langle|t|\rangle$.


ZEUS
$\alpha_{IP}(t)$
ZEUS PHP 96-00
$\alpha_{IP}(t) = (1.200 \pm 0.009) + (0.115 \pm 0.018\ GeV^{-2})\times t$
ZEUS (prel.) 98-00 ($< Q^2 > = 6.8\ GeV^2$)
$\alpha_{IP}(t)=(1.22\pm0.03)+(0.12\pm0.05\ GeV^{-2})\times t$
DL soft
2
1.8
1.6
1.4
1.2
1
0.8
0.6
-2
-1.75
-1.5
-1.25
-1
-0.75
-0.5
-0.25
0

What could be done: generalized parton densities (GPD): Burkhardt

$$g(x,Q^2) \to g(x,t,Q^2) \to g(x,b,Q^2)$$

A place to look: deeply virtual Compton scattering (DVCS)



Other methods: differential cross sections of diffractive vector production.
Fourier transform of $\frac{d\sigma}{dt}$ in diffractive ρ production: Mueller, Stasto, Munier

$$\int \frac{d^2q}{(2\pi)^2}\sqrt{\frac{d\sigma}{dt}}e^{-i\vec{q}\vec{b}},\ \ t=-\vec{q}^{\,2}$$

Fourier transform of cross section for J/ψ production: → Strikman's talk
radius smaller than proton radius. (for $g(x,b,Q^2)$)

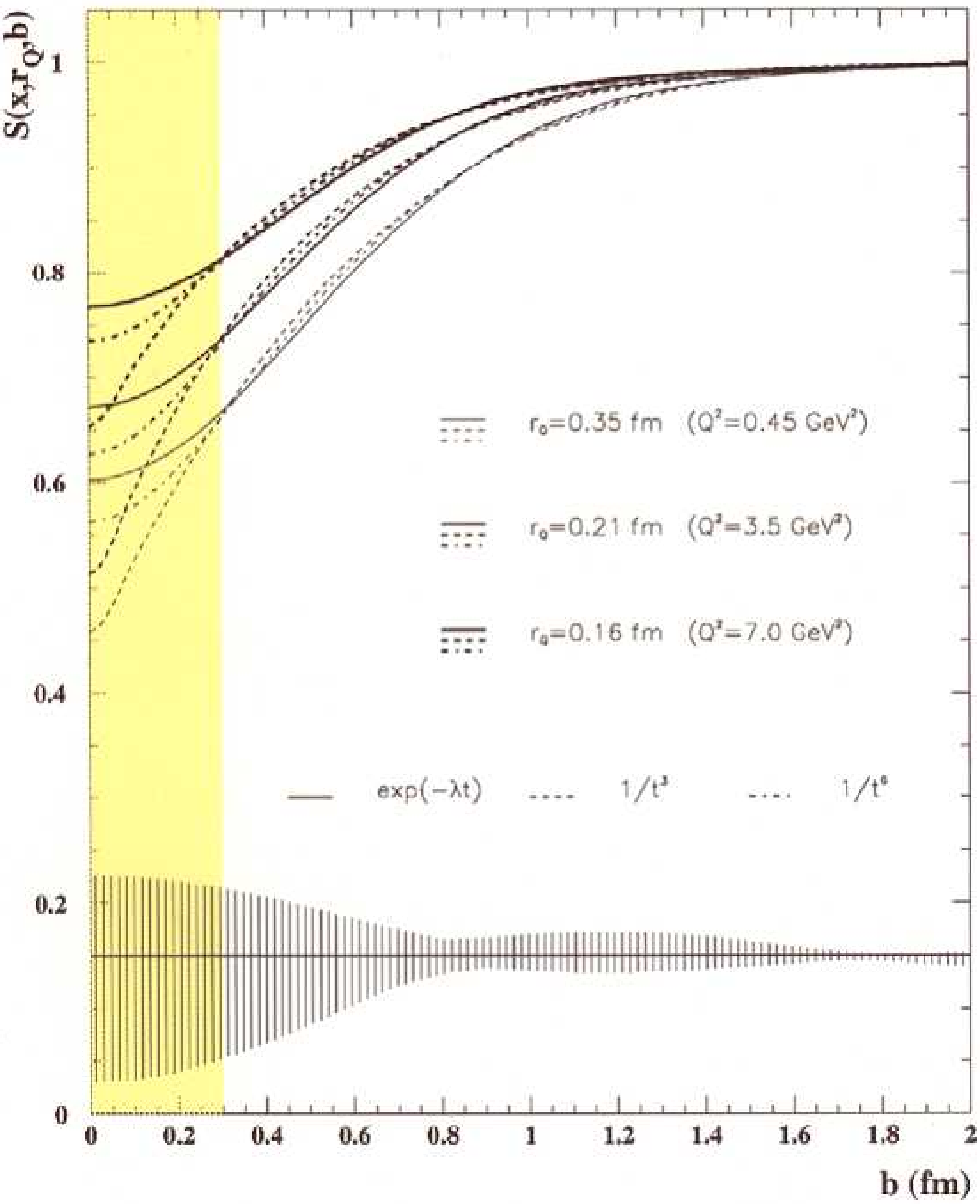


Figure 6: S-matrix for dipole-proton scattering as a function of impact parameter b. Three different Q^2 are considered, corresponding to three typical values for the size of the interacting dipole (which are estimated according to $r_Q \equiv e^{<\log r>_P}$). For each value of Q^2, the curves corresponding to three extrapolations of the data for $t > 0.6\ \mathrm{GeV}^2$ are shown. The shaded band indicates the region of impact parameter b where the choice of this extrapolation is crucial, and thus where our extraction is not reliable. The hashed band on the bottom is an estimate of the errors due to the experimental uncertainties on $d\sigma/dt|_{t=0}$ and on $B(t)$ (for $t < 0.6\ \mathrm{GeV}^2$).

Theory: BFKL

What has HERA contributed to 'proof of existence' of BFKL:

- (i) BFKL in F_2 (?)
- (ii) BFKL in forward jets — Mueller ...
- (iii) BFKL in diffractive vector production at large t — Ryskin, Forshaw ...

Ad (ii): Data show rise of the jet cross section (weaker than LO BFKL).
So far: LO calculation only
Status of NLO corrections:

- jet vertex: NLO calculations finished — JB, Colferai, Vacca
  novel type of pQCD calculation:
  interface between collinear and BFKL approximation
  (subtraction of collinear poles and central region)
  Wait for numerical analysis: Mueller-Navelet jets at Tevatron and at LHC.
- photon impact factor: virtual and real corrections — JB, Gieseke, Kyrieleis, Qiao
  analytic part complete; numerics of real corrections
  need to simplify virtual corrections — Fadin, Ivanov, Kotsky

Ad (iii): successfull agreement of LO BFKL with data.

The dependence of the NLO γ^* impact factor on the energy scale s_0 is of particular interest and shall be addressed now. The impact factor can be written as $\Phi_{\gamma^*} = g^2\Phi^{(0)}_{\gamma^*} + g^4\Phi^{(1)}_{\gamma^*}$, where $g^2 = 4\pi\alpha_s$. Since we only know the real corrections in the moment, we define:

$$\Phi'_{\gamma^*} = g^2\Phi^{(0)}_{\gamma^*} + g^4\Phi^{\rm real}_{\gamma^*}.$$

We set $e^2e_f^2 = 1$. As photon virtuality we choose $Q^2 = 15$ GeV as a typical value in $\gamma^*\gamma^*$ scattering. This choice only effects the strong coupling: $\alpha_s(Q^2) = 0.18$ or $g^2 = 1.5$. Fig.4 compares Φ'_{γ^*} to the LO impact factor $g^2\Phi^{(0)}_{\gamma^*}$ as function of $\hat{r}$ at different values of $\hat{s}_0$.

JB, Kyrieleis

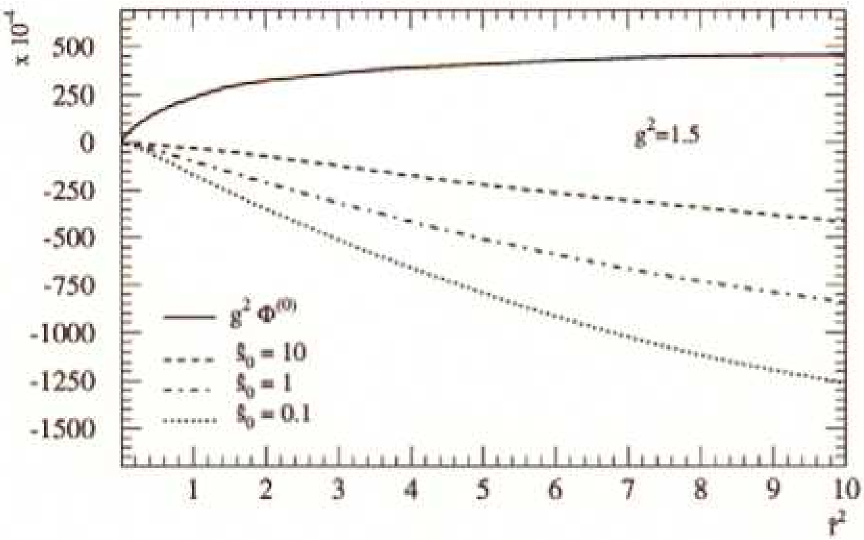
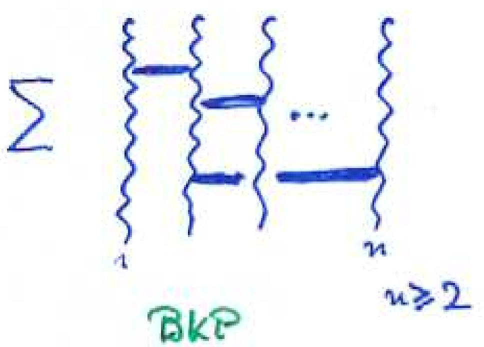


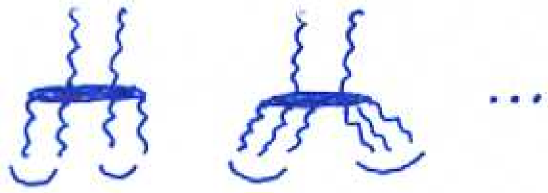
FIG. 4. Φ'_{γ^*} at different different values of $\hat{s}_0$

The ratio of Φ'_{γ^*} and $g^2\Phi^{(0)}_{\gamma^*}$ is shown in fig.5. The real corrections are negative and rather large. However, this is of not so much significance, because we have considered only part of the NLO corrections. More important, Φ'_{γ^*} decreases towards smaller values of $\hat{s}_0$. Since we included all $\hat{s}_0$ dependent terms in Φ'_{γ^*}, this implies that the γ^* impact factor gets smaller with decreasing $\hat{s}_0$. This behaviour has a simple explanation and supports the reliablilty of our calculation. Starting from the n-gluon production amplitude in the LLA, the emissions of an additional gluon is counted as NLO corrections to either the impact factor or the BFKL kernel. The parameter s_0 is introduced to separates the two contributions; the cross section in NLO BFKL is independent of s_0. The contribution from the BFKL Green function to the cross section is proportional to $(s/s_0)^\omega$ and thus increases towards smaller values of s_0. The impact factor therefore has to get smaller with decreasing s_0 to provide an s_0 independent cross section.

BFKL Field Theory:

BFKL Pomeron is the simplest piece of a 2 + 1 dimensional field theory
with the reggeized gluon playing the role of the 'elementary field':
BFKL Pomeron can split into two BFKL Pomerons





Triple Pomeron vertex in pQCD, defines the nonlinear term in the BK-equation.

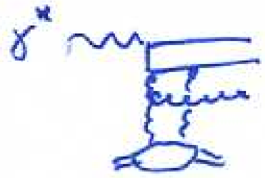
Very important feature: conformal symmetry (= invariance under Moebius transformations) in transverse coordinates

$$r \to \frac{a\,r\,+\,b}{c\,r\,+\,d},\ \bar{r} \to \frac{\bar{a}\,\bar{r}\,+\,\bar{b}}{\bar{c}\,\bar{r}\,+\,\bar{d}}$$

$$(r = r_1 + ir_2,\quad \bar{r} = r_1 - ir_2)$$

Very restrictive; integrability of BKP equation (at large N_c).

Vertex has been tested in diffractive jet production.

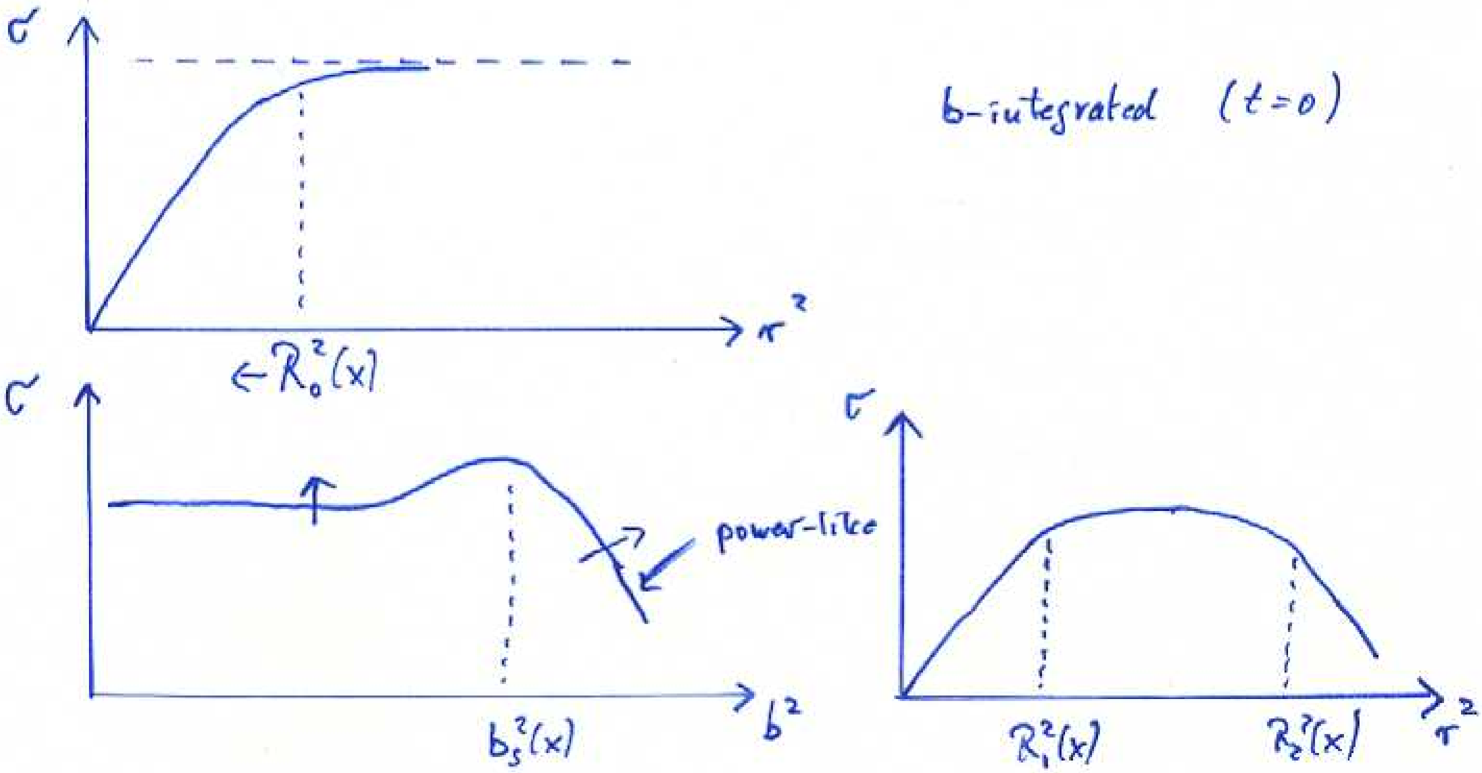


What is needed: saturation model for b-dependent dipole cross section.
Dipole formalism:

$$\sigma_{tot}^{\gamma^* p}(W^2) = \int d^2r \int dz \psi(r,Q,z)^* \sigma(r,x) \psi(r,Q,z)$$

$$T^{\gamma^* p \to Vp}(W^2, t) = iW^2 \int d^2r \int dz \psi_V(r,Q,z)^* e^{i\vec{q}\vec{r}z} \sigma(r,x,t) \psi(r,z)$$

Qualitative picture of dipole cross sections:



Attention: large-r, large-b region.

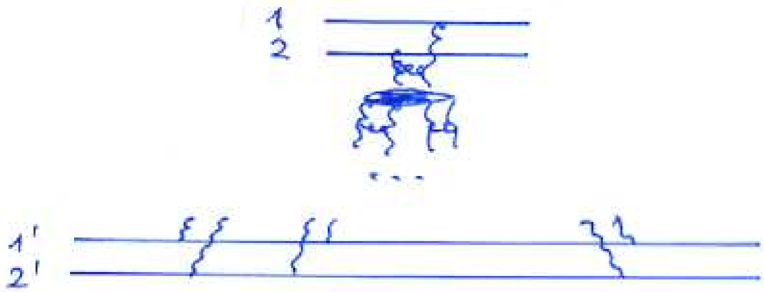
Teaney, Kowalski

A model:

$$\frac{d\sigma}{d^2b} = 2\left(1 - e^{-\frac{\pi^2 r^2 \alpha_s x g(x,\mu^2)}{2N_c} T(b)}\right)$$

Desirable: more comparison with data.

How to 'derive saturation': Balitsky-Kovchegov equation. JB, Ryskin, Vacca
How to apply to γ^* scattering on a single nucleon:
assume a single 'parent' dipole



$b = \frac{1}{2}(\rho_1+\rho_2) - \frac{1}{2}(\rho_1'+\rho_2')$

Conformal ansatz:

$\phi_{BFKL} = e^{Y\chi_{BFKL}}$

$$f(r_1, r_2, r_1', r_2', y) = \sum_n \int d\nu w(n,\nu) \int d^2 r_0 \cdot E_{n,\nu}(r_{10}, r_{20})^* E_{n,\nu}(r_{1'0}, r_{2'0}) \phi(n,\nu;y)$$

$$= \sum_n \int C(z)\phi(n,\nu;y), \quad z = \frac{r_{12} r_{1'2'}}{r_{11'} r_{22'}}$$

Nonlinear equation ($n=0$, $\gamma = 1/2 + i\nu$):

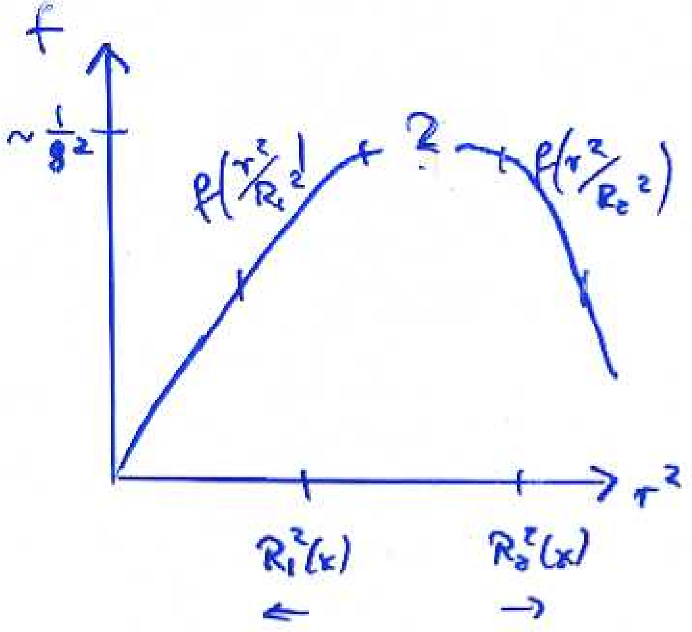
$$\frac{d\phi(\nu)}{dy} = \chi_{BFKL}(\nu)\phi(\nu,y) - \int d\nu_1 \int d\nu_2 V(\nu,\nu_1,\nu_2)\phi(\nu_1,y)\phi(\nu_2,y)$$

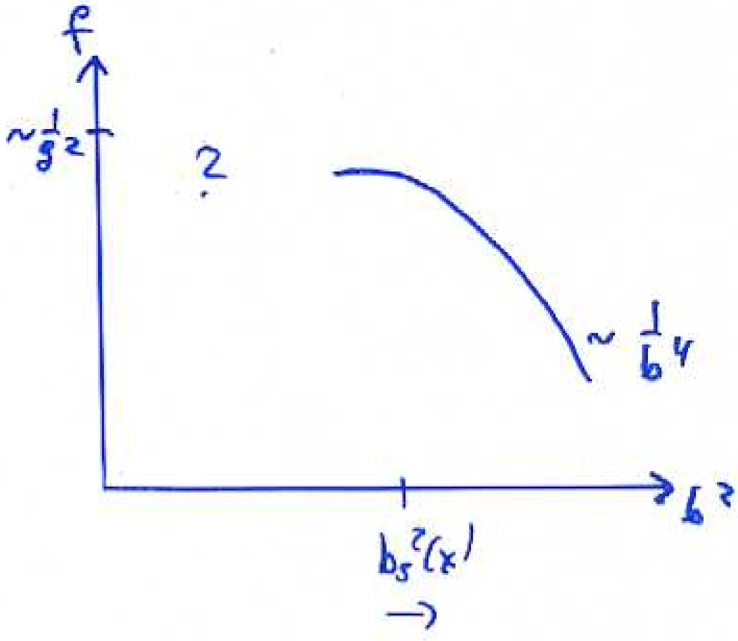
Quasifree-behavior (scaling) encoded in behavior near $i\nu + 1/2 = 0$:
- fixed b: at small r and and at large r
- fixed r: large b

But: integration over b problematic.
Impression: conformal invariance not valid at large b, needs npQCD.

numerical solutions:

Braun
Golec-Biernat, Stasto
Golec-Biernat, Stasto
Salgado





all radii powerlike in $1/x$

Summary

Accomplished:

- Evidence for saturation: gluons at small x are becoming dense
- DIS Diffraction: distribution in transverse space
- Theory: from small b towards large b

Wanted:

- Measurement of F_L
- DIS on nuclei
- polarized structure functions at small x
- More theoretical work

HERA III, eRHIC